BEARING CAPACITY PREDICTIONS OF SAND OVERLYING CLAY 
BASED ON LIMIT EQUILIBRIUM METHODS

MITSU OKAMURA, JIRO TAKEMURA and TSUTOMU KIMURA

ABSTRACT

A dense sand layer is usually assumed to be a bearing stratum in foundation design. It is, however, difficult for engineers to judge if a sand layer with limited thickness can be used as a bearing stratum when it is underlain by a thick soft clay deposit. Many important parameters must be considered in this problem, for example, the thickness of the sand, strength of clay and width, shape and embedment of footing. Several methods have been proposed to calculate the bearing capacities of the sand overlying clay, in which limit equilibrium of forces acting on an imaginary sand block between the base of the footing and the sand/clay interface was considered. The bearing capacities calculated based on existing methods are not the same for different assumptions adopted in each method. The shape of the sand block and the forces acting on the surface of the block are the determining factors in the bearing capacity calculation. In this paper, these factors as well as calculated bearing capacities are compared with the results of well-conditioned centrifuge tests (Okamura et al., 1997) to verify the validity of the methods. It has been confirmed that reasonable assumptions in which the variation of the shape of the sand block and the forces related to the parameters are taken into consideration are important to obtain a reasonable prediction. An alternative method is proposed and the bearing capacity charts are presented for easy reference.

Key words: bearing capacity, failure mechanism, limit equilibrium method, punching failure, sand overlying clay, shallow foundation (IGC: E3)

INTRODUCTION

Shallow foundations resting on a relatively thin sand layer overlying a deep clay bed sometimes break through the sand layer into the clay as shown in Fig. 1 (Vesic, 1975; Brown and Paterson, 1964; Craig and Chua, 1990). In order to estimate the bearing capacity of the two-layered subsoil profile with potential for punching shear failure, several limit equilibrium methods have been proposed (Yamaguchi, 1963; Meyerhof, 1974; Hanna and Meyerhof, 1980; Kraft and Helfrich, 1982; Baglioni et al., 1982; Myslivec and Kysela, 1978). Failure mechanisms on the basis of observations of small scale model tests in a gravitational field were determined. In these methods, an imaginary sand block like a column or truncated cone, resting on the interface between the upper sand layer and the lower clay, was assumed to be pushed down with the foundation together with stresses acting on the block as shown in Fig. 2. The bearing capacity is obtained from the equilibrium condition of the sand block. Two determining factors in the assumptions used in the limit equilibrium method, that is, angles of side surface of the sand block to the vertical, $\alpha$ (Fig. 2(a)), and stresses on the sand block differ for the different methods, which results in large differences in the calculated bearing capacities. Therefore, experimental verification of the assumptions is necessary in order to confirm the validity of the methods.

Craig and Chua (1990) conducted model tests both in 100 g centrifugal acceleration field and gravitational (i.e. 1 g) field using almost the same models and experimental devices. They reported that the failure mechanism and the bearing capacities observed in centrifuge tests were considerably different from those in 1 g tests. This difference was considered to arise from the difference of stress levels between the centrifuge and 1 g models. Since centrifuge testing provides nearly identical stress levels in a model to that in the prototype, results of centrifuge tests are more realistic than those of small scaled 1 g tests. In the absence of adequate observations of real founda-

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model tests on the bearing capacity of shallow footings resting on a sand layer with underlying clay. Attempts were also made to measure the stresses acting on the sand block (Okamura et al., 1997). In this paper a direct comparison is made between the assumptions in the existing limit equilibrium methods and the results of the centrifuge tests in order to examine the validity of the existing methods. An alternative method is proposed on the basis of the experimental observations and bearing capacity charts are given for easy reference.

EXISTING METHODS

The existing bearing capacity analyses of the sand overlying clay using a limit equilibrium method can be classified into two groups according to the shape of the sand block and the shearing resistance along the side of the block. The first group is the so-called projected area method which was first proposed by Yamaguchi (1963) and the second is the method by Meyerhof (1974) and Hanna and Meyerhof (1980). In this section of the paper these methods are reviewed.

Methods of Analysis

In the projected area method, it is assumed that vertical stresses propagate in sand from foundation to the clay surface at a constant angle α to the vertical as shown in Fig. 2(a). A rigid block of truncated cone or trapezoidal shape under the foundation was assumed. The bearing capacity of footing $q_f$ was determined from the ultimate bearing stress of the underlying clay and the base area of the block, as follows;

for strip footing;

$$q_f = \left(1 + \frac{H}{B} \tan \alpha\right)\left(c_u N_c + p_0\right)$$  \hspace{1cm} (1)

for circular footing;

$$K_s:$$ punching shear coefficient

Fig. 2. Mechanisms adopted for existing methods of analysis: (a) projected area method; (b) Hanna and Meyerhof (1980)
where $H$ is the thickness of the sand below the footing, $B$ is width or diameter of footing, $p_0$ is effective overburden pressure at the level of the footing base without considering vertical settlement of the footing due to loading, $c_0$ is undrained strength of underlying clay, $N_c$ and $s_c$ are bearing capacity factor and shape factor ($=1.2$ for circular footing). In this method, the shearing resistance of sand along the side of the sand block is neglected with an exception (Yamaguchi, 1963). The side angles of the block $\alpha$ proposed by various researchers are different from each other; for example, $30^0$ for Yamaguchi (1963), $\tan^{-1}(1/2)$ for Kraft and Helfrich (1983), $30^0$ and $45^0$ for Myslivec and Kysela (1982) and $\phi'$ for Bagloni et al. (1982). These angles $\alpha$ are assumed to be constant irrespective of the strengths of the soils and geometric conditions, except for that proposed by Bagloni et al. (1982).

Meyerhof (1974), Hanna and Meyerhof (1980) and Hanna (1981) proposed a method with a failure mechanism including a vertical side block (i.e. $\alpha = 0$), as shown in Fig. 2(b). The shearing resistance along the side surface of the block is taken into account in this method. They also made use of a concept of equilibrium of forces on the block and gave following formulae, for strip footing;

$$q_f = c_0 N_c + \frac{1}{B} (\gamma' H^2 + 2 H p_0) K_s \tan \phi' + p_0$$

(3)

for circular footing;

$$q_f = c_0 N_c + s_c + \frac{1}{B} (\gamma' H^2 + 2 H p_0) K_s \tan \phi' + p_0$$

(4)

where $\gamma'$, $\phi'$ and $K_s$ are the effective unit weight and angle of shearing resistance of sand, and punching shear coefficient respectively. In this method, $K_s$ is given in charts as a function of $\phi'$ and the ratio of the bearing capacity of uniform sand to that of uniform clay, $q_{f} (=0.5yBN_c s_c)$ to $q_{f} (=c_0 N_c s_c)$, where $s_c$ denotes the shape factor of footing (Vesic, 1973).

In all the other methods, vertical stresses on the base of the sand block are assumed to be the ultimate bearing stresses of the clay. Centrifuge test results showed that this assumption is adequate or somewhat conservative (Okamura et al., 1997). In the following discussions, influences of the two determining factors, that is, the angle $\alpha$ and the shearing resistance along the side surface of the sand block, on the bearing capacity of footing are focussed on. These assumptions are compared with the

Table 1. Summary of the centrifuge test for circular footing on sand layer without embedment

<table>
<thead>
<tr>
<th>Width of footing $B$ (m)</th>
<th>Depth of sand $H$ (m)</th>
<th>$H/B$</th>
<th>$\gamma'B$ (kPa)</th>
<th>Strength at clay surface $c_0$ (kPa)</th>
<th>$N_c$</th>
<th>$\lambda_c$</th>
<th>$\frac{2q_f}{\gamma'B}$</th>
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<td>85.9</td>
<td></td>
<td></td>
<td>6.1</td>
<td>general shear</td>
</tr>
</tbody>
</table>

* $B$ and $H$ are shown in prototype scale. ** Centrifugal acceleration = 50 g.
centrifuge test results. Conditions and results of the centrifuge tests are summarized in Tables 1 to 4. The detailed information about the tests was reported elsewhere (Okamura et al., 1997).

**Comparison with Centrifuge Test Results**

In the calculations of the bearing capacities of the centrifuge models using Hanna and Meyerhof’s method, the angle of 45° was used as the angle of shearing resistance \( \phi' \) for the sand. In Hanna and Meyerhof’s method, \( \phi' \) corresponds to the angle of shearing resistance of sand in which general shear failure occurs, and \( \phi' \) is distinguished from the angle of shearing resistance along the side surface of the sand block. Therefore, the angle \( \phi' \) was computed from the observed bearing capacity factors \( N_r \) ranging between 179 and 252 for strip footings on uniform sand \( (H/B=\infty) \) and equations (5) and (6) proposed by Caquot and Kerisel (1953).

\[
N_r = 2(N_r' + 1) \tan \phi' \tag{5}
\]

\[
N_q = e^{\pi \tan \phi'} \tan \left( \frac{\pi}{4} + \frac{\phi'}{2} \right) \tag{6}
\]

It should be noted that the angle \( \phi' \) of 45° obtained here is considered to be the average of the mobilized angle of shear resistance in the failure zone and different from

<table>
<thead>
<tr>
<th>Table 2. Summary of the centrifuge tests for strip footing on sand layer without embedment</th>
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<tbody>
<tr>
<td>Width of footing ( B ) (m)</td>
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<tr>
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* \( B \) and \( H \) are shown in prototype scale. ** Centrifugal acceleration = 50 g.

<table>
<thead>
<tr>
<th>Table 3. Summary of the centrifuge tests for circular footing on sand layer with embedment</th>
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<td>6.48</td>
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<td>1.6</td>
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<td>3.36</td>
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<tr>
<td>4.4</td>
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<td>2.56</td>
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<td>4.8</td>
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* \( B \) and \( H \) are shown in prototype scale. ** Centrifugal acceleration = 40 g.

<table>
<thead>
<tr>
<th>Table 4. Summary of the centrifuge tests for strip footing on sand (a) Circular footing</th>
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<tbody>
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<td>0.8</td>
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<td>2.4</td>
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<td>2.4</td>
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(b) Strip footing

<table>
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<tr>
<th>( B ) (m)</th>
<th>Centrifugal acceleration ( n ) (g)</th>
<th>( y'B ) (kPa)</th>
<th>( \lambda_r )</th>
<th>( 2q_r/y'B )</th>
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<td>1.0</td>
<td>50</td>
<td>9.74</td>
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<td>252</td>
</tr>
<tr>
<td>2.0</td>
<td>50</td>
<td>19.5</td>
<td>0</td>
<td>179</td>
</tr>
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</table>

* \( B \) is shown in prototype scale.
Fig. 3. Variation of $\alpha_m$ with $\lambda_c$ and $H/B$: (a) footing without embedment; (b) circular footing

that mobilized along the side surface of the sand block. Normalized bearing capacities, $2q_c/\gamma' B(=s_p N'_c)$, for circular footings on uniform sand ranged between 127 and 160 as shown in Table 4. The depth of sand $H$ and footing width $B$ shown in Tables 1 to 4 are in the prototype scale. Although the range of prototype footing width tested was somewhat different between strip and circular footings, the ratios of the mean values of $N'$ for circular footings to those for strip footing, 0.7, was used as the shape factor for circular footings $s_c$.

The shape factor $s_c$ for circular footings and $N'_c$ used in the calculation were 1.2 and 5.1, respectively. For clay of which strength increases with depth, the use of the undrained strength at the clay surface $c_0$ in combination with $N'_c=5.1$, which corresponds to the factor for clay with a uniform strength profile, must yield a conservative bearing capacity prediction. Hence, the representative undrained strength of the underlying clay $c_u$ given by equations (7) and (8) was utilized in this study, namely:

for strip footing: $c_u = \frac{N'_c}{5.1} c_0$  \hspace{1cm} (7)

for circular footing: $c_u = \frac{N'_c}{5.1 \times 2} c_0$  \hspace{1cm} (8)

where, $N'_c$ is a bearing capacity factor for clay with strength increasing linearly with depth calculated from slip line method. This $N'_c$ value is determined from the non-dimensional parameter $kB/c_0$, where $k$ is strength increasing ratio with depth (Davis and Booker, 1973; Housby and Wroth, 1983).

Fig. 4. Normalized horizontal earth pressure acting on the side of a and block

Angles $\alpha_m$ obtained from the centrifuge tests are plotted against normalized bearing capacity of underlying clay $c_u N'_c/\gamma' B(=\lambda_c)$ and $H/B$ in Fig. 3 (Okamura et al., 1997). The angle $\alpha_m$ was determined based on observations of both the vertical stress distribution on the surface of underlying clay and the deformation of the subsoil. The $\alpha_m$ angles increase with relative thickness $H/B$ and normalized overburden pressure $p_b'/\gamma' B(=\lambda_p)$, and decrease with increasing $\lambda_c$. The angles are not constant but vary from 3 to 19° in the tests, which is inconsistent
with the assumption in the existing methods. Relationships between normalized horizontal stresses $p'_h$ / $(y'z + p'_s)$ observed in the sand with small pressure cells at locations close to the side of sand block at the peak load and $\lambda_c$ for the cases with circular footing are shown in Fig. 4, where $p'_h$ and $z$ denote effective horizontal stress and depth of pressure cell from the level of the footing base respectively. The punching shear coefficient $K_s$, proposed by Hanna and Meyerhof (1980), which is identical to the normalized horizontal stress, is also shown in this figure. It can be seen from the figure that the normalized horizontal stresses observed increase with increasing

Fig. 5. Relationship between $2q_f / \gamma' B$ and $\lambda_c$, calculated using existing method
It can be considered that the higher strength of clay, the larger the lateral squeeze of sand which takes place beneath the footing. This may cause the higher normalized horizontal stress for the higher $\lambda_c$. It can also be seen that the normalized horizontal stress for the footing with embedment ($\lambda_p=0$) is smaller than that without embedment ($\lambda_p=0$). In the deeper footing for which the strength of sand is higher than that for the shallower footing, punching shear failure is more likely to occur and the lateral squeeze of the sand beneath the footing becomes less marked. In Hanna and Meyerhof's method, the coefficient $K_r$ is given independent of $\lambda_p$ or the depth of footings. For the footing without embedment, $K_r$ agrees well with the normalized horizontal stresses observed in the tests, while for the footing with embedment $K_r$ is greater than the observation.

Normalized bearing capacity $2q_f/\gamma'B$ of footings without embedment calculated from projected area method and Hanna and Meyerhof's method are plotted against $\lambda_p$ in Fig. 5 together with the results of the centrifuge tests. The calculated bearing capacities are also compared with the test results in Fig. 6. In the projected area method the angle $\alpha$ of $30^\circ$ was used, which is one of the smallest values from the existing methods mentioned above. The projected area method overestimates the measurements for the circular footings with $\lambda_p$ greater than 5 (Figs. 5(a), 6(a)) but underestimates for strip footings in all cases (Fig. 5(c)). Since, in this method, the magnitude of $\alpha$ used in the computation is considerably larger than actual as shown in Fig. 3(a), overestimation is inevitable for cases with higher $\lambda_p$, in which the reaction at the clay surface plays a dominant role in equilibrium of the forces. Fig. 7 shows the relationships between the ratios of the base area of sand block to that of footing and the angle $\alpha$. The ratio for circular footing increases more rapidly than that for strip footing with increasing $\alpha$. This implies that even a small increase in $\alpha$ causes a large increase in the base area of sand blocks under circular footings. This must also be the cause of the fact that overestimation becomes greater for circular footings than for strip footings.

In the cases of $\lambda_p=0$, Hanna and Meyerhof's method gives the higher normalized bearing capacity for the narrower footing with same $H/B$ and $\lambda_c$, while little difference in bearing capacities can be found in the tests with different footing width (Fig. 5(b)(d)). As shown in Fig. 6(b), however, the bearing capacities calculated from Hanna and Meyerhof's method compared well with the observation or are somewhat conservative. The extent of underestimation is greater for the smaller $\lambda_c$ and the higher $H/B$ (Figs. 5(b)(d), 6(b)). Since the punching shear coefficient $K_r$ agrees well with the observations for circular footings without embedment, the differences between calculated and observed bearing capacities can be considered to be caused by the assumption of $\alpha=0$.

From the results of centrifuge tests, it was confirmed that the superposition given by Eq. (9) can be applied for evaluating the bearing capacity of footings with embedment (Okamura et al., 1997),

$$q_f(\text{embedment}) = q_f(\text{surface}) + p_s \left( \frac{\Delta q_f}{\Delta p_s} \right)$$  \hspace{1cm} (9)

where $q_f(\text{embedment})$ and $q_f(\text{surface})$ are bearing capacities of the footing with and without embedment respectively and $\Delta q_f$ is bearing capacity increment due to increment of overburden pressure $\Delta p_s$. Ratios of $\Delta q_f/\Delta p_s$ obtained from the tests with circular footings and calculation with the two methods are plotted against $H/B$ in Fig. 8. The numbers in parentheses in the figure refer to $\lambda_c$. The ratios become equal to the bearing capacity factor $N_q$ when the subsoil consists of a single uniform material. The results of centrifuge tests indicated that this ratio ranges from 1 for uniform clay ($H/B=0$) to some 110 for uniform sand ($H/B=\infty$). It should be noted that the ratios observed seem to be independent of $\lambda_c$. Since Hanna and Meyerhof's method gives the higher $\Delta q_f/\Delta p_s$ for the higher $\lambda_c$, however, the overestimation of the effect of overburden pressure on the bearing capacity becomes
marked as $\lambda_c$ increases. The projected area method underestimates this effect. Judging from these results as well as the observed horizontal stresses shown in Fig. 4, it may be said that the shearing resistance along the side surface of the sand block should be reasonably taken into account in order to estimate the effect of overburden pressure on bearing capacity.

Normalized bearing capacities of circular footings with embedment calculated from the two methods are compared with the test results in Fig. 9. The Hanna and Meyerhof's method overestimates the observations and the extent of the overestimation increases with both $\lambda_c$ and $\lambda_p$, whereas for the projected area method, the results of calculations compare well with the observations. The underestimation of the effect of overburden pressure counterbalances the overestimation of the bearing capacity of footings without embedment for the projected area method. The discussions above lead to the conclusion that differences between observed and calculated bearing capacities arise from the discrepancy in the shapes of the sand block or forces on the sides of the block between the assumptions in the methods and the actual situation. A more realistic shape (or angle $\alpha$) and evaluation of forces are necessary in the assumptions to obtain reasonable predictions from the limit equilibrium methods.

Fig. 7. Variation of base area ratio with angle of the side of sand block

Fig. 8. Bearing capacity increment related to increment of effective overburden pressure

Fig. 9. Relationship between calculated and observed normalized bearing capacity for footing with embedment
PROPOSED METHOD

In this study, a new limit equilibrium method has been proposed in order to overcome the problems which exist in the assumptions made in existing methods. The failure mechanism adopted in the proposed method, which is similar to those in the existing methods, is illustrated schematically in Fig. 10. Vertical effective stresses acting on the base of the sand block, \( q_{\text{clay}} \), are assumed to be the ultimate bearing stresses of a rigid footing with rough base on clay subject to the effective surcharge pressure (i.e. \( p'_o + \gamma' H \)) in this problem, which is given by

\[
q_{\text{clay}} = p'_o + \gamma' H + c_u N_c s_c \tag{10}
\]

As was discussed earlier, the normalized horizontal earth pressures observed decrease with the embedment depth of the footing. In addition, the shearing resistance along the side of the block becomes dominant in equilibrium of the forces acting on the block as the footing depth increases. In this method, therefore, Rankine's passive coefficient \( K_p \), which corresponds to a lower limit of observations (as seen in Fig. 4), was adopted. The normal stress of \( K_p \) times the vertical stress is assumed to act on the side of the sand block. Consideration of equilibrium of the forces acting on the block, including self weight of the block, yields bearing capacity formulae as follows:

for a strip footing:

\[
q_r = \left(1 + 2 \frac{H}{B} \tan \alpha_c \right) (c_u N_c + p'_o + \gamma' H) + \frac{K_p}{\cos \phi' \cos \alpha_c} \left( \frac{H}{B} (p'_o + \gamma H) - \gamma' H \left(1 + \frac{H}{B} \tan \alpha_c \right) \right) \tag{11}
\]

for a circular footing:

\[
q_r = \left(1 + 2 \frac{H}{B} \tan \alpha_c \right) (c_u N_c s_c + p'_o + \gamma' H) + \frac{4K_p}{\cos \phi' \cos \alpha_c} \left( \frac{p'_o + \gamma' H}{2} \frac{H}{B} \right) \tag{12}
\]

Assuming that typical small elements \( A \) and \( B \) just above and below the sand/clay interface as shown in Fig. 10, are in the limit state of stress, the Mohr's circles of stresses for the two elements can be illustrated as shown in Fig. 11. Normal and shear stresses are normalized by the representative strength of the underlying clay \( c_u \) in the figure. It should be noted that the Mohr's circle for the clay element \( B \) in the figure is represented in terms of total stress excluding static water pressure, whereas for the circle for sand element \( A \) in terms of effective stress.

The direction of the slip surface in sand represented by a straight line \( P-C \) in Fig. 11 is assumed to be that of the side of the sand block. The angle of the side to the vertical \( \alpha_c \) can be obtained from the figure as a function of \( \phi' \) and normalized mean normal stress of the clay element \( B, \sigma_{mc} / c_u \), as follows.

\[
\alpha_c = \tan^{-1} \left( \frac{\sigma_{mc}/c_u - \sigma_{se}/c_u (1 + \sin^2 \phi')}{\cos \phi' \sin \phi' \sigma_{mc}/c_u + 1} \right) \tag{13}
\]

\[
\frac{\sigma_{mc}}{c_u} = N_c s_c \left( 1 - \frac{1}{\lambda_e} \frac{H}{B} + \frac{\lambda_e}{\lambda_c} \right) \tag{14}
\]

\[
\frac{\sigma_{se}}{c_u} = \frac{\sigma_{se}/c_u - \sqrt{\sigma_{se}/c_u^2 - \cos^2 \phi' ((\sigma_{mc}/c_u)^2 + 1)}}{\cos^2 \phi'} \tag{15}
\]

The angle of shearing resistance mobilized on the surface of a sand block should be estimated and adopted as \( \phi' \) in the proposed method. It is widely recognized that the angle \( \phi' \) indicates a stress level dependency (e.g. Bishop, 1966). In addition, strength anisotropy and progressive failure make it difficult to determine the angle \( \phi' \) for the bearing capacity calculation even for uniform sand. Fortunately, however, influences of principal stress rotation in sand and progressive failure of sand overlying clay on the bearing capacity may be less noticeable com-

![Fig. 10. Failure mechanism assumed in proposed method](image)

![Fig. 11. Mohr's circle of stress for soil elements A and B](image)
pared to a uniform sand. In this study, only the influence of stress level on $\phi'$ is taken into account in determining $\phi'$. The middle point of the side of sand block is chosen as a representative point at which Rankine's passive state is assumed, that is, the minor principle stress is $p_1 + \gamma' H/2$ and major principle stress equals $(p_1 + \gamma' H/2)K_s$. The angles $\phi'$ are determined by the use of the relationship between $\phi'$ and the mean principle stress, as shown in Fig. 12, which was obtained from both triaxial test and plane strain tests on sand used in the model tests (Fukushima and Tatsuoka, 1984; Tatsuoka et al., 1986; Fujii, 1976; Okamura et al., 1993). The angles $\phi'$ of 38° and 40° for circular footings with and without embedment respectively are adopted on the basis of triaxial test results and for strip footings $\phi'$ of 47° is adopted according to the results of plane strain tests.

For clay with increasing strength with depth, the bearing capacity factor $N'_f$ proposed by Davis and Booker (1973) and Houlsby and Wroth (1983) may usually be used, which increases with a gradient of the strength increase and with a decrease in $c_0$. The factor $N'_f$ is maximum for a normally consolidated clay and decreases with an increase in both $H/B$ and $\lambda_c$ (Okamura et al., 1997). In the case of $\lambda_c=0$ and $H/B \leq 1.0$, $N'_f$ is less than 6.4 for strip footing and 6.8 for circular footing (Davis and Booker, 1973, Houlsby and Wroth, 1982; Okamura et al., 1997), which is not significantly large compared to that for footings on uniform clay. A constant value $N'_f$ of 5.1 and shape factor $s_c$ of 1.2 for circular footing, irrespective of the gradient of strength increase, therefore can be adopted in this method.

The angle $\alpha_c$ calculated from this proposed method is plotted against the ratio $\sigma_{mc}/c_u$ in Fig. 13, together with that observed in the tests, $\alpha_m$. Fig. 14 shows variations of the angles with $\lambda_c$ and $H/B$ for circular footing without embedment. $\alpha_c$ is somewhat higher than $\alpha_m$, especially in the case with a small $\sigma_{mc}/c_u$ or large $\lambda_c$. The proposed method, however, appears to give a better approximation of the observed angle compared to the existing method.

The calculated angles $\alpha_c$ as well as those observed $\alpha_m$, increase with $\lambda_c$ and $H/B$ and decrease with increasing $\lambda_c$. But $\alpha_c$ is less sensitive to the changes of these parameters than $\alpha_m$.

Figure 15 shows a comparison between calculated and observed normalized bearing capacities for footings without embedment. For circular footings (Fig. 15(a)), the bearing capacity calculated compares well with the observations for smaller $\lambda_c$ irrespective of $H/B$. But the proposed method begins to overestimate the observations to some extent as $\lambda_c$ increases. This is caused by the fact that the gap between the estimated angle $\alpha_c$ and the
observed angle $\alpha_n$ becomes greater for the higher $\lambda_c$ as shown in Fig. 14. Relatively large differences between calculated and observed angles can also be seen for smaller $H/B$ in Fig. 14, but it does not result in an apparent overestimate of the bearing capacity. The reason for this is that the base area of the sand block is not sensitive to a change in $\alpha_n$ for small $H/B$. The proposed method gives somewhat conservative prediction for strip footings (Fig. 15(b)). Figure 16 indicates displacement vectors observed in the tests for footings without embedment (Okamura et al., 1997). Relatively large horizontal displacement of sand just outside the sand block can be seen for the strip footing, whereas there is a small horizontal displacement of sand for a circular footing. Although measurements of horizontal stress of sand were not conducted for the cases of strip footings, however, it can be expected from the displacement vectors that horizontal stresses on the side of the sand block under the strip footing may be larger than that under a circular footing. This may cause the underestimated of bearing capacity for a strip footing.

Relationships between observed normalized bearing capacities and those calculated by the proposed method are shown in Fig. 17. Excellent agreement can be seen for the cases of a circular footing without embedment ($\lambda_p=0$). The proposed method also appears to yield good predictions for circular footings with embedment except for the cases where $\lambda_c \geq 25$ and $\lambda_p \geq 30$. The calculated bearing capacities tend to be larger than the observations for the case with the higher $\lambda_c$ and $\lambda_p$. The larger $\alpha_n$ in the case with the higher $\lambda_c$ as shown in Fig. 14 is considered to be a reason for this. The dashed line in Fig. 8 corresponds to the predicted value of $\Delta q_x/\Delta P_l$ from the method for circular footing. The calculated value of $\Delta q_x/\Delta P_l$ is slightly larger than the observations. This must also be responsible for the overestimation of the bearing capacity for the larger $\lambda_p$. Although the proposed method gives an overestimate for circular footings as $\lambda_c$ and $\lambda_p$ increase, it can, however, predict the bearing capacity for any of the tests presented in this study with a maximum error of 19% for the cases without embedment and 32% for the cases with embedment. This method is believed to be applicable for predicting the bearing capacity of a sand layer overlying clay for $\lambda_c \leq 26$ and $\lambda_p \leq 4.8$. For the cases of strip footing, the proposed method predicts the bearing capacity on the safe side with a maximum error of 31%.

Normalized bearing capacity charts for circular and strip footings are given in Figs. 18 and 19 for easy reference. It should be noted that the bearing capacity for a limited depth of sand overlying a weaker clay layer does not exceed that of a deep sand bed ($H/B = \infty$). Hence, the value obtained from the charts and that from formula for the deep sand bed (e.g. Terzaghi, 1943) must be
Fig. 17. Relationship between calculated and observed normalized bearing capacity

CONCLUSIONS

In this study, the validity of existing methods for estimating the bearing capacity of a sand layer overlying a deep clay deposit were verified by comparing them with the results of a series of centrifuge tests. Not only the bearing capacity but also assumptions adopted in the methods were evaluated. The following conclusions are drawn:

1. The projected area method in which a constant side angle $\alpha$ of the sand block is assumed tends to overestimate the bearing capacity of the footing without embedment for the higher normalized bearing capacity of underlying clay $\lambda_c$, where the reaction at the clay surface plays a dominant role in equilibrium of the forces acting on the sand block. The extent of overestimation is greater for circular footings than for strip footings. Since the projected area method neglects the shearing resistance of sand, an underestimate of the bearing capacity increase due to an increase in the footing embedment depth is inevitable.

2. The method proposed by Meyerhof (1974) and Hanna and Meyerhof (1980), in which the vertical side sand block is utilized and the shearing resistance of sand is taken into account, gives a conservative value for the bearing capacity for a footing without embedment. For a
footing with embedment, however, overestimation becomes noticeable as the depth of the footing increases. The use of a higher horizontal stress acting on the side of the sand block is responsible for this. An assumption of a realistic value of the horizontal stress on the side is necessary in the calculation in order to accurately predict the effects of footing depth on the bearing capacity.

3) A new limit equilibrium method was proposed in which the variation of the side angle of the sand block with the parameters including $H/B$, $\lambda_e$ and $\lambda_p$ is taken into account. Bearing capacities calculated from the proposed method compared well with the observations, irrespective of shape, depth of footing and strength of the underlying clay.

4) Bearing capacity charts are presented based on the proposed method for easy reference.

NOTATIONS

The following symbols are used in this paper:

$B$: width or diameter of footing
$\phi'_c$: undrained strength at surface of underlying clay
$\phi'_r$: representative undrained strength of underlying clay
$H$: depth of sand below footing base
$K_p$: Rankine’s passive earth pressure coefficient
$K_p':$ punching shear coefficient proposed by Hanna and Meyerhof (1980)
$N_{o}$, $N_{o}'$: bearing capacity factors for uniform thick bed
$N_{o}'$: bearing capacity factor for strip or circular footings on clay with strength linearly increasing with depth
$\rho_o$: effective overburden pressure at the level of footing base
$\rho_o'$: horizontal effective stress at peak load observed in sand at locations close to the side of sand block
$\phi$: bearing capacity
$\alpha', \phi'$: shape factors for circular footing
$\alpha_{c}'$: angle of side of side of sand block to the vertical observed in the test
$\alpha_e$: angle of side of sand block calculated from proposed method
$\lambda_e$: normalized bearing capacity of underlying clay ($=c_eN_o/\gamma'B$)
$\lambda_p$: normalized overburden pressure ($=\rho_o/\gamma'B$)
$\phi$: angle of shear resistance of sand
$\gamma'$: effective unit weight of sand
$\sigma_{sc}$: mean normal stress of clay element just beneath the sand block subtracting static water pressure
$\Delta p_o$: overburden pressure increment
$\Delta q$: bearing capacity increment due to the increment of overburden pressure $\Delta p_o$

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