EFFECT OF THE SHAPE OF CYCLIC LOADING ON DAMPING RATIO
AT SMALL STRAINS

MLADEN VUCETIC, GIUSEPPE LANZO and MACAN DOROUDIAN

ABSTRACT
A series of 69 cyclic tests was conducted to study the damping properties of two reconstituted sands and three laboratory-made clays at small cyclic shear strain amplitudes $\gamma_c = 0.001$-0.04%. A recently developed constant-volume equivalent-undrained direct simple shear device for small-strain testing was employed. The specific effect of the shape of cyclic strain-time history on the equivalent viscous damping ratio, $\lambda$, was investigated. The results show that $\lambda$ can be significantly affected by the shape of the cyclic strain-time history because of the viscous nature of soil material response and the associated effects of creep and relaxation. As the shape of the cyclic strain-time history is changed from triangular to sinusoidal, and further from trapezoidal to square, $\lambda$ at small strains may increase by a factor of two or more. The test results show that this effect was largest for clays, smaller for silty sand and negligible for clean sand.

Key words: clay, creep, cyclic loading, damping, laboratory testing, rate of deformation, sand, silt content, simple shear test, viscosity (IGC: D1/D7)

INTRODUCTION AND OBJECTIVES
Reliable and accurate damping characteristics of soils at small strains are necessary for the solution of many soil dynamics problems, such as vibration of machine foundations, response of soil deposits and earth structures to earthquake loads, response of offshore soils and supported structures to ocean wave loads, evaluation of traffic vibration, etc. In these problems, typically involving cyclic loading, the stress-strain behavior of soils is described by cyclic loops, similar to the idealized loops in Fig. 1. Based on such loops, the damping characteristics are commonly evaluated by the equivalent viscous damping ratio, $\lambda$, derived originally by Jacobsen (1930):

$$\lambda = \frac{1}{4\pi} \frac{\Delta W}{\gamma_c \tau_c}$$ (1)

In Eq. (1) and Fig. 1, $\Delta W =$ the area inside the loop, $\tau_c =$ cyclic shear stress amplitude, $\gamma_c =$ cyclic shear strain amplitude, $G_{max} =$ maximum shear modulus at small strains, and $G_s =$ secant shear modulus corresponding to $\tau_c$ and $\gamma_c$.

In current practice, the cyclic loops are recorded during cyclic laboratory testing and $\lambda$ is obtained by measuring $\Delta W$. However, the cyclic loading tests can be performed with various shapes of either loading-time or deformation-time histories. For example, in resonant column tests reported by Kim et al. (1991) and cyclic simple shear tests reported by Dobry and Vucetic (1987) and Vucetic (1990), the shape of cyclic loading or deformation was typically sinusoidal; Andersen et al. (1980), on the other hand, applied a trapezoidal shape of loading during their extensive cyclic characterization of a clay; while Thiers and Seed (1968) and Shibuya et al. (1995) applied triangular shape of cyclic deformation on the soils they tested. As explained below, these different shapes of either cyclic loading or deformation can yield for the same soil different values of $\Delta W$ and thus $\lambda$. Therefore, the shape of cyclic loading or deformation must be properly considered in design.

The idealized cyclic loops corresponding to triangular and trapezoidal shapes of cyclic straining for small and large cyclic strain amplitude, $\gamma_c$, are presented in Figs. 1(a) and 1(b), respectively. The associated cyclic strain-time histories are presented in Fig. 1(c). The loops corresponding to the triangular shape of cyclic straining are plotted by solid lines, while the loops corresponding to the trapezoidal shape of cyclic straining are plotted by dashed lines.

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It can be seen in Fig. 1 that due to the viscous relaxation of the soil between points $b$ and $c$, and $e$ and $f$, $\Delta W$ is much larger for the trapezoidal shape of cyclic straining than for the triangular shape. It can also be noticed that at small cyclic strains (Fig. 1(b)), where soil behavior is close to linear, the relative increase of $\Delta W$ due to the change of the shape from triangular to trapezoidal is much larger than at large cyclic strains (Fig. 1(a)). This is because $\Delta W$ for the triangular shape of cyclic straining at large $\gamma$, is already large (Fig. 1(a)), due to the high non-linearity of the soil, and consequently it cannot be increased considerably in relative terms. At small $\gamma$, on the other hand, $\Delta W$ for the triangular shape of cyclic straining is very small (Fig. 1(b)), so the relaxation between
points $b$ and $c$, and $e$ and $f$, can increase $\Delta W$ significantly in relative terms.

Figure 2 presents the variation of $\lambda$ with $\gamma_c$ and the plasticity index of the soil, PI, obtained from the results of many tests conducted on a broad range of soils in a number of different laboratories using different shapes of cyclic strain-time histories (Dobry and Vucetic, 1987; Vucetic and Dobry, 1991). Lines showing approximate range and average values of the threshold volumetric cyclic shear strain, $\gamma_{\text{th}}$, for different values of PI are also included (Vucetic, 1994). The cyclic shear strain amplitude $\gamma_s$ defines the threshold above which the soil microstructure starts to change permanently due to the cyclic loading, and below which such changes are negligible (Dobry et al., 1982).

It can be seen in Fig. 2 that an approximate relation between $\lambda$, $\gamma_c$ and PI could be developed only for $\gamma_c > 0.005\%$. Until now there has been a general consensus among researchers and practitioners that $\lambda$ is relatively well defined for most soils at moderate and large values of $\gamma_c$, while more experimental investigations are needed in the range of small $\gamma_c$ to clarify the influence of plasticity index (PI), silt content, effective vertical consolidation stress ($\sigma_{vc}$), overconsolidation ratio (OCR), void ratio ($e$), frequency ($f$), shape of cyclic strain-time history, and possibly other factors (Dobry and Vucetic, 1987; EPRI, 1991; Kim et al., 1991; Silvestri, 1991; Kokusho et al., 1994; Toki et al., 1994; Tatsuoka et al., 1978, 1995).

According to the above discussion in connection with Fig. 1, it seems that the correlation in Fig. 2 cannot be extended to small cyclic strains unless the effect of the shape of cyclic straining is taken into account. As already explained, at small $\gamma_c$ the shape of cyclic straining can influence strongly the value of $\Delta W$ and thus $\lambda$, while at large $\gamma_c$ this influence is relatively small. In fact, some Japanese researchers, such as Nakajima et al. (1994) and Toki et al. (1994), pointed out recently that the influence of the shape of cyclic loading or deformation on $\lambda$ at small cyclic strains can be quite significant, and therefore it should be investigated. Their suggestions and the data are in agreement with the test results presented below in this paper.

The specific objective of this paper is to attempt a clarification of the effect of the cyclic strain-time history on $\lambda$ at small cyclic strains on the basis of a comprehensive testing program. To meet this objective, a series of 69 small-strain constant-volume equivalent-undrained direct simple shear cyclic tests was conducted in a device recently designed by Doroudian and Vucetic (1995). In this new device, two sands and three clays were tested in the approximate range $\gamma_c = 0.001$–$0.04\%$ with the following shapes of cyclic straining: triangular, sinusoidal, trapezoidal and almost perfectly square. During the same investigation the effects of PI, silt content, $\sigma_{vc}$, OCR and $f$ on $\lambda$ were determined and are described by the writers elsewhere (Vucetic et al., 1998; Lanzo, 1995).

**SOILS TESTED, TESTING APPARATUS AND TESTING PROCEDURE**

Five different soils tested are listed in Table 1 along with their basic physical properties and classification characteristics. They are two reconstituted sands with different silt contents, Santa Monica (SM) sand and Antelope Valley (AV) sand, and three laboratory-made clays having different plasticity indices, labeled A, B and C. The grain size distribution curves for the two sands are shown in Fig. 3.

The new testing apparatus is called the double speci-

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*Fig. 2. Influence of PI on $\lambda$ versus $\gamma_c$ curves (Vucetic and Dobry 1991), with approximate range of $\gamma_{th}$ (Vucetic 1994)*

<table>
<thead>
<tr>
<th>Soil</th>
<th>Initial water content $w_i$ (%)</th>
<th>Initial void ratio $e_0$</th>
<th>Liquid limit $w_L$</th>
<th>Plastic limit $w_p$</th>
<th>Plasticity index PI</th>
<th>Initial liquidity index LI</th>
<th>Silt content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Santa Monica Sand (SM sand)</td>
<td>—</td>
<td>0.549</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0</td>
</tr>
<tr>
<td>Antelope Valley Sand (AV sand)</td>
<td>—</td>
<td>0.886</td>
<td>—</td>
<td>—</td>
<td>0</td>
<td>—</td>
<td>13</td>
</tr>
<tr>
<td>Clay A</td>
<td>48.0</td>
<td>1.382</td>
<td>52</td>
<td>30</td>
<td>22</td>
<td>0.82</td>
<td>—</td>
</tr>
<tr>
<td>Clay B</td>
<td>58.4</td>
<td>1.570</td>
<td>66</td>
<td>28</td>
<td>38</td>
<td>0.80</td>
<td>—</td>
</tr>
<tr>
<td>Clay C</td>
<td>63.0</td>
<td>1.729</td>
<td>125</td>
<td>50</td>
<td>75</td>
<td>0.17</td>
<td>—</td>
</tr>
</tbody>
</table>
placement of the middle cap between the specimens was applied manually via a system of micrometer, spring, horizontal piston and load cell. Such a manually driven system was used because any type of hydraulic, pneumatic or electrical motor would introduce into the DSDSS device undesirable vibrations. As a result, very small strains and stresses could be applied and measured in a controlled manner.

All five soils were tested under the constant volume conditions, the sands in dry state (oven dry) and the clays fully saturated. The specimens were cylindrical, 6.6 cm in diameter and 2 cm high. They were all prepared and tested following the original Norwegian Geotechnical Institute (NGI) constant-volume equivalent-undrained direct simple shear procedure (Bjerrum and Landva, 1966). To maintain the constant volume, the specimens were confined during cyclic straining in the wire-reinforced rubber membranes which restrict or almost completely prevent radial strains, while at the same time the specimen's height was maintained constant. Considering that the constant volume conditions in the NGI simple shear device can simulate undrained conditions (Dyvik et al., 1987), the tests results presented in this paper are applicable to the undrained conditions.

**TESTING PROGRAM**

The testing program of the entire investigation is summarized in Table 2. For each soil a pair of specimens was subjected to several steps of vertical loading and unloading, except for Clay C which was not unloaded. For Clays A and B, these loading and unloading consolidation steps are marked in Fig. 5 on their $e$-$\log\sigma'$ curves. At each step, the specimens were subjected to the vertical stress, $\sigma'_{ry}$, and overconsolidation ratio, OCR, listed in Table 2 until the completion of primary consolidation. The only exception was Clay C, for which the time of consolidation was not sufficient for the completion of primary consolidation. Consequently, the values of OCR could not be determined for Clay C and are not reported in Table 2. After being consolidated, the specimens were subjected under the same $\sigma'_{ry}$ and OCR to several consecutive cyclic strain-controlled tests with different levels of constant $\gamma'$, different shapes of cyclic straining and somewhat different frequencies.

The cyclic strain amplitude, $\gamma'$, was varied from test to test between approximately 0.001% and 0.04%, which is either below or just slightly above $\gamma'$ (see Fig. 2), and no more than 10 cycles of $\gamma'$ were applied in each cyclic test. Consequently, the tests were essentially nondestructive, and it was found that $\lambda$ in a given test was practically independent of the cycle at which it was measured. The frequency, $f$, was between 0.01 and 0.1 Hz, but its effect was found negligible for this range.

The effect of the shape of cyclic strain-time history was examined and reported below only for the levels of $\sigma'_{ry}$ and OCR that are marked in Table 2 by asterisks. Therefore, this investigation covers the range of $\sigma'_{ry}$ between 30 and 80 kPa and OCR between 1 and 9.5.
### Table 2. Summary of the testing program

<table>
<thead>
<tr>
<th>Soil</th>
<th>Vertical load sequence</th>
<th>Effective vertical consolidation stress $\sigma'_c$ (kPa)</th>
<th>Void ratio $e$</th>
<th>Liquidity index LI</th>
<th>Overconsolidation ratio OCR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Santa Monica Sand (SM sand)</td>
<td>Loading</td>
<td>30</td>
<td>0.548</td>
<td>—</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>80</td>
<td>0.546</td>
<td>—</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150</td>
<td>0.544</td>
<td>—</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Unloading</td>
<td>80</td>
<td>0.544</td>
<td>—</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30*</td>
<td>0.545</td>
<td>—</td>
<td>$5^*$</td>
</tr>
<tr>
<td>Antelope Valley Sand (AV sand)</td>
<td>Loading</td>
<td>30</td>
<td>0.885</td>
<td>—</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>80*</td>
<td>0.875</td>
<td>—</td>
<td>$1^*$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150</td>
<td>0.861</td>
<td>—</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Unloading</td>
<td>150</td>
<td>—</td>
<td>—</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>80</td>
<td>—</td>
<td>—</td>
<td>1.9</td>
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<td></td>
<td></td>
<td>30</td>
<td>—</td>
<td>—</td>
<td>5</td>
</tr>
<tr>
<td>Clay A</td>
<td>Loading</td>
<td>$30^*$</td>
<td>1.361</td>
<td>0.81</td>
<td>$3.3^*$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$80^*$</td>
<td>1.326</td>
<td>0.78</td>
<td>$1.25^*$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150</td>
<td>1.249</td>
<td>0.70</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>264</td>
<td>1.166</td>
<td>0.59</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Unloading</td>
<td>150</td>
<td>1.166</td>
<td>0.59</td>
<td>1.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>80</td>
<td>1.169</td>
<td>0.60</td>
<td>3.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30*</td>
<td>1.173</td>
<td>0.61</td>
<td>$9.46^*$</td>
</tr>
<tr>
<td>Clay B</td>
<td>Loading</td>
<td>$30^*$</td>
<td>1.561</td>
<td>0.78</td>
<td>$4.33^*$</td>
</tr>
<tr>
<td></td>
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<td>$80^*$</td>
<td>1.545</td>
<td>0.77</td>
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<td>150</td>
<td>1.509</td>
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<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>264</td>
<td>1.404</td>
<td>0.63</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Unloading</td>
<td>150</td>
<td>1.454</td>
<td>0.68</td>
<td>1.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>80</td>
<td>1.469</td>
<td>0.69</td>
<td>3.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30*</td>
<td>1.477</td>
<td>0.70</td>
<td>$9.46^*$</td>
</tr>
<tr>
<td>Clay C</td>
<td>Loading</td>
<td>$30^*$</td>
<td>1.711</td>
<td>0.16</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td></td>
<td>80</td>
<td>1.660</td>
<td>0.14</td>
<td>—</td>
</tr>
</tbody>
</table>

![Figure 5. Consolidation steps for Clays A and B](image)

\[
\theta = \frac{1}{T\gamma_s} \int |\gamma| \, dt, \tag{2}
\]

where the integration is performed over one cycle of straining. The meaning of $\theta$ is illustrated in Fig. 6(a), where the integral in Eq. (2) is the sum of the two shaded areas, $t=t_1$ and $T$=period of cyclic straining. Figures 6(b) to 6(f) show the values of $\theta$ for the following five simple shapes of cyclic straining: impulse, triangular, sinusoidal, trapezoidal and square. During the impulse load (Fig. 6(b)), the soil is subjected to the cyclic strain for a very short time, and hence $\theta \approx 0$. For the perfectly square shape (Fig. 6(f)), the soil is subjected to the cyclic strain amplitude, $\gamma_s$, for the entire period $T$, and hence $\theta = 1$. The cases of triangular ($\phi = 0.5$), sinusoidal ($\phi = 0.64$) and trapezoidal ($\phi = 0.75$) shapes fall in between these two extremes. It can be seen that $\theta$ is defined for given $\gamma_s$ and $T$ in such a way that it increases if the soil is subjected to a single cycle to higher strain for a longer period of time. The reason for such a definition, and why $\lambda$ should increase as $\theta$ increases, is explained below.

Soils are viscous materials and are therefore significantly affected by creep and relaxation, which can cause considerable changes in the shape of the stress-strain curve (Wood, 1982). How much and why the shape of a loop and its area $\Delta W$ can change due to the effect of relaxa-
tion, if the viscous soil is subjected to larger strains for a longer period of time, has already been shown and explained in Fig. 1 for the triangular versus trapezoidal shape of cyclic straining.

Furthermore, it has been explained earlier (Dobry and Vucetic, 1987; Vucetic, 1990) that in the case of the cyclic straining of viscous clay with the sinusoidal shape, the effects of the relaxation and creep are responsible for the rounded shape of the tips of the cyclic loops, as compared to the pointed tips obtained for the triangular shape of cyclic straining. In the case of the sinusoidal shape of cyclic straining, the tips of the loops are rounded because around them the magnitude of strain, \( \gamma = \gamma_c \sin(2\pi t/T) \), varies around its maximum \( \gamma_c \) while the strain rate \( \dot{\gamma} = dy/\dot{t} \) varies around zero. Such exposure to relatively large strains and corresponding stresses while \( \gamma = 0 \) enables significant relaxation and creep of the soil. On the other hand, in the case of the triangular shape of cyclic straining, the absolute magnitude of the strain rate, \( |\dot{\gamma}| \), is constant throughout a cycle, while at the tips of the loops \( \dot{\gamma} \) just changes the sign. Consequently, during the triangular cyclic straining there is no time for soil to relax or creep at the tips of the loops, and thus the tips are pointed.

Considering that the rounded tips make the area of the loop, \( \Delta W \), larger than the pointed tips, the value of \( \lambda \) obtained from sinusoidal shape of cyclic straining should be larger than from the triangular shape. Similarly, if the shape of the cyclic straining is trapezoidal, even more time will be available for the relaxation and creep around the tips of the loops, resulting in even larger \( \Delta W \) and

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Fig. 6. Shapes of cyclic strain-time history

![Fig. 6. Shapes of cyclic strain-time history](image)

Fig. 7. Shear strain-time history and associated cyclic stress-strain loops for Clay A for: (a) the shape of cyclic straining between triangular and sinusoidal; (b) the shape of cyclic straining similar to trapezoidal

![Fig. 7. Shear strain-time history and associated cyclic stress-strain loops for Clay A](image)
thus $\lambda$. Figure 7 confirms that for Clay A. For the shape between triangular and sinusoidal having $\theta=0.61$, $\lambda=4.36\%$, while for the shape similar to trapezoidal having $\theta=0.80$, $\lambda$ practically doubles to $\lambda=8.29\%$. Furthermore, Fig. 8 shows that for clean SM sand, which has a very small viscosity and is therefore not susceptible to considerable creep and relaxation, $\lambda$ is not affected by $\theta$.

It must be noted that the shapes of cyclic straining in the tests displayed in Figs. 7 and 8 and the rest of the investigation were not perfectly triangular, sinusoidal, trapezoidal or square, because in the DSDSS device the cyclic shear strains were applied and controlled manually. This shortcoming had consequences that have to be clarified. For example, in the test displayed in Fig. 7(b), the shear strain between points 1 and 4, and points 1' and 4', was not constant but slowly changed because the applied strain-time history was not perfectly trapezoidal. Consequently, both the relaxation and creep took place between these points. Had the strain-time history been perfectly trapezoidal, only a pure relaxation of the type sketched in Fig. 1(b) would have taken place.

To clarify further the behavior around the tips of the loops, the strain-time history and stress-strain curve between points 1 and 4 in Fig. 7(b) are presented in a much larger scale in Fig. 9. It can be seen in Fig. 9(a) that before point 1 the rate of shear straining was high, and consequently the stresses were increasing at a high rate too. Between points 1 and 2, the shear strains were still increas-

![Fig. 8. Shear strain-time history and associated cyclic stress-strain loops for Santa Monica sand for: (a) the shape of cyclic straining between triangular and sinusoidal; (b) the trapezoidal shape of cyclic straining](image-url)
This decrease of stresses between points 2 and 3 was due to the relaxation of stresses, which dominated the stress-strain behavior during this short period. Beyond point 3, the strains started to decrease at a relatively high rate, which caused a significant decrease of stresses.

At the end, it must be noted that the effect of $\theta$ on $\lambda$ can be rigorously examined only if the period $T$ is the same among the tests compared, such as in the case shown in Fig. 7. It is obvious that if the shape of cyclic straining is perfectly square, $\Delta W$ will increase as $T$ increases due to the increased effect of relaxation, and thus for the same $\theta = 1$ different values of $\lambda$ will be obtained. The same is true for the trapezoidal shape and may also be true for other shapes of cyclic straining. However, in this study the effect of $T$ alone was found negligible, because a very clear trend of increasing $\lambda$ with $\theta$ was consistently obtained for $T$ varying between 10 and 100 seconds.

TEST RESULTS

In this study, the shape of cyclic straining varied from almost purely triangular, corresponding to $\theta = 0.53$, to almost perfectly square, corresponding to $\theta = 0.89$. The results showing the variation of $\lambda$ with $\theta$ are presented in Figs. 10 through 13.

It can be seen in Fig. 10(a) that in the clean SM sand the variation of $\theta$ from 0.59 to 0.89 did not cause any consistent change of $\lambda$, as already indicated in Fig. 8. However, as shown in Fig. 10(b), in the AV sand containing 13% silt, $\theta$ has a consistent and considerable effect on $\lambda$, i.e., $\lambda$ increases as $\theta$ increases. This effect is caused by a more viscous character of the silt in silty sand, as compared to a clean sand of very low viscosity. Figures 11, 12 and 13 show further that for Clays A, B and C, respectively, which are significantly more viscous than the AV silty sand, $\lambda$ is even more affected by $\theta$. In all three clays, $\lambda$ increased approximately by a factor of two when $\theta$ increased from approximately 0.60 to 0.80.

No consistent trend of $\lambda$ with OCR was obtained for an approximately constant change of $\theta$. To examine the relation between $\lambda$, $\theta$ and OCR more tests are needed.

![Diagram](image1)

![Diagram](image2)

Fig. 11. Effect of the shape of cyclic straining on $\lambda$ for Clay A

![Diagram](image3)

![Diagram](image4)

Fig. 12. Effect of the shape of cyclic straining on $\lambda$ for Clay B

Fig. 10. Effect of the shape of cyclic straining on $\lambda$ for: (a) Santa Monica clean sand; (b) Antelope Valley silty sand
Accordingly, Figs. 10 to 13 generally confirm, as expected, that the effect of $\theta$ is more significant for clays than sands. This is because clays are more viscous than sands and the effects of the creep and relaxation are generally increasing as the viscosity of the soil increases. Based on this observation, it can be expected that the effect of $\theta$ in clays should become greater as the soil indices PI and LI increase, because they are generally associated with higher viscosity (Mitchell, 1993). However, such trends cannot be clearly observed in Figs. 11 to 13. Evidently, many more different clays must be tested in order to examine whether there is any correlation between $\lambda$, $\theta$, and PI and LI.

**DISCUSSION AND CONCLUSIONS**

A series of 69 cyclic strain-controlled tests on two reconstituted sands and three laboratory-made clays was conducted in a newly designed constant-volume equivalent-undrained direct simple shear device to measure the effect of the shape of cyclic strain-time history on the equivalent viscous damping ratio, $\lambda$, at small cyclic shear strain amplitudes, $\gamma_s$. The amplitude $\gamma_s$ varied in the approximate range of 0.001% to 0.04%. One sand was clean, while the other had a silt content of 13%. The clays had different plasticity indices, PI = 22, 38 and 75. The tests were conducted at two vertical effective consolidation stresses, $\sigma'_{oc}$ = 30 kPa and 80 kPa, different over-consolidation ratios, OCR, between 1 and 9.5, and somewhat different frequencies, $f$, between 0.01 Hz and 0.1 Hz.

The shapes of the cyclic strain-time histories applied were approximately triangular, sinusoidal, trapezoidal and square. To investigate systematically the effect of these shapes, a new parameter called the strain-time history shape parameter, $\theta$, is introduced. For perfectly triangular shape $\theta=0.5$, for sinusoid shape $\theta=0.64$, for trapezoidal shape $\theta=0.75$, and for a perfectly square shape $\theta=1.0$. In the tests conducted, $\theta$ varied from approximately 0.55 (close to triangular) to approximately 0.90 (close to square).

For the applied ranges of $\theta$, $\gamma_s$, $\sigma'_{oc}$, OCR and $f$, the test results show that $\lambda$ increased consistently and significantly with $\theta$ in all soils, except in the clean sand. In many cases, when the shape was changed from the triangular to trapezoidal, and further to approximately square, $\lambda$ increased by a factor of two or even more. Evidently, the shape of cyclic straining has a very strong effect on $\lambda$ at small strains. This effect is the consequence of the viscous nature of soil, which is manifested through the effects that creep and relaxation can have on the area of the cyclic loop, $\Delta W$. Accordingly, the effect is significant for viscous clays but small or negligible for sands. At large cyclic strains, however, this effect does not seem to be important, because the soil nonlinearity dominates the value of $\Delta W$ and can thus overshadow the effect of the shape of cyclic straining.

In conclusion, when the equivalent viscous damping ratio, $\lambda$, is evaluated for practical purposes, the shape of the cyclic strain-time or stress-time history should be well defined and clearly specified. This is particularly important for the evaluation of $\lambda$ in the range of small cyclic shear strains.

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