WAVE-INDUCED SEABED INSTABILITY: DIFFERENCE BETWEEN LIQUEFACTION AND SHEAR FAILURE

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ABSTRACT

The mechanism of wave-induced instability in a permeable seabed have been studied for more than two decades. The distinction between shear failure and liquefaction, however, has not been clearly defined. This paper presents a fundamental study on the differences in two failure modes for a fully saturated seabed of both finite and infinite thickness. The wave-induced effective stresses and pore pressure, obtained from an analytical solution of Biot's pore-elastic consolidation theory, were employed to examine the failure modes under a two-dimensional plane strain condition. A case study is presented to examine the failure modes with respect to several parameters, such as excess pore pressure, seepage flow, seepage force, failure areas and stress path in the seabed. The conclusions obtained from this study were as follows; (1) the thickness of a permeable seabed affects the pore pressure and effective stress response to ocean waves and the failure mode of the seabed, (2) either a liquefaction or shear failure, or both, occur in the seabed, even in the saturated seabed, (3) the Mohr-Coulomb's failure criterion, when combined with elastic stresses, can not be employed to estimate the liquefaction failure in the seabed, (4) the liquefaction can be evaluated by a criterion in terms of the excess pore pressure, (5) The liquefied zone in the seabed is significantly different from the shear failure zone. The shape beneath the seabed surface for the former is almost identical to the contour where the upward seepage flow is concentrated.

Key words: effective stress, failure, liquefaction, ocean wave, pore pressure, seepage, stress path (IGC: E7/E8)

INTRODUCTION

Seabed instability is one of the important factors affecting the safety of facilities in ocean areas, such as marine pipelines, oil storage tanks, oil production platforms, and coastal shore protection structures such as rubble mound and other types of breakwaters. When a wave is propagated on the ocean surface, it produces a pressure field extending down to the porous seabed, accompanied by variation in stresses. Traditionally, the instability induced in the seabed by the wave has been examined by either total stress or effective stress concepts. In both analyses, the wave-induced water pressure on the seabed surface is considered as an external force. The physical meaning of this external force, however, is different for these two approaches. In the total stress approach, the wave-induced water pressure is treated as a surface force acting on the seabed surface, whereas in the effective stress analysis, it is converted to the seepage force which

is a body force acting on the soil skeleton. One of the possible reasons for such a difference is attributed to the drainage condition in the seabed. In the case of an impermeable seabed consisting of clay or mud, total stress analysis can be applied (Henkel, 1970). In order to investigate the failure mechanism in a permeable seabed, however, effective stress analysis is preferable, because it is closely related to the deformation and failure of the soil skeleton.

Associated with the temporal and spatial variations in the wave-induced water pressure on the seabed, a field of seepage flow is generated within the soil skeleton. It induces a seepage force and variation in the effective stress field. There are two procedures for calculating the seabed response, namely, the analytical and the numerical solution. In this paper, the analytical solution is employed, because it is helpful to clarify the fundamental seabed response more easily than the numerical solution and it can be utilized to check the numerical solution. From the

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analytical solutions already developed for the wave-induced soil response (Yamamoto, 1977; Madsen, 1978; Okusa, 1985; Hsu et al., 1993; Hsu and Jeng, 1994), the effective stresses can be calculated, which can be employed to determine a shear failure by means of the Mohr-Coulomb's failure criterion. Yamamoto (1977) suggested the possibility of shear failure and liquefaction in a seabed on the basis of this failure criterion without defining the mechanisms of liquefaction. Okusa (1985) presented two criteria for evaluating soil liquefaction in which the effective normal vertical stress and effective mean normal stress during a storm were compared. The distinction between shear failure and liquefaction was not, however, identified. More recently, Zen and Yamazaki (1990a, b) pointed out the importance of the excess pore pressure in relation to soil liquefaction, from which a criterion was proposed. They also proposed a liquefaction experiment in the laboratory and verified it using the laboratory data available (Zen and Yamazaki, 1991).

There are two different mechanisms for wave-induced liquefaction, namely residual and transient liquefaction, depending on the way that the excess pore pressure is generated (Zen and Yamazaki, 1990a). The former is caused by the residual or progressive nature of the excess pore pressure, occurring after a certain number of cyclic waves, while the latter is generated by the transient or oscillatory nature of the excess pore pressure, which occurs periodically; many times during a storm sequence, responding to each wave. In this study, only the transient liquefaction was considered.

It is anticipated that the mechanism of a liquefaction failure is different from that of a shear failure. The former is considered as a type of quick sand or boiling which may be closely related to vertical seepage flow. Once the liquefaction occurs, the seabed becomes fluid state and the shear resistance can no longer be expected. For a shear failure the seabed mostly retains a solid state and the shear resistance is mobilized along the shear plane during a failure.

It is very important, therefore, to clearly identify the different failure modes and their governing criteria in order to evaluate the stability of a seabed.

This paper presents the fundamental failure mechanisms induced by waves in a fully saturated permeable seabed of finite and infinite thicknesses, respectively. The requirements of shear failure and liquefaction were examined analytically by means of effective stress and excess pore pressure on the basis of the Mohr-Coulomb's failure criterion under a two-dimensional plane strain condition. Typical analytical results for the excess pore pressure, seepage flow, seepage force, failure area and stress path in the seabed have been presented graphically. The differences between these two failure modes in a porous seabed are also discussed in detail.

THEORETICAL BACKGROUND

Governing Equations

Many different types of marine soil can be found in nature. In order to derive an applicable solution for the wave-induced soil response, however, it is necessary to restrict the present analysis to certain types of soil, for which the following assumptions should be made about the soil properties:

- (1) the soil skeleton and the pore fluid are compressible;
- (2) the soil skeleton satisfies Hooke's law;
- (3) the flow in the porous bed obeys Darcy's law;
- (4) the seabed is elastic, porous, non-cohesive, horizontal and of finite thickness;
- (5) the seabed is isotropic and homogeneous;
- (6) the wave is a two-dimensional progressive wave.

On the basis of the assumptions stated above, together with the consolidation theory of Biot (1941) and the storage equation of Verruijt (1969), the governing equation for the wave/soil interaction problem in a compressible porous seabed with compressible pore fluid is given by

$$\frac{k_x}{k_z} \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} - \frac{\gamma_w n' \beta}{k_z} \frac{\partial P}{\partial t} = \frac{\gamma_w}{k_z} \frac{\partial \varepsilon}{\partial t} \quad (1)$$

where x and z are the horizontal and vertical co-ordinates, respectively; k_x and k_z are the permeabilities of the soil in the x - and z -directions, respectively; P is the wave-induced pore pressure, γ_w is the unit weight of the pore-water; n' is the soil porosity; t is time; ε is the volumetric strain normally defined in terms of the displacement gradients of the soil; and β is the compressibility of the pore-fluid.

From the effective stress concept and Hooke's law, force equilibrium within the soil skeleton relates the pore pressure gradient to soil displacements and volume strain (Biot, 1941). This provides

$$G \nabla^2 \xi + \frac{G}{1-2\mu} \frac{\partial \varepsilon}{\partial x} = \frac{\partial P}{\partial x} \quad (2)$$

and

$$G \nabla^2 \chi + \frac{G}{1-2\mu} \frac{\partial \varepsilon}{\partial z} = \frac{\partial P}{\partial z} \quad (3)$$

in which ξ and χ are the soil displacements in the x - and z -directions, respectively. The shear modulus of the soil G is related to Young's modulus E by the Poisson's ratio μ in the form of $E/2(1+\mu)$.

The relationship between effective stress and soil displacement may be expressed as

$$\sigma'_x = 2G \left[\frac{\partial \xi}{\partial x} + \frac{\mu}{1-2\mu} \varepsilon \right] \quad (4)$$

$$\sigma'_z = 2G \left[\frac{\partial \chi}{\partial z} + \frac{\mu}{1-2\mu} \varepsilon \right] \quad (5)$$

$$\tau'_{xz} = G \left[\frac{\partial \xi}{\partial z} + \frac{\partial \chi}{\partial x} \right] = \tau'_{zx} \quad (6)$$

where the shear stress in double subscripts, τ'_{rs} , denotes the stress in the s -direction on a plane perpendicular to the r -axis.

Pore Pressure and Effective Stresses

Based on the governing Eqs. (1)–(3), together with the boundary conditions at the impermeable bottom and the seabed surface, the expression for the wave-induced pore pressure resulting in a porous seabed of finite thickness is given by (Hsu and Jeng, 1994),

$$P = \frac{p_0}{1-2\mu} \{ (1-\kappa-2\mu)(C_2 e^{\lambda z} - C_4 e^{-\lambda z}) + (1-\mu)(\delta^2 - \lambda^2)(C_5 e^{\delta z} + C_6 e^{-\delta z}) \} e^{I(\lambda x - \omega t)} \quad (7)$$

where λ is the wave number ($=2\pi/L$, in which L is the wave length), ω is the angular frequency of the wave ($=2\pi/T$, T is the wave period), I denotes the square root of -1 , and C coefficients were presented in the literature by Hsu and Jeng (1994).

The p_0 in Eq. (7) is the amplitude of the wave pressure at the seabed surface, using the first-order theory, such as

$$p_0 = \frac{\gamma_\omega H}{2 \cosh \lambda d} \quad (8)$$

where H is the wave height and d is the water depth above the seabed surface (sometimes designated “mud line”).

The two parametric coefficients δ and κ are defined by

$$\delta^2 = \lambda^2 \frac{k_x}{k_z} - \frac{I\omega\gamma_\omega}{k_z} \left\{ n'\beta + \frac{(1-2\mu)}{2G(1-\mu)} \right\} \quad (9)$$

and

$$\kappa = \frac{(1-2\mu) \left[\lambda^2 \left\{ 1 - \frac{k_x}{k_z} \right\} + \frac{I\omega\gamma_\omega n'\beta}{k_z} \right]}{\lambda^2 \left\{ 1 - \frac{k_x}{k_z} \right\} + \frac{I\omega\gamma_\omega}{k_z} \left\{ n'\beta + \frac{1-2\mu}{G} \right\}} \quad (10)$$

It should be noted herein that κ is a non-zero parameter for an unsaturated anisotropic soil, but $\kappa=0$ for soil in the saturated and isotropic conditions. The parameter, $\kappa=0$, corresponds to the compressibility of the pore-fluid, $\beta=0$. This is the case in which the pore-fluid is completely incompressible. When the compressibility of the pore-fluid of 0.45 (1/GPa) is applied to Eq. (10), as saturated water without air, the value of the parameter, κ ,

becomes 0.0013 for the parameters shown in Table 1, which can be considered nearly equal to 0 in Eqs. (11) to (15). In this paper, therefore the value of the parameter, κ , is assumed to be 0.

The effective normal stresses (positive denotes compression) are given by

$$\begin{aligned} \sigma'_x = p_0 & \left[\left\{ (C_1 + C_2 \lambda z) + \frac{2\mu\kappa}{1-2\mu} C_2 \right\} e^{\lambda z} \right. \\ & + \left\{ (C_3 + C_4 \lambda z) - \frac{2\mu\kappa}{1-2\mu} C_4 \right\} e^{-\lambda z} \\ & + \left\{ \lambda^2 - \frac{\mu(\delta^2 - \lambda^2)}{1-2\mu} \right\} (C_5 e^{\delta z} + C_6 e^{-\delta z}) \left. \right] e^{I(\lambda x - \omega t)} \quad (11) \end{aligned}$$

$$\begin{aligned} \sigma'_z = -p_0 & \left[\left\{ (C_1 + C_2 \lambda z) - \frac{2\kappa(1-\mu)}{1-2\mu} C_2 \right\} e^{\lambda z} \right. \\ & + \left\{ (C_3 + C_4 \lambda z) + \frac{2\kappa(1-\mu)}{1-2\mu} C_4 \right\} e^{-\lambda z} \\ & + \frac{1}{1-2\mu} \{ \delta^2(1-\mu) - \lambda^2 \mu \} (C_5 e^{\delta z} + C_6 e^{-\delta z}) \left. \right] e^{I(\lambda x - \omega t)} \quad (12) \end{aligned}$$

and effective shear stress is

$$\begin{aligned} \tau'_{xz} = -Ip_0 & \left[\{ C_1 + (\lambda z - \kappa) C_2 \} e^{\lambda z} - \{ C_3 + (\lambda z + \kappa) C_4 \} e^{-\lambda z} \right. \\ & + \lambda \delta (C_5 e^{\delta z} - C_6 e^{-\delta z}) \left. \right] e^{I(\lambda x - \omega t)} \quad (13) \end{aligned}$$

where the additional coefficients C_1 and C_6 were reported by Hsu and Jeng (1994) as mentioned above.

Seepage Force

If a differential water pressure gradient exists in the seabed, seepage flow is generated, being accompanied by the seepage force on the soil skeleton in the direction of the flow. The x - and z -components of the seepage force are given by

$$\begin{aligned} j_x &= -\frac{\partial P}{\partial x} \\ &= -\frac{Ip_0}{1-2\mu} \{ (1-\kappa-2\mu)(C_2 e^{\lambda z} - C_4 e^{-\lambda z}) \\ &+ (1-\mu)(\delta^2 - \lambda^2)(C_5 e^{\delta z} + C_6 e^{-\delta z}) \} e^{I(\lambda x - \omega t)} \quad (14) \end{aligned}$$

$$\begin{aligned} j_z &= -\frac{\partial P}{\partial z} \\ &= -\frac{p_0}{1-2\mu} \{ (\lambda(1-\kappa-2\mu)(C_2 e^{\lambda z} + C_4 e^{-\lambda z}) \\ &+ \delta(1-\mu)(\delta^2 - \lambda^2)(C_5 e^{\delta z} - C_6 e^{-\delta z}) \} e^{I(\lambda x - \omega t)}. \quad (15) \end{aligned}$$

The resultant total seepage force, j , and its direction, θ , can be calculated by

$$j = \sqrt{j_x^2 + j_z^2} \quad (16)$$

and

$$\theta = \arctan \left(\frac{j_z}{j_x} \right) \quad (17)$$

where θ is the angle measured in a counter-clock-wise direction from the positive x -direction. If the seepage

Table 1. Input data for case analysis

Wave period T	12.0 s
Water depth d	10.0 m
Wave length L	113.2 m
Wave height H	6.0 m
Permeability $k_x = k_z$	10^{-4} m/s
Shear modulus G	10^7 N/m ²
Poisson's ratio μ	1/3
Porosity n'	0.3
Submerged unit weight γ'	9.8 kN/m ³
Degree of saturation S_r	1.0
Internal friction angle ϕ'_f	35°

flow is quasi-steady and Darcy's law is appropriate, the hydraulic gradient, i , and the velocity of the seepage flow, v can be defined as

$$i = \frac{j}{\gamma_w} \quad (18)$$

$$v = k_z i. \quad (19)$$

Effective Stress Path

In order to evaluate the failure mode in the soil skeleton, it is necessary to obtain the description of the stress path experienced during the passage of waves. There are several expressions for the stress path available in the literature. In this study, however, a $\bar{p}'$ - $\bar{q}'$ plane is used, under a two dimensional plane strain condition where the parameters, $\bar{p}'$ and $\bar{q}'$ are defined by

$$\bar{p}' = (\bar{\sigma}_1' + \bar{\sigma}_3')/2 \quad (20)$$

and

$$\bar{q}' = (\bar{\sigma}_1' - \bar{\sigma}_3')/2 \quad (21)$$

in which $\bar{\sigma}_1'$ and $\bar{\sigma}_3'$ are the effective principal stresses in the seabed. The effect of the intermediate principal stress is disregarded, herein.

In the sections stated above, only the wave-induced variational stresses in soils from the initial equilibrium state under a calm sea have been considered. The stresses due to the self-weight of soil have to be superimposed to the wave-induced stresses, therefore, in order to obtain the $\bar{\sigma}_1'$ and $\bar{\sigma}_3'$.

The initial stresses for calm sea conditions without wave action, σ'_{z0} and σ'_{x0} , can be related to

$$\sigma'_{z0} = \gamma' z \quad (22)$$

and

$$\sigma'_{x0} = K_0 \gamma' z \quad (23)$$

where K_0 is the coefficient of earth pressure at rest and is related to the Poisson's ratio, μ , as $K_0 = \mu/(1-\mu)$. The total effective stresses, $\bar{\sigma}_z'$ and $\bar{\sigma}_x'$, may be given by;

$$\bar{\sigma}_z' = \sigma'_{z0} + \sigma_z' \quad (24)$$

and

$$\bar{\sigma}_x' = \sigma'_{x0} + \sigma_x' \quad (25)$$

where the second terms on the right-hand-side of Eqs. (24) and (25) represent the wave-induced effective stresses from Eqs. (11) and (12), respectively.

Since the shear stress on the horizontal and vertical planes is zero for the initial equilibrium state, the shear stress $\bar{\tau}_{xz}$ is given as

$$\bar{\tau}_{xz}' = \tau_{xz}'. \quad (26)$$

The effective principal stresses, $\bar{\sigma}_1'$ and $\bar{\sigma}_3'$, are generally represented by;

$$\bar{\sigma}_1' = \frac{\bar{\sigma}_x' + \bar{\sigma}_z'}{2} + \sqrt{\left(\frac{\bar{\sigma}_x' - \bar{\sigma}_z'}{2}\right)^2 + (\bar{\tau}_{xz}')^2} \quad (27)$$

and

$$\bar{\sigma}_3' = \frac{\bar{\sigma}_x' + \bar{\sigma}_z'}{2} - \sqrt{\left(\frac{\bar{\sigma}_x' - \bar{\sigma}_z'}{2}\right)^2 + (\bar{\tau}_{xz}')^2}. \quad (28)$$

Substituting Eqs. (27) and (28) into Eqs. (20) and (21), $\bar{p}'$ and $\bar{q}'$ can be obtained to draw the stress path. The inclination of a point on the stress path is defined as the stress angle ϕ' , such that

$$\phi' = \arcsin \left(\frac{\bar{q}'}{\bar{p}'} \right). \quad (29)$$

Shear Failure Criterion

Based on Mohr-Coulomb's failure criterion, as shown in Fig. 1, the limiting condition for shear failure in a soil skeleton may be given by

$$\bar{\tau}_f = \bar{\sigma}_f \tan \phi_f' \quad (30)$$

where ϕ_f' denotes the internal friction angle of the soil, τ_f' and σ_f' represent the shear stress and effective normal stress on the failure plane, respectively. In general, ϕ_f' depends on the soil type, for example, 30–35 degrees for sand and 40–45 degrees for gravel.

When the stress reaches the failure envelope, the stress angle becomes identical to the internal friction angle of the soil. Thus, the failure criterion for shear failure at a given point and instance may be defined as

$$\phi' \geq \phi_f'. \quad (31)$$

Introducing the principal stresses, $\bar{\sigma}_1'$ and $\bar{\sigma}_3'$, into Eq. (31), the criterion of shear failure becomes

$$\phi' = \arcsin \left(\frac{\sqrt{(\bar{\sigma}_z' - \bar{\sigma}_x')^2 + 4(\bar{\tau}_{xz}')^2}}{\bar{\sigma}_x' + \bar{\sigma}_z'} \right) \geq \phi_f'. \quad (32)$$

Excess Pore Pressure and Liquefaction Criterion

Since both the hydrostatic pressure at the seabed surface and the oscillatory pore pressure in the seabed vary temporarily and spatially, the excess pore pressure, u_e can be calculated as the excess component beyond the

Fig. 1. Mohr's circle diagram for wave-induced effective stresses (After Yamamoto, 1977)

hydrostatic pressure. One such relationship was introduced by Zen and Yamazaki (1990s), as

$$u_e = -(P_b - P) \quad (33)$$

where P_b denotes the wave-induced pressure at the seabed surface.

In order to apply the concept of excess pore pressure to the seabed, a schematic drawings of the pore pressure and effective stress distribution are illustrated in Fig. 2. The solid curves in Fig. 2(a) indicate the pore pressure beneath the wave trough and the wave crest. The excess pore pressure expressed by Eq. (33) is transient in nature, because P_b and P are oscillatory and periodical as mentioned above. Consequently, the effective stress varies periodically in accordance with the change of the excess pore pressure as shown in Fig. 2(b). Since the decrease in the effective stress may be regarded as result of the build-up of the excess pore pressure, the liquefaction can be evaluated by comparing the initial mean effective stresses under calm sea with the excess pore pressure. As shown previously, the excess pore pressure is defined by Eq. (33), while the initial effective mean stress, σ'_{m0} , is given by

$$\sigma'_{m0} = \frac{(1+2K_0)}{3} \gamma' z. \quad (34)$$

The liquefaction criterion can, therefore, be expressed as

$$\frac{(1+2K_0)}{3} \gamma' z + (P_b - P) \leq 0. \quad (35)$$

Unlike the case of shear failure for the two dimensional plane strain condition as shown by Eq. (32), the reason why the mean effective stress is used for the liquefaction criterion represented by Eq. (35) is that the excess pore pressure acts multi-directionally and isotropically in the soil skeleton and the effective stress in any direction should be equal to zero at the moment of liquefaction, namely at the suspended state of soil particles. When compared with the case in which only the vertical effective stress, $\gamma' z$, is adopted in place of the effective mean stress, σ'_{m0} , Eq. (35) gives a conservative side in the evaluation of liquefaction potential, for a coefficient of earth pressure at rest, K_0 , less than 1.0. Eq. (35), when simplified to a one-dimensional case, is identical

Fig. 2. Schematic diagram showing definition of liquefaction based on excess pore pressure (After Zen and Yamazaki, 1990a)

cal to that proposed by Zen and Yamazaki (1990a). It is beneficial to relate the liquefaction criterion to the pore pressure, such as P_b and P , because they can be readily measured in the laboratory and the field (Zen and Yamazaki, 1991).

CASE ANALYSIS AND DISCUSSION

Input Data

The input data used in the case analysis are listed in Table 1. The geotechnical properties of the soil considered correspond to a fine sand. Although the degree of saturation may significantly affect the seabed response, a fully saturated soil only is presented. All geotechnical parameters were assumed to be constant throughout the soil skeleton. In order to demonstrate the effect of seabed thickness, seabeds of infinite depth ($h/L = \infty$, where h is the total thickness) and a finite depth ($h/L = 0.2$) are considered.

Pore pressure and Excess Pore Pressure

As indicated in the theoretical consideration presented above, the spatial and temporal variations in pore pres-

Fig. 3. Contours of pore pressure P/p_0 versus wave phase $\lambda x - \omega t$ for seabed thickness of (a) $h/L = \infty$, (b) $h/L = 0.2$

sure affect the excess pore pressure, effective stress and seepage flow within the soil skeleton. The pressure distribution is significantly dependent upon the soil thickness, as shown in Fig. 3. Different patterns of pressure variation exist with the repetitive cell bounded between the consecutive wave crest and trough, from $(\lambda x - \omega t)/2\pi = 0$ to 0.5 and from 0.5 to 1, for example.

This pattern is not symmetrical with the wave trough in a soil of finite thickness (see Fig. 3(b)), despite it being symmetrical in the case of an infinite thickness (see Fig. 3(a)). In addition, for a soil with infinite thickness, the pore pressure decreases progressively with depth, with distributions centering on the wave crest and the trough. For a soil of finite thickness, the pore pressure in the vicinity of the wave crest and trough decreases initially from the seabed surface to a certain depth, and is then followed by an increase. This may be attributed to the effects of the bottom boundary and the partial drainage in the permeable seabed. In the phases other than the wave crest and trough, the pressure shows almost constant value in the vertical direction.

The vertical variations in the non-dimensional excess pore pressure, u_e/p_0 , as defined by Eq. (33), are illustrated in Fig. 4, where the excess pore pressure under the action of wave crest is denoted by $\omega t = 0^\circ, 360^\circ$, and for wave trough at 180° , etc. The oscillatory pattern of the excess pore pressure varies with the seabed thickness. By adding the net submerged weight of the soil to the excess pore pressure, a picture of liquefaction potential can readily be drawn.

Seepage Forces and Direction

Substituting the differential pore pressure gradients in the x - and z -directions into Eqs. (14) and (15) gives the seepage forces, j_x/p_0 and j_z/p_0 , in each direction (Figs. 5 and 6). It was observed that contours of the seepage forces vary with the wave phase $\lambda x - \omega t$ and, over all, with soil thickness. The horizontal seepage forces are greatest at one-quarter and three-quarters of the wave cycle, and become negligible under the wave crest and trough (Fig. 5). For the vertical seepage forces given in Fig. 6(a), a symmetrical pattern can be found for a soil of

Fig. 4. Vertical distribution of the normalized excess pore pressure, u_e/p_0 , for various stages of the wave cycle, ωt , for seabed thickness (a) $h/L = \infty$, (b) $h/L = 0.2$

Fig. 5. Contours of seepage force j_x/p_0 in x -direction, for seabed thickness (a) $h/L = \infty$, (b) $h/L = 0.2$

infinite thickness, similar to that of the horizontal seepage force, but with its maximum seepage force shifting toward the crest and trough. Skewed distribution patterns exist in the vertical seepage force in soil of finite thickness ($h/L=0.2$), as in Fig. 6(b). The maximum seepage force in the vertical direction (350 units) for soil of finite thickness is found to be predominantly 7 times larger than the other three cases (about 50 units, as shown in Figs. 5 and 6(a)).

Figure 7 shows the magnitude and resultant direction of the total seepage force for the case of $h/L=0.2$. Since soil instability occurs within a relatively thin layer beneath the seabed surface, only a small portion of the seabed is shown. The contour of the resultant seepage force exhibits a pattern of skewed cells, with its centres of maximum magnitude (350 units) offset from the position of the wave crest and trough, as in Fig. 7(a), while Fig. 7(b) defines a strong tendency showing the seepage forces pushing the soil up at locations near the wave trough and pressing down near the wave crests. It may therefore be considered that the upward seepage forces are concentrated near the seabed surface and play an important role

in soil instability.

Shear Failure and Liquefaction

The non-dimensional effective normal stresses and shear stress calculated from Eqs. (11)–(13) are presented in Fig. 8. The contours of stress as well as the contours of pore pressure shown in Fig. 3, exhibit a repetitive pattern and skewed distributions for the finite depth condition. They do not provide a direct insight into the mechanism of the shear failure nor liquefaction so clearly that the distribution of the computed stress angle, ϕ' , is drawn in Fig. 9. For a soil of finite thickness, $h/L=0.2$, the contours of stress angle are non-symmetrical, as shown in Fig. 9(b), in contrast to the symmetrical pattern for the case of infinite thickness shown in Fig. 9(a).

A commonly accepted practice in applying the concept of the stress angle is to determine the shear failure condition at a particular depth where the stress angle is equal to or greater than the internal friction angle specified for the soil (Yamamoto, 1977 and 1981). If the internal friction angle for the soil is 35 degrees, for example, the shear failure may be considered to occur in the region

Fig. 6. Contours of seepage force j_z/p_0 in z -direction, for seabed thickness (a) $h/L=\infty$, (b) $h/L=0.2$

Fig. 7. (a) Spatial distribution of total seepage force j/p_0 , and (b) vector directions, for soil thickness of $h/L=0.2$

Fig. 8. Contour of (a) wave-induced effective horizontal normal stress (b) wave-induced effective horizontal normal stress (c) wave-induced effective shear stress, for seabed thicknesses of $h/L = \infty$ and 0.2, respectively

bounded between the seabed surface and the stress angle contour of 35 degrees. It appears in Figs. 9(a) and 9(b) that the shear failure occurs over a thin layer of less than about 1.0 m immediately beneath the seabed surface, irrespective of the thickness of permeable seabed. When

Figs. 9(a) and 9(b) were compared, however, it was found that the shape of the anticipated shear failure zone is quite different. It is symmetrical to wave trough for the case of infinite thickness as shown in Fig. 9(a) but is asymmetrical for the case of finite thickness as shown in Fig.

Fig. 9. Distribution of the computed stress angle in degrees for soil thickness of $h/L = \infty$ and 0.2, respectively. Dashed lines indicate zone of potential liquefaction

9(b). This tendency is common to the contours of the pore pressure, seepage force and effective stresses already presented in Figs. 3, 7 and 8, respectively. In addition, the depth of the shear failure zone becomes shallower as shown in Fig. 9(b) than that in Fig. 9(a), especially for the phase, $(\lambda x - \omega t)/2\pi$, between 0 to 0.2 and 0.5 to 1.0. The thickness of permeable seabed definitely affects the depth and shape of the shear failure zone.

When the angle of internal friction is 30 degrees, the shear failure zone extends to a wider areas. Namely, in Fig. 9(a), the shear failure zone spreads deeply into the seabed up to a maximum depth of about 2.5 m in the shear failure zone under the wave crest. The tendency that the shear failure zone spreads widely and the maximum depth of the shear failure zone appears under the wave crest was also observed in Fig. 9(b). Although the shear failure in the sandy seabed takes place mostly in a shallow depth beneath the seabed surface, the shear failure zone is deepened in the vicinity of the wave crest, depending on the angle of internal friction of the soil. Referring to Fig. 7, this may be attributed to the press-

down action under the wave crest and push-up action under the wave trough created by the seepage force in the seabed.

The liquefied zone, determined by Eq. (35), is also presented in Fig. 9. There is no possibility of liquefaction for the case of $h/L = \infty$, while a liquefied zone with a maximum depth of 1.0 m is likely for $h/L = 0.2$, as denoted by the dotted line in Fig. 9(b). The liquefied zone is limited to a shallow portion closely related to the wave trough, and its shape is significantly different from that of the shear failure zone calculated. It should also be noted that the shape of the liquefied zone is similar to that of the vertical seepage force given in Fig. 6(b). For a monochromatic wave propagating the ocean, some areas in the seabed are found to be in a state of the shear failure or liquefaction at different phase of the wave passage, or even in the combined state of shear failure and liquefaction.

Based on the above analysis, four possible zones may be identified; 1) the zone with liquefaction and shear failure (marked by "1" in Fig. 9(b)), 2) the zone with liquefaction only (marked by "2"), 3) the zone with shear failure only, i.e., at location outside the liquefied zone and bounded by the region with stress angle greater than 35 degrees, for example, and 4) the stable zone, i.e., area outside the other three zones mentioned.

Effective Stress Path to Failure

In order to further evaluate the failure mode, the Mohr's circles of stress at the depths corresponding to the Point Nos. 1, 2 and 3 marked on Fig. 9(b) are drawn in Fig. 10. In Fig. 10, the Mohr's circles not only at the phase of $(\lambda x - \omega t)/2\pi = 0.57$, but also for the several typical phases are presented to look into the variation of the circles raised in the seabed in accordance with the passage of waves. Note that these circles are ones when a progressive wave acts on the seabed surface periodically and steadily with a constant wave period and wave height. The straight lines in Fig. 10 are the Coulomb's failure envelopes drawn provided that the angle internal friction of the soil is 35 degrees. As clearly shown in Fig. 10, the shear failure occurs for all phases at the Point No. 1. At the Point Nos. 2 and 3 no shear failure is observed, though the Mohr's circle at the phase of 0.2 at the Point No. 2 is considerably close to the failure envelope. In Fig. 10(a), the negative value of $\bar{\sigma}'$ are observed at the phases of 0.4 and 0.57. Whether or not the liquefaction occurs, however, can not be definitely estimated.

The alternative stress plot in a $\bar{p}'$ - $\bar{q}'$ plane is given in Fig. 11. The computed stress path for the Point No. 1 shows an elliptical orbit and is always located at a position beyond the failure envelope. Since the stress path can not pass beyond the failure envelope, it is thought that the path may move along the failure envelope. Such behavior is beyond the scope of elastic solution treated in this study. The stress path for the Point No. 2 reveals a little distorted shape at the bottom of the ellipse, because of the complexity of stress angle contours at the horizontal plane across the Point No. 2 shown in Fig. 9(b). The

Fig. 10. Mohr's circle of stress, showing τ' versus σ' for (a) Point No. 1, (b) Point No. 2 and (c) Point No. 3 marked on Fig. 9(b)

stress path for Point No. 3 is very simple indicating a straight line and lies always below the failure envelope. It is interesting to note in these three patterns of stress path that the value of $\bar{p}'$ hardly reaches the origin of the axis before the path comes in contact with the failure envelope, even at the Point No. 1 where the shear failure occurs at any phases. This means that the Mohr-Coulomb's failure criterion when combined with the elastic stresses can not be employed to estimate the liquefaction failure in the seabed. A liquefaction criterion in terms of the excess pore pressure generated by ocean waves and initial effective mean stress was introduced as already shown in

Fig. 11. Effective stress path on a p' - q' plane for Point Nos. 1, 2 and 3 marked on Fig. 9(b)

Eq. (35). The liquefied zone estimated using Eq. (35) and presented in Fig. 9 shows a significant difference in its shape and position from the shear failure zone.

Generally speaking, when the elastic analysis is employed for the calculation of stress in a ground, sometimes the tensile stresses such as a negative value of vertical stress are found. This does not necessarily mean the occurrence of liquefaction unless the excess pore pressure is appropriately evaluated. For this reason, it can be suggested that a liquefaction criterion should be employed to evaluate the liquefaction failure apart from the Mohr-Coulomb's shear failure criterion, when the seabed is treated as an elastic half-space.

CONCLUSIONS

The differential gradients for the wave-associated water pressure and pore pressure generate a field of hydraulic gradient for seepage flow in the seabed. Consequently, the seepage force on the soil skeleton and variations in effective stress and excess pore pressure in the seabed are produced. Under some combination of wave and soil conditions, seabed instability (shear failure or liquefaction or both) may occur, depending on the magnitude of the effective stress and excess pore pressure. When liquefaction occurs, shear resistance is no longer anticipated because the phenomenon is considered as a type of boiling or quick sand. Whereas shear resistance is believed to be mobilized during shear failure. Shear failure and liquefaction failure should, therefore, be clearly distinguished in the stability analysis of the seabed.

Based on the results of the case analysis presented in this study and the theoretical evaluation of the different failure modes, the following conclusions can be drawn:

- (1) The thickness of a permeable seabed significantly affects the distribution of pore pressure, effective stress and the failure mode of the seabed. Symmetrical patterns of these parameters can be found for a seabed of infinite thickness, but asymmetric patterns for a finite thickness.
- (2) Either a liquefaction or shear failure, or both occur in a saturated seabed, depending on the relative posi-

tion of a stress path on the $\bar{p}'$ - $\bar{q}'$ plane and the excess pore pressure generated by ocean waves.

- (3) The Mohr-Coulomb's failure criterion, when combined with elastic stresses, can not be employed to estimate the liquefaction failure in the seabed. Liquefaction can favorably be evaluated by a criterion in terms of the excess pore pressure.
- (4) The liquefied zone is distinctly different from the shear failure zone. The former corresponds to the region where the upward seepage force is predominant, normally in the vicinity of the wave trough, but the latter appears in a thin layer beneath the seabed surface. In the vicinity of the wave crest, the shear failure zone is deepened depending on the internal friction angle.

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