BEARING CAPACITY OF SHALLOW FOUNDATIONS

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ABSTRACT

A new concept for the calculation of the bearing capacity of shallow foundations is presented. The Terzaghi-Buisman equation was transformed and extended to include all possible load cases. The shape ratio was rigorously defined. Torsional moment factors considering eccentric horizontal loads were proposed and defined for the first time, in order that complete 6-dimensional interaction diagrams could be derived. The effect of embedded depth was also investigated and considered for special factors. The influence of cohesion was derived from Caquot's theorem of the corresponding states of stress. Experimental investigations with some interesting results are presented. A new calculation scheme is presented at the conclusion of the paper.

Key words: bearing capacity, eccentric horizontal load, experimental investigation, theorem of Caquot, torsional moment, (IGC: E3/E14)

INTRODUCTION

In order to ensure the stability of foundations, their bearing capacity on a special subsoil must be determined. The base failure resistance of shallow foundations can be calculated according to many national codes of practice e.g., the Danish code (Brinch Hansen, 1970).

Most of the former experimental investigations account for only simple load cases (e.g., eccentricity or inclination) and some analyses neglect the necessity to combine these special load cases.

There are existing load cases which were not considered by simple calculation schemes like most of the national codes of practice. None of these codes take the eccentricity of a horizontal load into account (Malcharek and Smolctzcyk, 1981). For this reason, a new concept has been worked out that relies on the former schemes as far as possible but introduces some fundamental improvements. It relies on an idealized plasticity theory using the Mohr-Coulomb failure criterion as did e.g., Brinch Hansen (1970). This theory has been recognized to be sufficient for practical cases. With an extensive test program that considers the effects of different types of loading on the failure load some deficiencies in the calculation of the bearing capacity of shallow foundations can be overcome.

PARAMETER

For the calculation of bearing capacity, several parameters are required. The shear strength parameters \(\phi\) and \(c\) and the unit weight of the soil below the base \(\gamma\) should first be mentioned. The lateral load \(q\) above the base which results from the laterally bedded soil is also important. Finally the lateral lengths of the foundation should be considered. According to the orientation of a system of coordinates they are here defined with \(B_2\) (in \(x_2\)-direction) and \(B_1\) (in \(x_1\)-direction). The foundation geometry and loading is shown in Fig. 1.

In order to estimate the bearing capacity, the relevant loading is required which can be described by a force

\[ \text{Fig. 1. Foundation geometry and loading} \]

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with its components $F_1, F_2, F_3$ and a moment with its components $M_1, M_2$ and $M_3$. The style of the system of coordinates selected herein is based upon the conventional notation for the calculation of bar systems.

The ultimate bearing capacity of a rectangular foundation is not determined if a function $F$, which has yet to be determined, satisfies the following condition.

$$ F(\phi, c, \gamma, q, B_2, B_3, F_1, F_3, M_1, M_2, M_3) > 0 $$

Such equations are known from plasticity theory by the formulation of the yield criterion (Chen, 1975).

**DIMENSIONAL ANALYSIS**

With some abbreviations shown in Eqs. (2)–(7) the problem can be simplified by application of dimensional analysis.

$$ \tilde{\epsilon} = \frac{c}{B_3 \gamma}, \quad \tilde{q} = \frac{q}{B_3 \gamma}, \quad \tilde{f}_1 = \frac{F_1}{\gamma B_3^2} $$(2), (3), (4)

$$ f_2 = \frac{F_2}{F_1} \quad \text{and} \quad f_3 = \frac{F_3}{F_1} $$ (5a), (5b)

$$ m_1 = \frac{M_1}{F_1 B_2}, \quad m_2 = \frac{M_2}{F_1 B_3}, \quad m_3 = \frac{M_3}{F_1 B_2} $$ (6a), (6b), (6c)

The shape ratio $\bar{b}$ is of special importance. It depends only on the chosen position of the system of coordinates and does not depend on a comparison of the lateral lengths as in most of the well-known codes of practice (Brinch Hansen, 1970). It is therefore not considered important if $B_2$ or $B_3$ is the length or the width.

$$ \bar{b} = B_2 / B_3 $$

(7)

Using dimensional analysis the relationship between the twelve dimensioned parameters (1) can now be transformed into an equivalent relationship (8) between ten dimensionless parameters.

$$ \tilde{F}(\phi, \tilde{\epsilon}, \tilde{q}, \bar{b}, f_2, f_3, f_1, m_1, m_2, m_3) > 0 $$

(8)

This implicit presentation can be transformed into an explicit and more convenient presentation (Peran, 1995).

$$ f(\phi, \tilde{\epsilon}, \tilde{q}, \bar{b}, f_2, f_3, m_1, m_2, m_3) - f_1 > 0 $$

(9)

**CONCEPT FOR A FORMULA**

The function studied to calculate the value $f(\tilde{\epsilon})$ according to (9) which depends on nine input parameters must be determined. Here a statement shall be chosen which is based upon the well-known Terzaghi-Buisman equation (Buisman, 1940).

$$ f(\tilde{\epsilon}) = \begin{cases} 
N_{10} \cdot s \cdot \mu \cdot i \cdot \tau \gamma \\
\tilde{q} \cdot N_{10} \cdot s \cdot \mu \cdot k \cdot \tau q \\
+ \tilde{\epsilon} \cdot N_c 
\end{cases} $$

(10)

Other evolutions of Eq. (9) are possible. The main advantage of this equation is that several components are already known in principle. The expressions for the single coefficients correspond to the expressions for common schemes as much as possible. $N_{10}$ and $N_{00}$ are the bearing capacity factors of the $y$- and $q$-terms. Both of them depend only on $\phi$. The shape factors $s$, and $s_0$ and the inclination factors $i_1$ and $i_2$ are known. In addition, the new terms, bending moment factors $\mu$, and $\mu_0$ and torsional moment factors $\tau_\gamma$ and $\tau_\phi$ are introduced.

A special treatment will be used to evaluate the $c$-term. It will be shown later that it is very convenient to include the effects of shape ratio and different types of loading into the factor $N_c$.

**EXPERIMENTAL INVESTIGATIONS**

The different parameters in Eq. (10) can be determined by experiments or calculations according to the classical plasticity theory. The bearing capacity factors $N_{10}$ and $N_{00}$ usually result from theoretical calculations. They have been verified by full scale tests of the Degebo (Weiss, 1978).

The absolute size of dimensionless failure loads $f(\tilde{\epsilon})$ for full scale and model tests, even under identical load conditions, is not the same. The phenomenon is called 'scale effect', however, comparisons between the results $f(\tilde{\epsilon})$ of model and full scale tests have shown that the reduction factors like $i_1$ or $\mu$, which were determined from model tests correspond very well to those from full scale tests (Steinfelt, 1979 or Gotardi et al., 1994). This indicates that the dimensionless failure load $f(\tilde{\epsilon})$ of a model test is affected by the dimensionless load components $f_1, f_2, f_3, m_1, m_2$ and $m_3$ in the same way as in a full scale test. Model tests are therefore considered appropriate to investigate the influence of different types of loading.

A test program was drafted that contains 153 model tests. In order to verify the model tests the experimental program includes simple load cases which were already investigated by full scale tests (Weiss, 1978).

The tests, here, were made on a very dense medium to coarse sand, which had been drizzled by an adjustable box from 80 cm height. Model foundations with lateral lengths between 4 cm and 20 cm were placed on the sand. The experimental apparatus is shown in Fig. 2. Initially the vertical load is applied on the foundation by using a crossbar and hanging dead-weights. In a second step the horizontal force was applied from a cable with a hanging dead-weight. The cable was fixed 1.2 cm above the base of the foundation, so the overturning moment had to be neutralized by a small eccentric dead-weight. At any time, displacements of the foundation were measured, so that the horizontal forces could be increased appropriately.

In addition to the large number of possible load combinations, the shape ratio $\bar{b}$ was varied. Each value of the coefficients studied from (10), now, resulted from two tests with an identical model foundation, which had been carried out on the same packing. The first of these tests, a reference test was performed with an exclusive working concentric vertical load. The second test was performed with the special load combination which was investigated. The quotient of the two vertical load components
gave the respective reduction factor.

The test program was divided into the series (A-K) shown in Table 1. The result of one series was either a specified coefficient or the result of a simple multiplication of factors according to (10) correctly considering the reduction of the failure load. Some interesting results of this test program as well as the leading ideas for the analysis are presented below.

**SHAPE FACTOR**

A special case for concentric vertically loaded foundations without embedded depth on cohesionless soils results from (10).

\[ f^r = f^r_s(\phi, \bar{b}) = N_{\phi}(\phi) \cdot s_r(\bar{b}, \phi) \]  

(11)

The definition of the shape ratio according to (7) requires a specific proof of the invariance of the failure load \(F_1\) to the orientation of the system of coordinates; that means the failure load must not depend on the eligible position of the system of coordinates (see Fig. 3).

For the establishment of the shape factor \(s_r\), this implies that both Eqs. (12) and (13) must provide the same failure load.

\[ F_1 = B_1 \cdot B_3^3 \cdot \gamma \cdot N_{\phi}(\phi) \cdot s_r(B_2 / B_1, \phi) \]  

(12)

\[ F_1 = B_2 \cdot B_3^3 \cdot \gamma \cdot N_{\phi}(\phi) \cdot s_r(B_2 / B_3, \phi) \]  

(13)

From these two equations and (7) follows with elementary mathematical transformations Eq. (14) which has to be satisfied by the shape factor \(s_r\) in any case.

\[ \bar{b} \cdot s_r(\bar{b}, \phi) = s_r \left( \frac{1}{\bar{b}}, \phi \right) \]  

(14)

By inclusion of strip foundations it also follows:

\[ s_r(\bar{b}, \phi) = 1 \]  

for \( \bar{b} = 0 \)  

(15)

Series A results in a possible definition of the shape factor according to (16) as proposed by Caquot and contained in the French code of practice (DTU).

\[ s_r(\bar{b}, \phi) = \frac{1}{1 + \bar{b}} \]  

(16)

This equation serves the two conditions shown in Eqs.
(14) and (15). Therefore the necessary invariance of (11) is proven. The effect of $\phi$ has been recognized as negligible for practical problems.

**BENDING MOMENT FACTOR**

Many national codes of practice e.g., in Denmark or France, consider the load eccentricity by the concept of a fictitious reduced base area (Malcharek and Smolczyk, 1981). In this concept the lateral lengths appear reduced so that the vertical load now acts in the centre of the fictitious base area. This proceeding has many disadvantages (Perau, 1995). So it does only work if the loading can be reduced to an resultant force which works on one point of the base area. There is no way to calculate the bearing capacity if the horizontal load has other eccentricity than the vertical load. A second problem of the reduced-area concept is that eccentricities unfortunately influence the shape of the foundation, the shape factors and, as can be seen later, other loading factors from (10).

A new concept was determined which considers the influence of eccentricity due to a bending moment factor $\mu_r$. This factor depends on $m_2$ and $m_3$ and perhaps on the shape ratio $b$. It has to be mentioned that the signs of $m_2$ and $m_3$ must not have any effect on $\mu_r$. The results of series B which dealt only with the influence of $m_2$ are shown in Fig. 4.

The results of series B show a great dispersion but the dependence on $m_3$ is very clear. A relevant dependence on the shape ratio $b$ could not be found. So $\mu_r$ can be determined by Eq. (17):

$$\mu_r = \frac{1}{3} - 2.5 \cdot |m_2|$$

(17)

Series C was made to examine the effect of biaxial eccentricity. The results, as shown in Fig. 5, lead to the following Eq. (18) which coincides with a simple multiplication of the respective factors for uniaxial eccentricity in the $x_2$- and $x_3$-directions.

$$\mu_r = (1 - 2.5 \cdot |m_2|) \cdot (1 - 2.5 \cdot |m_3|)$$

(18)

**INCLINATION FACTOR**

The results of series D (Fig. 6) and E (Fig. 7) indicate that the factor $i_\gamma$ is not so much influenced by the shape of the foundation and the direction of the horizontal load. The factor does depend only on the load inclination.

As can be seen in Figs. 6 and 7 experimental results from foundations with parallel or not parallel horizontal load both fit Eq. (19) very well so that in every case this equation can be used.

$$i_\gamma \approx (1 - \sqrt{f_2^2 + f_3^2})^{2.5}$$

(19)

**TORSIONAL MOMENT FACTOR**

Series F dealt with foundations which were loaded by a concentric vertical load and a torsional moment $M_t$. This loading is not normally observed in practice but it is required as an extreme case for loadings with eccentric
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The experimental investigations of the series G, H, J and K (see Table 1) have shown that the presented equation supplies results which are usually correct and rather safe (Perau, 1995). In extreme situations, however, the simple equation could be uneconomic. So, if the signs of \( f_2 \) and \( m_1 \) (respective \( f_1 \) and \( m_2 \)) are not identical—Gottardi, G. and Butterfield, R. (1993) call it "negative eccentricity"—the experimental values of failure loads exceed the one from simple multiplication from (18) and (19) according to (10). This "unsymmetrical behaviour" was also stated by Gottardi, G. and Butterfield, R. (1993). But, if just one or both of \( |f_2| \) and \( |m_1| \) remain under 0.3 the experimental values exceed the calculated one by a factor from at most 1.5. Considering the extreme loading and the safe side of the calculated value that should be accepted.

**EMBEDDED DEPTH**

After the reduction factors of the first part of the bearing capacity Eq. (10) were investigated in detail by experiments, the coefficients for the second part had to be determined. This term looks after the additional effect of an embedded depth of the foundation or a lateral load \( q \). This term must naturally depend also on the shape of the foundation and the working load. The well-known bearing capacity factor \( N_0 \) was derived by calculations according to plasticity theory and verified by full scale tests (Weiss, 1978). The effects of shape, load inclination and bending moments in combination with vertical loads was investigated by experiments so that \( s_{nr} \), \( \mu_q \) and \( i_\gamma \) could be defined (de Beer, 1970; Weiss, 1978). The result of a new evaluation of this experiments (Perau, 1995) are presented in Table 2. The still remaining factor \( i_\gamma \) obviously never has been investigated in detail. A first approach should be that \( i_\gamma \) is similar to \( i_\gamma \). Because all load reduction factors from \( q \)-term exceed the respective factors from \( \gamma \)-term this assumption should be on the safe side. A more economic solution can be found by additional experimental investigations.

**COHESION**

The terms mentioned before do not include the favorable effect of subsoil cohesion. According to (10) an additional term has to be developed which overcomes this deficiency.

The cohesion term in the base failure equation can be found from Caquot's theorem of the corresponding states of stress. This theorem transfers a static \( c,\phi \)-parameter problem to a more simple \( \phi \)-parameter problem. There are additional normal stresses \( \Delta \sigma = c / \tan \phi \) acting on this substitutional system. In the calculation of bearing capacity these fictitious stresses appear as additional lateral loading and also additional foundation loading. With this assumption the bearing capacity factor \( N_c \) can be formulated more generally.
\[ N_c = \frac{1}{\tan \phi} \left( N_{q0} \cdot s_q \cdot \mu_q \cdot i_q \cdot i_{\gamma} - 1 \right) \]  \hspace{1cm} (22)

A separate establishment of further coefficients for the cohesion-term is therefore not necessary. Because the additional normal stress due to the theorem of Caquot is put on the subsoil in the base area, all coefficients which intervene the vertical load \( F_i \) have to be modified so that \( F_i \) is replaced by the expression:

\[ F_i^* = F_i + B_2 \cdot B_3 \cdot \frac{c}{\tan \phi} \]  \hspace{1cm} (23)

For example, the formula \( i_r \) (19) has to be supplied in the following way after using (5a) and (5b):

\[ i_r = \left( 1 - \sqrt{\frac{F_2}{F_1 + B_2 \cdot B_3 \cdot \frac{c}{\tan \phi}}} \right)^2 + \left( \frac{F_3}{F_i + B_2 \cdot B_3 \cdot \frac{c}{\tan \phi}} \right)^{2.5} \]  \hspace{1cm} (24)

Bearing capacity factors and the appertaining coefficients for initial states with the shear parameter \( \phi_0 = 0 \) and \( c_e \) could be derived from a \( \phi_0 \)-analysis. The results are presented in the right column of Table 2.

**CALCULATION OF THE BEARING CAPACITY**

According to the applied safety concept the design loads \( (F_1, F_2, F_3, M_1, M_2, M_3) \) must be calculated first. The same shall apply to the shear parameters \( \phi \) and \( c \) (effective shear parameters or parameters for the initial state). With the lateral lengths \( B_2 \) and \( B_3 \) the following parameters have to be evaluated: The shape ratio \( \overline{b} \) equals zero for strip foundations, the dimensionless forces are \( f_1, f_2, f_3 \) and the dimensionless moments are \( m_1, m_2, m_3 \).

The bearing capacity can then be calculated with Table 2. Usually the factor \( f_c \) is unequal to one. An iteration

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### Table 2. Calculation scheme

<table>
<thead>
<tr>
<th></th>
<th>Angle of internal friction</th>
<th>( \phi \neq 0 )</th>
<th>( \phi = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Bearing capacity factors</td>
<td>( N_{q0} = \frac{1 + \sin \phi}{1 - \sin \phi} \cdot \frac{1 + \tan \phi}{1 - \tan \phi} \cdot N_{c0} )</td>
<td>( N_{q0} = 2 + \pi )</td>
</tr>
<tr>
<td>3</td>
<td>Shape factors</td>
<td>( s_r = \frac{1}{1 + \overline{b}} )</td>
<td>( s_r = 1.6 \cdot \frac{\overline{b}}{1 + \overline{b}^2} )</td>
</tr>
<tr>
<td>4</td>
<td>Loading coefficients</td>
<td>( \mu_0 = (1 - 2.5 \cdot m_1 \cdot f_1) \cdot (1 - 2.5 \cdot m_1 \cdot f_2) \cdot (1 - 3.6 \cdot \tan \phi \cdot m_3) )</td>
<td>( \mu_0 = -3.6 \cdot (m_2 + m_3) )</td>
</tr>
<tr>
<td>4b</td>
<td>Inclination</td>
<td>( i_r = (1 - \sqrt{f_1^2 + f_2^2 + f_3^2}) \cdot \frac{1}{1 + \overline{b}} )</td>
<td>( i_r = (1 - 0.7 \cdot \sqrt{f_1^2 + f_2^2 + f_3^2}) \cdot \frac{1}{1 + \overline{b}} )</td>
</tr>
<tr>
<td>4c</td>
<td>Torsion</td>
<td>( i_{r_2} = 1 - \frac{13 \cdot \overline{b}}{1 + \overline{b}} \cdot m_1 \cdot f_1 )</td>
<td>( i_{r_2} = 1 - \frac{13 \cdot \overline{b}}{1 + \overline{b}} \cdot m_1 \cdot f_1 )</td>
</tr>
<tr>
<td>5</td>
<td>Bearing capacity factors for calculation</td>
<td>( N_c = (N_{q0} - 1) \cdot \tan \phi )</td>
<td>( N_c = N_{q0} \cdot s_q \cdot \mu_q \cdot i_q \cdot i_{\gamma} )</td>
</tr>
<tr>
<td>6</td>
<td>Base failure resistance</td>
<td>( f_1^* = N_c + q \cdot N_q + c \cdot N_c )</td>
<td>( N_c = N_{q0} + s_q + \mu_c )</td>
</tr>
</tbody>
</table>
might therefore be necessary with $f_c = 1$ as the first step. This iteration step leads to an uneconomic result for $f_b^*$ because the realistic $f_c$ always remains under 1.0 and the values of the reduction factors in (10) and so the resistance $f_b^*$ decrease with increasing $f_c$.

In a second step the new calculated $f_b^*$ can be placed into Eq. (25). The second calculation leads to an unsafe value for the base failure resistance. A third step is therefore required. The result is a good approximation on the safe side. The bearing capacity, at last, is proven if the inequality (9) is fulfilled for any load case.

CONCLUSIONS

Most of the existing national codes of practice do not consider every possible load case, e.g., the eccentricity of horizontal loads. They normally examine only special load cases and neglect any combination of them. With introduction of a fixed system of coordinates the problem can be better described. A new definition of the shape ratio and the load components resulted automatically.

An equation has been formulated based on the classical Terzaghi-Buisman equation but takes all possible load combinations into account. Special factors have been derived which allow this calculation first of all for basic load cases. In order to consider a combination of these basic load cases the factors can be connected by a simple multiplication. The results are considered safe. The cohesion-term in the base failure equation was derived from Caquot's theorem of the corresponding states of stress. A separate establishment of coefficients is therefore not necessary. A complete calculation scheme for all possible load combinations has finally been presented which can be used rigorously.

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NOTATION

- $\bar{b}$: Shape ratio of a shallow foundation $B_L/B_h$ [l]
- $B_L, B_h$: Lateral lengths of a shallow foundation in $x_L$- and $x_h$-directions [m]
- $c$: Cohesion of the soil [kN/m$^2$]
- $\bar{c}, \bar{q}$: Dimensionless parameters for cohesion and lateral load [l]
- $D$: Embedded depth of a foundation [m]
- $f_s, f_d, f_t$: Dimensionless parameters for the components of force [l]

$\bar{f}_b^*$: Dimensionless parameter for bearing capacity [l]
$f_c$: Factor for consideration of the fictitious normal stress under the base of a foundation [l]
$F, \bar{F}$: Functions [l]
$F_1, F_2, F_3$: Components of a force in $x_r, x_t$- and $x_f$-directions [kN]
$i_{\rho}, i_\xi$: Inclination factors [l]
$i_{\tau_\rho}, i_{\tau_\xi}$: Torsional moment factors [l]
$m_r, m_\pi$: Dimensionless parameters for the components of a moment [kNm]
$N_r, N_\pi$: Components of a moment in $x_r, x_t$- and $x_f$-directions [kNm]
$N_r, N_\pi$: Bearing capacity factors ($\gamma$- and $q$-terms) [kN/m$^2$]
$q$: Unit weight of the soil below the base [kN/m$^2$]
$\gamma$: Bending moment factors ($\gamma$- and $q$-terms) [l]
$\phi$: Angle of internal friction [°]
$\Delta \sigma$: Fictitious normal stress (theorem of Caquot) [kN/m$^2$]

REFERENCES