DEGREE OF CONSOLIDATION AND REDUCTION COEFFICIENT CONCERNING DIFFERENTIAL CONSOLIDATION SETTLEMENTS

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ABSTRACT

The author has proposed a reduction coefficient of differential settlements in a soil-structure interaction and has shown that it is very useful for predicting differential settlements. In such a prediction, the soil model has been assumed to be elastic and linear and until now consolidation settlements were evaluated as equivalent elastic settlements. It is very significant, however, to extend the elastic model to a consolidation model, because practical cases are frequently related to consolidation settlements in reclaimed land.

This paper, therefore, attempts to formulate the characteristics of consolidation settlements in soil-structure interaction, in order to incorporate the consolidation settlements into the prediction method of differential settlements suggested by the author.

Terzaghi's one-dimensional consolidation was interpreted mechanically to be expressed as an infinite linear summation of Kelvin type viscoelastic models. The soil consisting of an infinite linear summation of Viscoelastic Pasternak Models was then proposed. Using this model, the degree of differential consolidation settlements was introduced and the reduction coefficient of differential consolidation settlements was formulated. Some characteristics of differential settlements are discussed. The results indicate that differential consolidation settlements occur faster than the average settlements and the final differential consolidation settlements correspond to equivalent elastic differential settlements.

Key words: consolidation, foundation, settlement, soft ground, viscoelasticity (IGC: E2/E12)

INTRODUCTION

The author has proposed a reduction coefficient of differential settlements in order to formulate the reduction effects on differential settlements through a soil-structure interaction of a Timoshenko Beam on a Pasternak Model (Tanahashi, 1994a). It has also been shown that the reduction coefficient of differential settlements is very useful for predicting differential settlements reduced by the rigidity of soils and structures. In such a prediction, the soil model has been assumed to be elastic and linear and until now consolidation settlements were evaluated as equivalent elastic settlements. It is very significant, however, to extend the elastic model to a consolidation model, because practical cases are frequently related to consolidation settlements in reclaimed land consisting of a reclaimed sandy layer with a clay layer beneath it.

This paper, therefore, attempts to formulate the characteristics of consolidation settlements in a soil-structure interaction, in order to incorporate the consolidation settlements into the prediction method of differential settlements suggested by the author (Tanahashi, 1994a).

In order to achieve this goal: 1) Terzaghi's one-dimensional consolidation was interpreted mechanically to be expressed as an infinite linear summation of Kelvin type viscoelastic models. 2) A Viscoelastic Pasternak Model, i.e., a shear layer on a Kelvin type viscoelastic model, was introduced and the soil model consisting of an infinite linear summation of Viscoelastic Pasternak Models was proposed.

Using this model, the degree of differential consolidation settlements was introduced and the reduction coefficient of differential consolidation settlements was formulated. Some characteristics of differential settlements are discussed. As a result, it was shown that differential consolidation settlements occur faster than the average settlements and the final differential consolidation settlements correspond to equivalent elastic differential settlements.

VISCOELASTIC PASTERNAK MODEL

A classical Winkler Model is often used in soil-structure interaction analyses, where the shear rigidity of soils

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Fig. 1. Viscoelastic Pasternak Model

is ignored. The shear rigidity of soils, however, has some reduction effects actually on differential settlements due to non-homogeneity of the soil properties and loads. The author et al. (Tanahashi et al., 1992) proposed to use a Pasternak Model as an improved and refined foundation model in order to evaluate the reduction effects of the shear rigidity of soils on differential settlements.

The Pasternak Model is regarded as a shear layer on a Winkler Model. The Winkler Model, in this case, was replaced with a Kelvin (or Voigt) type viscoelastic model. A Viscoelastic Pasternak Model was therefore introduced as shown in Fig. 1. It should be noted that this model has no viscous shear layer and is a little different from the Viscoelastic Pasternak Foundation with a viscous shear layer suggested by Kerr (1961, 1964).

INTERPRETATION OF ONE-DIMENSIONAL CONSOLIDATION

The consolidation settlement is assumed to be on the basis of Terzaghi's one-dimensional consolidation theory, or Mikasa's more general theory with a constant coefficient of consolidation (Mikasa, 1963). Secondary consolidation therefore was ignored in this discussion.

As is well known, the degree of consolidation U for a uniform initial excess pore pressure distribution is expressed as:

$$U = 1 - 2 \sum_{m=0}^{\infty} \frac{\exp(-M^2 T_v)}{M^2} \quad (1)$$

where

$$M = (2m+1)\pi/2: \quad m=0, 1, 2, \dots,$$

$$T_v = c_v t / (H/2)^2: \quad \text{time factor,}$$

$$t: \quad \text{time,}$$

$$c_v: \quad \text{coefficient of consolidation,}$$

$$H: \quad \text{depth of a clay layer under double drainage conditions.}$$

M^2 determines the velocity of consolidation and an eigenvalue corresponding to the eigenfunction of consolidation (Tamura, 1980).

Using the formula,

$$2 \sum_{m=0}^{\infty} \frac{1}{M^2} = 1 \quad (2)$$

Eq. (1) can be rewritten as follows,

$$U = 2 \sum_{m=0}^{\infty} \frac{1 - \exp(-M^2 T_v)}{M^2} \quad (3)$$

Fig. 2. Time-displacement relationship in Kelvin type viscoelastic model

In a Kelvin type viscoelastic model, the relationship between displacement y and time t is expressed based on the condition of time-independent load q as follows (Freudenthal et al., 1957):

$$y = \frac{q}{k_0} \left\{ 1 - \exp\left(-\frac{t}{\tau}\right) \right\} \quad (4)$$

where k_0 is the spring constant or the coefficient of subgrade reaction and τ is the retardation time when the residual settlement reaches $1/e$ of the final settlement $\mu = q/k_0$, as shown in Fig. 2.

By comparing Eq. (3) with Eq. (4), the one-dimensional consolidation can be readily interpreted to be an infinite linear summation of Kelvin type viscoelastic models with a retardation time of $1/M^2$.

LOAD-DISPLACEMENT RELATIONSHIP IN THE VISCOELASTIC PASTERNAK MODEL

In general, the reaction term $k_0 \tau \partial y / \partial t$, which is proportional to the displacement velocity, appears in the governing differential equation of the Kelvin type viscoelastic model. The differential equation of the Viscoelastic Pasternak Model is therefore written, adding the reaction term to that of the Pasternak Model under the plane strain condition (Tanahashi, 1994b) as,

$$G \frac{\partial^2 y}{\partial x^2} - k_0 \left(y + \tau \frac{\partial y}{\partial t} \right) = -q \quad (5)$$

where G is the shear rigidity of the shear layer in the vertical direction.

Using the characteristic value

$$\gamma_s = \sqrt{k_0 / G}$$

Eq. (5) yields,

$$\frac{\partial^2 y}{\partial x^2} - \gamma_s^2 \left(y + \tau \frac{\partial y}{\partial t} \right) = -\frac{\gamma_s^2 q}{k_0} \quad (6)$$

The time-independent load distribution is given by a trigonometric function as:

$$q = q_0 (1 + c \cos \omega x) \quad (7)$$

where

- c : amplitude of the load variation (constant),
- $\omega = 2\pi/L$: wave number,
- L : wavelength.

The solution of Eqs. (6) and (7) was obtained using the same procedure by Freudenthal et al. (1957) for a Bernoulli-Euler Beam on a Kelvin type viscoelastic model as follows.

$$y = \mu \left\{ 1 - \exp \left(-\frac{t}{\tau} \right) + \alpha_s(\omega, t) c \cos \omega x \right\} \quad (8)$$

where

$$\mu = q_0 / k_0: \text{ average final settlement,} \quad (9)$$

$$\alpha_s(\omega, t) = \alpha_s(\omega) \left\{ 1 - \exp \left(-\frac{t}{\alpha_s(\omega)\tau} \right) \right\}$$

$$\alpha_s(\omega) = \frac{1}{1 + \omega^2 / \gamma_s^2}. \quad (10)$$

$\alpha_s(\omega)$ is the reduction coefficient of differential settlements for the Pasternak Model (Tanahashi, 1994a) and $\alpha_s(\omega, t)$ has been defined as the reduction coefficient for the Viscoelastic Pasternak Model (Tanahashi, 1994b).

Since $\alpha_s(\omega, \infty) = \alpha_s(\omega)$, the final settlement y_F is given by,

$$y_F = \mu \{ 1 + \alpha_s(\omega) c \cos \omega x \}. \quad (11)$$

Equation (11) shows that both the final average settlement and the final differential settlement coincide with those of the Pasternak Model respectively. Equation (9), however, shows that the consolidation with respect to the differential settlement proceeds faster than that of the average settlement because $\alpha_s(\omega) < 1$.

DIFFERENTIAL SETTLEMENT COMPONENT IN CONSOLIDATION SETTLEMENTS

The shear layer on top of the consolidation layer is considered based on Terzaghi's one-dimensional consolidation theory. If the consolidation layer is regarded as an infinite linear summation of Kelvin type viscoelastic models as mentioned above, then the shear layer on the consolidation layer is regarded as an infinite linear summation of Viscoelastic Pasternak Models, as presented in the following Eqs. (12) through (16). This procedure is reasonable because eigenfunctions are orthogonal and independent of one another. It is also one type of modal analyses as illustrated conceptually in Fig. 3.

In the author's previous study (Tanahashi, 1994b), a one-dimensional consolidation layer was modeled as a layer of the Viscoelastic Pasternak Model. In general, this approximation is accurate enough except for the early stage of consolidation, because higher order modes of consolidation finish rapidly. It has been noted that, in the present study, a one-dimensional consolidation layer is modeled as an infinite linear summation of Viscoelastic Pasternak Models, which is a mechanically equal to Terzaghi's one-dimensional consolidation.

One-dimensional consolidation modeling has some shortcomings in the analysis of differential settlements, because they are essentially of two or three-dimensional problem. Further studies are therefore necessary in order to confirm the accuracy of the modeling. This modeling

Fig. 3. An interpretation of shear layer on consolidation layer

is a more improved and higher approximation than elastic modeling for explicit formulation of the soil-structure interaction with respect to consolidation.

Initially, the load distribution q is expressed in a series of q_m as follows:

$$q = \sum_{m=0}^{\infty} q_m = \sum_{m=0}^{\infty} \frac{2q_0}{M^2} (1 + c \cos \omega x). \quad (12)$$

A Viscoelastic Pasternak Model with retardation time $\tau = 1/M^2$ and characteristic value γ_s which is loaded with q_m is then governed by the following equation:

$$\frac{\partial^2 y_m}{\partial x^2} - \gamma_s^2 \left(y_m + \frac{1}{M^2} \frac{\partial y_m}{\partial T_v} \right) = -\frac{\gamma_s^2 q_m}{k_0} \quad (13)$$

where $M = (2m+1)\pi/2$: $m=0, 1, 2, \dots$.

Consequently, the displacement y is also expressed in a series of y_m as:

$$\begin{aligned} y &= \sum_{m=0}^{\infty} y_m \\ &= \sum_{m=0}^{\infty} \frac{2\mu}{M^2} \left\{ 1 - \exp(-M^2 T_v) + \alpha_s(\omega) \right. \\ &\quad \times \left. \left[1 - \exp\left(-\frac{M^2 T_v}{\alpha_s(\omega)}\right) \right] c \cos \omega x \right\} \\ &= \mu \left\{ 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} \exp(-M^2 T_v) + \alpha_s(\omega) \right. \\ &\quad \times \left. \left[1 - \sum_{m=0}^{\infty} \frac{2}{M^2} \exp\left(-\frac{M^2 T_v}{\alpha_s(\omega)}\right) \right] c \cos \omega x \right\} \\ &= \mu \{ U + \alpha_s(\omega) U_D c \cos \omega x \} \\ &= \mu \{ U + \alpha_s(\omega, T_v) c \cos \omega x \} \end{aligned} \quad (14)$$

where

$$U_D = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} \exp\left(-\frac{M^2 T_v}{\alpha_s(\omega)}\right) \quad (15)$$

$$\alpha_s(\omega, T_v) = \alpha_s(\omega) U_D. \quad (16)$$

Using the non-dimensional wave number $\Omega = \omega/\gamma_s$, Eqs. (15) and (16) yield,

$$U_D = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} \exp\left(-\frac{M^2 T_v}{\alpha_s(\Omega)}\right) \quad (17)$$

$$\alpha_s(\Omega, T_v) = \alpha_s(\Omega) U_D \quad (18)$$

where

$$\alpha_s(\Omega) = \frac{1}{1 + \Omega^2}. \quad (19)$$

Equation (14) shows the average settlements occur according to the degree of consolidation of Eq. (1), while U_D expresses the degree of consolidation with respect to the differential settlement component. Consequently U_D is defined as the degree of differential consolidation settlements and is shown in Fig. 4. The curve of $\Omega=0$ is the same as U , which corresponds to the case when the shear rigidity of soils is ignored, i.e., $G=0$.

$\alpha_s(\Omega, T_v)$ is also defined as the reduction coefficient of differential consolidation settlements, and is expressed as a function of non-dimensional wave number Ω and time factor T_v in Fig. 5. The final value of $\alpha_s(\Omega, T_v)$, i.e.,

Fig. 5. Reduction coefficient of differential consolidation settlements

Fig. 4. Degree of differential consolidation settlements

$\alpha_s(\Omega, \infty)$ agrees with $\alpha_s(\Omega)$.

CHARACTERISTICS OF DIFFERENTIAL CONSOLIDATION SETTLEMENTS

Some conclusions concerning differential consolidation settlements are drawn from Eq. (14) through (19) and Figs. 4 and 5 as follows:

Equation (14) shows the average settlements occur according to the degree of consolidation U and the differential settlement components occur according to the degree of differential consolidation settlements U_D . This means that the differential settlements reach the final state faster than the average settlements.

Figure 4 shows the larger the wave number Ω is, the faster the differential settlements occur. This means that the larger the shear rigidity of soils is or the shorter the wavelength is, the faster the differential settlements converge to the final values.

According to Fig. 5, the reduction coefficient of differential consolidation settlements reaches that of the Pasternak Model when $T_V = \infty$. The final state of consolidation settlement coincides with the equivalent elastic settlement.

Figure 5 also shows that after the time $T_V = 0.05$ has elapsed, short waves of differential settlements of $\Omega \geq 3$ rarely occur. In short waves of $\Omega \geq 3$, the reduction coefficient of differential consolidation settlements is fairly small and almost the same as that of elastic settlements.

The actual data were evaluated according to these results.

Since the characteristic values in Port Island of Kobe, Japan, are as (Tanahashi, 1994a),

$$\gamma_s = 0.014 \sim 0.061 \text{ m}^{-1}$$

the wavelengths for $\Omega = \omega / \gamma_s = 3$ are obtained as,

$$L = 2\pi / \omega = 2\pi / (3\gamma_s) = 34 \sim 149 \text{ m.}$$

If $T_V = 0.05$ is converted to the actual time t , using the depth of the consolidation layer $H = 11 \sim 16 \text{ m}$ and the coefficient of consolidation $c_V = 40 \sim 90 \text{ cm}^2/\text{day}$ under double drainage conditions, then,

$$t = T_V(H/2)^2 / c_V = 168 \sim 800 \text{ days.}$$

These results reveal that waves shorter than $34 \sim 149 \text{ m}$ rarely occur after $168 \sim 800$ days from the completion of reclamation.

CONCLUSIONS

The consolidating reclaimed land was modeled as an infinite linear summation of Viscoelastic Pasternak Models. The degree of differential consolidation settle-

ments and the reduction coefficient of differential settlements were formulated. The characteristics of consolidation settlements are thus discussed. The main results are summarized based on the condition that loads vary as follows:

- (1) The average settlements occur according to the degree of consolidation U and the differential settlement components occur according to the degree of differential settlements U_D .
- (2) The differential settlements reach the final state faster than the average settlements. The larger the shear rigidity of the soils is or the shorter the wavelength is, the faster the differential settlements converge to the final values.
- (3) The final state of consolidation settlement coincides with the equivalent elastic settlement. The prediction method of differential settlements suggested by the author is therefore applicable for the final state of consolidation settlements.
- (4) After the time $T_V = 0.05$ has elapsed, short waves of $\Omega \geq 3$ rarely occur. In short waves of $\Omega \geq 3$, the reduction coefficient of differential settlements is almost the same as that for elastic differential settlements.

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