3D AND 2D MODEL TESTS AND NUMERICAL ANALYSES OF SETTLEMENTS AND EARTH PRESSURES DUE TO TUNNEL EXCAVATION

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ABSTRACT

Three-dimensional (3D) and two-dimensional (2D) model tests of tunnel excavation problems with two different depths were carried out to confirm the analytical results obtained previously. 3D and 2D elastoplastic finite element analyses corresponding to the model tests were also performed. The model test results were compared with the results obtained from numerical analyses. Through comparison supported by the model tests and numerical analyses, it was determined that a clay foundation results in a large surface settlement and a much wider surface settlement curve than a sand foundation with the same displacement. It is also shown that the settlements are different whether or not the construction procedure is taken into account (3D or 2D) even if the displacements imposed are the same at the end of excavation. In addition, the earth pressures were predicted through numerical analyses. It was concluded that the distribution of earth pressures in the sandy subsoil near the bottom of lowering basement is influenced considerably by the smoothness of basement, dilatancy characteristic and the magnitude of the prescribed displacement.

Key words: clay, constitutive equation for soils, dilatancy, earth pressure, finite element method, model test, sand, settlement, three-dimensional analysis, tunnel (IGC: E2/E5/H5)

INTRODUCTION

Most geotechnical problems have been analyzed as two-dimensional (2D) (e.g. plane strain or axisymmetric condition). This is because of the limitation of processing speed and computer memory and the lack of a suitable constitutive model for evaluating quantitatively the soil behavior under general three-dimensional (3D) stress conditions. Although 3D FEM is sometimes applied for the analysis of geotechnical problems, soil mass is assumed to be elastic for simplicity. Elastic model cannot however reflect soil dilatancy and also can not take into account the effect of loading and/or displacement histories on soil behavior.

Elastoplastic constitutive models for clay and sand, named t_{ij} -clay model and t_{ij} -sand model, have been developed that can describe 3D stress-strain behavior of clay and sand with unified soil parameters (Nakai and Matsuoka, 1986; Nakai, 1989). These models have been applied to the analyses of 2D geotechnical problems (e.g. Nakai and Matsuoka, 1987; Nakai et al., 1991; Nakai et al., 1995). More recently, three-dimensional (3D) FE

analysis of lowering basement problems have also been performed (Nakai and Xu, 1995). In this investigation, we have concluded from computational results that the positive and negative dilatancy of geomaterials, and 3D effects are very important to predict the deformation and earth pressure of a foundation.

In this study, in order to confirm the dilatancy and 3D effects, 3D and 2D model tests of the lowering basement problems were carried out. Kaolin clay powder and dry Toyoura sand were used in the tests. It is experimentally confirmed that Kaolin clay powder exhibits only negative dilatancy in the way like a normally consolidated clay, and that Toyoura sand shows positive dilatancy at low confining pressure levels in model tests. 3D and 2D finite element analyses were also carried out using a new mesh to match the experimental displacement pattern of the lowering basement. In the analyses, normally consolidated Fujinomori clay and medium dense Toyoura sand were assumed as the geomaterials with positive and negative dilatancy, respectively. Through the model tests and numerical analyses, it was concluded that the results of the numerical analyses are supported by the model tests,

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and that it is necessary to carry out 3D FE analyses using constitutive models which can reflect dilatancy characteristic of a soil mass in three-dimensional stress. In addition, the prediction of earth pressures were also made using numerical analysis. It was demonstrated that the distribution of earth pressures in the lowering basement problems is influenced by soil dilatancy, the magnitude of displacement, the boundary condition (roughness of basement) and other factors, particularly in sand.

METHODS OF MODEL TESTS

Figure 1 shows a schematic diagram of the apparatus used in the model tests, and Photograph 1(a) shows a complete view of it. As shown in the figure and the photograph, a wooden frame is set on an iron table, and ten square blocks are buried along a central straight line on the table. The frame is 800 mm in length and 160 mm/80 mm in height. The blocks (80 mm $\times$ 80 mm) are movable in the upward and downward direction individually and/or simultaneously. So, the proportion of the depth D of the model subsoil to the width B of lowering basement is $D/B=2.0$ ($B=80$ mm: width of lowering basements) when the depth of soil foundation $D=160$ mm, and $D/B=1.0$ when $D=80$ mm. The soil foundations were prepared by air pluviation through a sieve (square opening of sieves are 0.42 mm for sand and 0.84 mm for clay powder). The initial void ratios were about 0.70 (sand) and 3.88 (clay). The unit weight of the sand and clay were 15.3 kN/m³ and 5.4 kN/m³ respectively. Photograph 1(a) shows the upper part of the apparatus in which the sample is set, and Photograph 1(b) the lower part of the apparatus (device controlling displacement of movable blocks). The displacement of each block can be applied by the screw beneath the blocks, and the handle in the left-hand side can give the same displacement to all of the blocks at the same time. The excavation process for tunneling can be simulated by giving a downward displacement to the blocks one by one from one end. Displacement of $d=3.2$ mm is imposed in all of the present model tests. The settlement of the foundation surface on cross section (A to A' in Fig. 1) can be measured continuously

(a)

(b)

Photo. 1. Equipment for model tests of 3D lowering basement

with a laser type displacement transducer. The laser type displacement transducer is mounted on the rods crossing over the frame box and can be driven by an electric motor. The position of the laser type displacement transducer is measured by a supersonic wave type transducer. All the surface settlement data and positions are then recorded in a floppy disc of a personal computer.

MECHANICAL PROPERTIES OF SOILS USED IN MODEL TESTS

Since the model tests are performed at 1 g, the confining pressures of model tests are fairly low (the depth of model ground is 160 mm or 80 mm). Toyoura sand and Kaolin powder are used as the materials which are supposed to exhibit positive and negative dilatancy respectively even at very low stress level. The following triaxial tests were carried out in order to demonstrate such dilatancy behavior for both materials. Figure 2 shows the triaxial test method schematically. Very low constant effective confining pressure is applied through the water head difference h_w between burettes A and B to the dry sample surrounded by a thin rubber membrane under atmosphere pressure. During testing, h_w is kept

Fig. 1. Schematic diagram of equipment

constant by adjusting the water head of burette B. The volume change in the sample can be measured by burette A.

Figure 3 shows the observed stress-strain-dilatancy curves for Kaolin clay powder and Toyoura sand under $\sigma'_r = 2.0$ kPa. Since the axial stresses on the top and bottom of the specimen are different due to self-weight of

the sample, stress-strain curves at both ends are plotted for each figure. The sample are prepared by air pluviation in the same way as those of the model tests. Toyoura sand shows a little volume contraction at the beginning, and then exhibits considerable dilation of volume. Kaolin clay powder shows only a negative dilatancy as usually observed in the normally consolidated clay.

Fig. 2. Triaxial compression equipment under low confining pressure

Fig. 3. Observed stress-strain-dilatancy relationship in triaxial compression test

CONSTITUTIVE MODELS USED IN NUMERICAL ANALYSES

The models used for the numerical analyses are t_{ij} -clay and t_{ij} -sand models which have been fully described elsewhere (Nakai and Matsuoka, 1986; Nakai, 1989). These models are capable of describing the following soil characteristics in particular:

- 1) Influence of intermediate principal stress on the deformation and strength of soils
- 2) Stress path dependency of flow rule

Figure 4 shows the influence of intermediate principal stress of clay schematically. (a) is the relationship between stress ratio (q/p), deviatoric strain (ϵ_d) and volumetric strain (ϵ_v) in triaxial compression, triaxial extension and plane strain tests, (b) the relationship between stress ratio (q/p) and plastic strain increment ratio ($-d\epsilon_v^p/d\epsilon_d^p$) in triaxial compression and extension tests, and (c) the direction of strain increment vectors on the octahedral plane in true triaxial tests. In these figures, q , p , ϵ_v and ϵ_d are the stress and strain parameters used in a lot of models, such as the Cam clay model. Models using these stress and strain parameters hence cannot describe the differences in deformation, strength and dilatancy depending on the relative magnitude of intermediate principal stress shown in figures (a) and (b) and the deviation of the direction of strain increment vectors on the octahedral plane with the increase of stress ratio shown in figure (c). The model used in this analysis describes such influence of intermediate principal stress precisely by employing the t_{ij} concept. In the model based on t_{ij} concept, a new modified stress t_{ij} is introduced in the formulation of the yield function which is defined as

$$t_{ij} = \sigma_{ik} a_{kj} \quad (1)$$

and it is assumed that the associated flow rule is not in σ_{ij} space but in t_{ij} space.

$$d\epsilon_{ij}^{p(AF)} = \lambda \frac{\partial f}{\partial t_{ij}} \quad (2)$$

In Eq. (1), a_{ij} is a non-dimensional symmetric tensor whose principal values are given by the direction cosines of a spatially mobilized plane. Next, Figure 5 shows the stress path dependency of flow rule (influence of stress path on stress-dilatancy relation). Such difference in Fig. 5 cannot be illustrated by usual elastoplastic models with single plastic potential (yield function). By assuming not only the plastic strain increment $d\epsilon_{ij}^{p(AF)}$ which satisfied Eq. (2) but also isotropic plastic strain increment $d\epsilon_{ij}^{p(IC)}$ under the stress path in which the mean stress increases, the above stress path dependency is properly taken into

consideration in the model.

$$d\epsilon_{ij}^{p(IC)} = K \frac{\delta_{ij}}{3} \quad (3)$$

(a) stress-strain curve

(b) stress-dilatancy relationship

(c) direction of strain increment on octahedral plane

Fig. 4. Influence of intermediate principal stress in clay: (a) stress-strain curve, (b) stress-dilatancy relationship, (c) the direction of strain increment on octahedral plane

Fig. 5. Stress path dependency on stress-dilatancy relationship in clay

Table 1. Soil parameters for clay and sand

Fujinomori clay		Toyoura sand	
$\lambda/(1+e_0)$	5.08×10^{-2}	C_t	0.84×10^{-2}
$\kappa/(1+e_0)$	1.12×10^{-2}	C_e	0.60×10^{-2}
$\phi'(\text{comp.})$	33.7°	m	0.3
α	0.70	R_f	4.7
ν_e	0.0	D_f	-0.6
		α	0.85
		ν_e	0.0

Fig. 6. Computed stress-strain-dilatancy relationship in triaxial compression test

$$d\epsilon_{ij}^p = d\epsilon_{ij}^{p(AF)} + d\epsilon_{ij}^{p(IC)} \quad (4)$$

It should also be noted that although the plastic strain increment is divided into two components, only one set of yield function and hardening parameters is used so that no new parameter is added.

In addition, t_{ij} sand model can describe

3) Positive dilatancy as well as negative dilatancy during strain hardening process

For this purpose, a plastic work $W^*p = \int t_{ij} d\epsilon_{ij}^p$ based on t_{ij} is used as a hardening parameter in place of plastic volumetric strain ϵ_v^p .

The values of soil parameters for Toyoura sand and Fujinomori clay, which are assumed in the analyses of lowering basement problems, are tabulated in Table 1. Figure 6 indicates calculated stress-strain-dilatancy curves for triaxial compression tests under constant minor principal stress $\sigma'_3 = 196$ kPa for Toyoura sand and Fujinomori clay. These analytical results correspond to the observed results shown in Fig. 3.

METHOD OF ANALYSES

Since the stress level of the model tests is fairly low as described before, it is very difficult to determine the soil parameters used in the analyses precisely. Therefore, analyses are performed at a scale of almost one hundred times as large as the model tests. Figure 7 shows the finite

element meshes used in the analyses. Since the lowering basement is symmetric, half of it is analyzed. Soil mass domain is discretized using 600 rectangular elements, each of which has 8 nodal points and $2 \times 2 \times 2$ Gauss quadrature. As shown in this figure, the meshes are refined over and in the vicinity of the lowering basement so as to simulate the displacement pattern of lowering basement in the model tests. This is because the displacement pattern in the previous analysis (Nakai and Xu, 1995) in which the meshes were not refined over and near lowering basement are somewhat different from that in the model tests. In the figure, L represents the length of excavation. The proportion of the analyses is $D/B = 2.0$ ($B = B_u + B_s = 10$ m) when the depth of the foundation $D = 20$ m, and $D/B = 1.0$ when $D = 10$ m. The lowering basement in the analysis has 8 m uniform displacement at the central part ($B_u = 8$ m) and 1 m sloping shape displacement on each end ($B_s = 2$ m). Displacement of 0.05 m and 0.2 m were imposed for all the analyses. The basement is assumed to be rough, unless otherwise stated. The soil foundation is assumed to be a medium dense Toyoura sand and a normally consolidated Fujinomori clay. In order to concentrate on the response behavior of the foundation due to different volume-change patterns for soil materials described above and to match the conditions of model tests, the fully drained condition was assumed for both materials. The mesh size dependency of the analytical results was discussed at the Appendix of

(a) Schematic diagram of lowering basement ($D=20$ m)

A1A2A3A4 : $Z=40$ m, B1B2B3B4 : $Z=44$ m, C1C2C3C4 : $Z=48$ m, D1D2D3D4 : $Z=80$ m

(b) half of model foundation for 3D FEM analysis ($D=20$ m)

(c) mesh in XY-plane ($D=20$ m)

(d) mesh in XY-plane ($D=10$ m)

Fig. 7. FEM mesh for 3D numerical analysis

paper (Nakai and Xu, 1995).

The initial stress for each element is calculated using the unit weight of soils (15.5 kN/m^3 (sand), 17.6 kN/m^3 (clay)) and the coefficient of earth pressure at rest (0.57 (sand), 0.47 (clay)). Figure 7(b) indicates that the excavation has proceeded from $L=0 \text{ m}$ to $L=24 \text{ m}$ with the same displacement (0.05 m or 0.2 m) and from $L=24 \text{ m}$ to $L+8=32 \text{ m}$, with a sloping face.

RESULTS AND DISCUSSIONS

Model Tests

Figure 8(a) shows the surface settlements observed ($D/B=2.0$) of 3D tests at the center of a measured line AA' in Fig. 1 compared to tunnel excavation length L . 2D test results are also represented on the right-hand side with arrows in the same figure. This figure shows that the surface settlements for 3D and 2D model tests in clay are much larger than those in sand, when the same displacement on the lowering basement is imposed. Furthermore, the surface settlement from a 3D test in sand was a little smaller than that of a 2D test for the same displacement. The 3D test results in clay (Kaolin powder) were considerably greater than that of the 2D test. It can therefore be seen from these test results that surface settlements depend not only on the soil properties but also on whether or not the influence of the construction procedure is taken into account (3D or 2D), even when the same displacement on lowering basement is imposed. These observed tendencies also correspond to those of analytical

Fig. 8. Observed surface settlements at the center of measured line against tunnel excavation length L

results presented before (Nakai and Xu, 1995), although the present test conditions are somewhat different from the previous analyses. Figure 8(b) shows the results with $D/B=1.0$. The same tendency as Fig. 8(a) is observed except that the 3D settlement in sandy soil is larger than 2D settlement, in the same way as in clay. Comparing Fig. 8(a) with Fig. 8(b), it can be seen that although the settle-

Fig. 9. Observed surface settlement profiles at cross section ($D/B=2.0$)

Fig. 10. Observed normalized surface settlement profiles at cross section ($D/B=2.0$)

ment in clay with $D/B=1.0$ is a little larger than that with $D/B=2.0$, the settlement in sand increases rapidly as the ratio D/B decreases. It was also found that the settlements at the center in the 3D tests of clay are larger than the displacement imposed on the basement (the dotted lines in the figures show the displacements imposed on the basement).

Fig. 11. Observed surface settlement profiles at cross section ($D/B=1.0$)

Fig. 12. Observed normalized surface settlement profiles at cross section ($D/B=1.0$)

Figure 9 shows the observed profiles ($D/B=2.0$) of surface settlements for 3D and 2D tests after completion of the displacing basement. Figure 10 also shows the profiles of surface settlements normalized with respect to surface settlements at the center (S_{max}). The prescribed pattern of the lowering basement is represented at the bottom of each figure. It can be seen from these figures that the settlement curve in clayey soil is flatter and extends to a wider range than that in sandy soil. Figure 11 shows the observed profiles with $D/B=1.0$. Figure 12 shows the profiles of surface settlements normalized by the maximum values of the surface settlements. The tendency of surface settlement profiles with $D/B=1.0$ is the same as that with $D/B=2.0$. It is seen from the comparison between Figs. 10 and 12 that the settlement in Fig. 12 with $D/B=1.0$ occurs more locally than those in Fig. 10 with $D/B=2.0$ in each case.

It can also be noted that tension hair cracks are observed on the free surface in clay but not in the sand.

Deformations of Numerical Analyses

The results computed using the mesh in Fig. 7 are shown in Figs. 13 to 17, which correspond to the test results in Figs. 8 to 12, respectively. Quantitative comparison between the test results and computed results can not be made, however because stress level and the material itself are different. The computed results describe fairly well the observed differences of magnitude and profile of surface settlement between sand and clay, between 3D and 2D, and between $D/B=2.0$ and $D/B=1.0$. The com-

Fig. 13. Computed surface settlements at point C4 versus tunnel excavation length L

puted contours of settlements in the subsoil profile normalized by the displacement of the lowering basement are indicated in Figs. 18 and 19. We can see that the settlement in sand occurs more locally than that in clay, and this tendency is more notable at large displacement ($d=0.2$ m) in sand. It is also clear that the maximum set-

tlement occurs just on the lowering basement in sand, but occurs at surface above the lowering basement in clay. Figures 20 and 21 show the contour of volumetric strain at section *B* in 3D analysis. Clay shows only negative dilatancy (volume contraction) and the contour lines of the volumetric strain are widely distributed, opposite-

Fig. 14. Computed surface settlement profiles at section C ($D/B=2.0$)

Fig. 16. Computed surface settlement profiles at section C ($D/B=1.0$)

Fig. 15. Computed normalized surface settlement profiles at section C ($D/B=2.0$)

Fig. 17. Computed normalized surface settlement profiles at section C ($D/B=1.0$)

ly, sand shows positive dilatancy (volume dilation) and the contour lines are concentrated on which the displacement is imposed. These facts support the difference of surface settlements between clay and sand. In addition, it is worth indicating that 3D tests and analyses can reflect volume change of soil much more precisely because it can

simulate successive excavation procedure. This is the reason why 3D settlement in clay is larger than 2D settlement, and 3D settlement in sand becomes much smaller with the increase of the ratio of D/B particularly. It is also noted, that even if elastic modulus is changed, the difference between clay and sand and between 3D and 2D can not be described by elastic analyses.

Earth Pressures of Numerical Analyses

Since the earth pressure was not measured in the model tests, only the results of computations are shown. Discussions on the influence of the soil dilatancy and the roughness of basement on the distribution of earth pressure can then be made on the basis of these results.

Figures 22 and 23 show the 3D and 2D analytical distributions of vertical stress normalized with initial stress near basements at section B when the displacement of 0.05 m and 0.2 m are applied, and the excavation is proceeded to the section $D(L=80\text{ m})$ in Fig. 7. Since the stress on the basement of $D/B=2.0$ is the two times as large as that of $D/B=1.0$, the vertical axis of $D/B=2.0$ in these figures are indicated at double scale. When a base-

Fig. 18. Computed contour lines of settlement normalized with imposed displacement (s/d) at section C ($D/B=2.0$)

Fig. 19. Computed contour lines of settlement normalized with imposed displacement (s/d) at section C ($D/B=1.0$)

ment displacement of 0.05 m is imposed, the tendency of stress distributions in all the cases is more or less the same. Namely, though the magnitude of stress concentration depends on the depth of foundation ($D/B=2.0$ or

1.0), sand or clay and 3D or 2D, the stress on the lowering basement decreases and that in the adjacent increases due to an arching effect. Beyond this region, the stress decreases gradually and then extend to the initial one. This tendency corresponds to the previous numerical results in which the mode of the basement displacements is a little different (Nakai and Xu, 1995) and the previous experimental results (e.g., Murayama and Matsuoka, 1971; Adachi et al., 1994). The results with $d=0.2$ m (Fig. 23) show that the stress distributions of sand are different from those of $d=0.05$ m. Stress in sand shown in Fig. 23 increases more sharply at the edge of lowering basement and decreases sharply again just outside of the zone of stress concentration, although there is not much difference of stress distribution depending on the magnitude of displacement in the clay. In order to confirm such mechanism of arching effect, numerical analysis with smooth basement was also carried out. The analytical results with smooth basement are shown in Figs. 24 and 25. In these cases the patterns of stress distribution in

Fig. 20. Computed contour lines of volumetric strain (%) at section B ($D/B=2.0$)

Fig. 21. Computed contour lines of volumetric strain (%) at section B ($D/B=1.0$)

Fig. 22. Pressure distributions near basement at section B with 0.05 m displacement imposed

Fig. 24. Pressure distributions near basement at section B with 0.05 m displacement imposed (smooth)

Fig. 23. Pressure distributions near basement at section B with 0.2 m displacement imposed

Fig. 25. Pressure distributions near basement at section B with 0.2 m displacement imposed (smooth)

sand and clay are similar, and those in sand are not influenced very much by the magnitude of displacements. Through these analytical results of stress distribution, it can be supposed that the arching effects in sand are more noticeable due to its positive dilatancy and are influenced by the roughness of basement and the magnitude of displacement imposed on the basement. Those in clay with negative dilatancy are not influenced very much by the roughness and the magnitude of displacement.

CONCLUSIONS

In order to verify 3D and dilatancy effects on settlements and earth pressures which are obtained from numerical analyses, 3D and 2D model tests were made. Elastoplastic finite element analyses have also been carried out in which refined meshes were used to appropriately simulate the displacement pattern of model tests. In both the model tests and numerical analyses, two foundation depths were assumed—the depth D is two times the tunnel width B ($D/B=2.0$) and is equal to B ($D/B=1.0$). 3D tests and numerical analyses can take into account the influence of the excavation process of tunneling on the ground movements, but the ones under plane strain condition (2D) cannot. Influence of soil dilatancy on subsoil movements have also been discussed assuming sand and clay as the geomaterials. The following results were obtained through the present experimental and numerical study:

- 1) Very good qualitative agreements were obtained on the surface settlements through the study of numerical analyses and model tests.
- 2) Surface settlement in a clay foundation is much larger and extend much wider than that in the sand foundation. The reason for this is that the clay shows negative dilatancy and sand shows positive dilatancy.
- 3) The difference mentioned above is more noticeable

in 3D than in 2D. Settlement in sand considering excavation process (3D tests and analyses) are smaller than that without consideration of it (2D tests and analyses), but vice versa in clay.

- 4) Analytical earth pressure distributions due to the arching effect correspond to the experiment ones reported before by other researchers. It was shown numerically that the arching effect in sand is more noticeable than that in clay, because of the positive dilatancy.

To predict the deformation and earth pressure in tunneling, it is necessary to carry out a 3D analysis in which the construction process and mechanical properties of the soil including soil dilatancy are properly taken into account.

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