A DAMAGE MODEL FOR SOIL-CEMENT MIXTURE

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ABSTRACT

In the frame of the continuous damage theory, a damage model for soil-cement mixture is proposed using the unified form with one set of parameters, based on experimental observations. The suggested model can reflect its main mechanical properties: as the confining pressure increases, the strength and strain at peak stress increase remarkably and the stress-strain relations change from strain-softening to strain-hardening while the volumetric strain changes from dilatation to contraction. All the parameters can be calibrated by a group of conventional triaxial tests. Hollow cylinder tests with complicated stress states were conducted to verify the damage model. The results predicted by the damage model agree well with the tests, which implies that the damage model can be applied to complicated stress states.

Key words: constitutive equation, damage model, laboratory tests, mechanical properties, soil-cement mixture (IGC: D10/E13)

INTRODUCTION

Soil-cement mixture is composed of cement, water, clay, sand, gravel and bentonite etc. It has apparent advantages over common concrete in building cutoff walls under earth-rockfill dams. In accordance with the test results, the main mechanical properties of soil-cement mixture under triaxial condition are as follows (Yu, Pu and Ugai, 1997): as the confining pressure increases, the strength and the strain at peak stress increase remarkably, the stress-strain relations change from strain-softening to strain-hardening and the volumetric strain changes from dilatation to contraction, but the initial modulus changes little. In addition, when the confining pressure is high, the tangent modulus in the middle of the stress-strain curves of some samples either has no obvious decrease over a wide range, or even increases somewhat, such as with curves corresponding to $\sigma_3=0.7$ and 1.2 MPa in the range of $\varepsilon_a=2-7\%$ in Fig. 2 (sample J5). In this study, a damage model is proposed to reflect such properties.

Classical damage mechanics was first established by Kachanov (1958) when he studied the one-dimensional creep problem, and was first applied by Dougill et al. (1976) to rock and concrete. Thereafter, it has been attracting the attention of many researchers. Especially in the last decade, it developed fast and many damage models for rock and concrete have been suggested.

From the view of the research approach, most of these models are phenomenological, i.e. the damage variable is the average measure in macroscale of the variations of the microstructure or the relevant parameters of the macro-mechanical behavior. The simplest damage variable is a scalar, whereas various order tensors have been employed to account for anisotropic damage, such as the first order (Krajcinovic et al., 1981), the second order (Voyiadjis et al., 1993) and the fourth order (Yazdani et al., 1988, 1990; Ortiz, 1985; Carol et al., 1991; Mazars et al., 1989).

Different damage variable and evolution rules have been applied to reflect the different mechanical properties and the mechanism itself. In this sense, established damage models for concrete and rock can be classified into groups such as: (1) brittle damage models (Krajcinovic et al., 1981, 1983, 1986; Nobile, 1986, 1991; Resende and Martin, 1984, 1988; etc.), which mainly reflect the brittle behavior, such as strain softening, dilatation and degradation of the elastic stiffness based on the mechanism of the nucleation and growth of microcracks; (2) elasto-plastic damage model (Lubliner et al., 1989; Oller et al., 1990; Klisinski et al., 1988; Ortiz, 1985; Yazdani and Schreyer, 1988, 1990; Frantziskonis and Desai, 1987a, b, c), which reflect the more general properties including plastic behavior, where most of them depend on the mechanism of plastic flow and the growth of microcracks; (3) other kinds of damage model,

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such as the bounding surface damage model (Voyiadjis et al., 1993), the probabilistic damage model (Breyse, 1990) and the microplane damage model (Carol et al., 1991), etc.

Geomaterials have the common behavior that, along with the increase of confining pressure, stress-strain curves change from strain softening to strain hardening, and volumetric strain changes from dilatation to contraction. For soil-cement mixture structures, in the normal range of lateral pressure, such phenomena will occur and are quite important for the analysis of deformation and seepage. But few of the already established models can reflect such behavior by the unified form and only one set of parameters.

In order to simulate strain-softening and strain-hardening, Klisinski and Mroz's (1988) assumed different yield surface and damage surface for different stress conditions; in the bounding surface damage model (Voyiadjis et al., 1993), different equations are adopted in different areas in stress space.

Frantziskonis and Desai (1987a,b,c) have suggested an elasto-plastic model with damage for strain softening geomaterials, in which the material is decomposed in two phases: the first component, termed topical, is elasto-plastic hardening (Desai, 1984) while in the second component, termed damaged, the behavior is considered as rigid perfectly plastic with zero yield strength. Desai (1992) further suggested that a linear elastic relation and elasto-plastic relation are employed for the topical part and the damaged part, respectively, to reflect strain-hardening of the whole material.

In this paper, a damage model for soil-cement mixture is constituted using the unified form with one set of parameters in the frame of the continuous damage theory. The proposed damage model is based on experimental observations and can reflect its main mechanical properties. The procedures to determine the parameters of the established damage model are described. Hollow cylinder tests with complicated stress states are conducted to verify the damage model.

CONSTITUTION OF THE DAMAGE MODEL FOR SOIL-CEMENT MIXTURE

Basic Assumptions

In light of the experimental observations and referring to research results on concrete and other geomaterials, it can be considered that the microscopic mechanism of the mechanical properties of a soil-cement mixture comprises the following two parts: (1) nucleation, growth and coalescence of microcracks; (2) frictional slip and interlock on the shear surface. The effects of the two factors on the mechanical properties vary with stress condition and stress level so that the observed macroscopic properties are also different.

Based on such a mechanism, the basic assumptions are made as follows: (1) the strength of a soil-cement mixture is composed of two parts, one is the structural strength and the other is the frictional resistance strength; both of

them satisfy the Mohr-Coulomb strength criterion; (2) the apparent stress in a cross section under a certain strain is divided into the structural stress and that caused by the frictional resistance (which is simply called frictional resistance stress in the following paragraphs); (3) the stress-strain relation of both parts is hyperbolic. The structural strength mainly results from the cementation of the mortar and the dissipation of energy by crack growth, whereas the frictional resistance strength mainly comes from the friction and interlock in the shear surface. In this paper, the superscript "s" signifies the structural part and "f" signifies the frictional resistance part.

Figure 1 shows the decomposition of the stress in a certain section, where F and A are the total force and the total area, respectively, and F^s , A^s and F^f , A^f are those of the structural part and frictional resistance part, respectively. Obviously,

$$F = F^s + F^f, \quad A = A^s + A^f \quad (1)$$

Along with the increase in strain, the cracks grow and the cohesion is destroyed gradually, so that the ratio of the area bearing structural stress decreases while that bearing frictional resistance stress increases. When the strain reaches a certain large value, the structural stress disappears completely and all the apparent stress is carried by the frictional resistance part.

The damage variable r is defined as,

$$r = A^f / A \quad (2)$$

From Eqs. (1) and (2) we can get,

$$\sigma = (1 - r)\sigma^s + r\sigma^f \quad (3)$$

where σ , σ^s and σ^f are the apparent stress tensor, struc-

Fig. 1. The decomposition of stress

tural stress tensor and frictional resistance stress tensor, respectively, and all of them are effective stresses.

The stress component is,

$$\sigma_{ij} = (1-r)\sigma_{ij}^s + r\sigma_{ij}^f \quad (4)$$

and its incremental form is

$$d\sigma_{ij} = (1-r)d\sigma_{ij}^s + rd\sigma_{ij}^f + (\sigma_{ij}^s - \sigma_{ij}^f)dr \quad (5)$$

The Stress-Strain Relation

In line with the basic assumptions mentioned above, structural stress and frictional resistance stress satisfy the following relations, respectively, when they achieve their limited strength independently:

$$\tau^s = c^s + \sigma^s \tan \phi^s \quad (6)$$

$$\tau^f = c^f + \sigma^f \tan \phi^f \quad (7)$$

where c^s and ϕ^s , c^f and ϕ^f are Mohr-Coulomb's strength indexes. From the previous assumptions, the tangent modulus for both parts is derived as follows (Duncan and Chang, 1970),

$$E_t^s = [1 - (1 - \sin \phi^s)(\sigma_1 - \sigma_3)^s / (2c^s \cos \phi^s + 2\sigma_3^s \sin \phi^s)]^2 E_i^s \quad (8)$$

$$E_t^f = [1 - (1 - \sin \phi^f)(\sigma_1 - \sigma_3)^f / (2c^f \cos \phi^f + 2\sigma_3^f \sin \phi^f)]^2 E_i^f \quad (9)$$

where E_t^s and E_t^f are the tangent modulus of the structural part and the frictional resistance part, respectively, and E_i^s and E_i^f are the corresponding initial modulus. Set the initial modulus of the total sample as E_{i0} , then,

$$E_i^s = E_{i0} \quad (10)$$

Although the assumption that,

$$E_i^f = E_{i0} \quad (11)$$

seems unreasonable, but it will be shown in the next section that it has little influence on the total constitution.

Poisson's Ratio

In accordance with experimental observations (Yu, Pu and Ugai, 1997), the Poisson's ratio of a soil-cement mixture has the following features: (1) the initial Poisson's ratio is dependent on the lateral pressure, i.e. the higher the lateral pressure, the less the initial Poisson's ratio; (2) when the lateral pressure is low, dilatation will occur i.e. $\mu < 0.5$ and along with the increase of lateral pressure, the volumetric strain changes from dilatation to contraction ($\mu < 0.5$); (3) no matter how high or low the lateral pressure is, the volumetric strain will not change ($\mu = 0.5$) after a certain large shear strain. In this paper, we adopt the hypothesis of strain equivalence, which was introduced by Lemaitre (1971): "the strain associated with a damaged state under the applied stress is equivalent to the strain associated with its undamaged state under the effective (net) stress." Therefore, the Poisson's ratio of the structural part, the frictional resistance part and the total sample are the same. On the basis of the above-mentioned features, the following equation is proposed to ex-

press the Poisson's ratio,

$$\mu = \mu_0(1-r)/(\sigma_3/p_0)^{n1} + r(1-r)[a1 + a2(\sigma_3/p_0)^{n2}] + 0.5r \quad (12)$$

where p_0 is a unit pressure set at 0.1 MPa, and μ_0 , $n1$, $n2$, $a1$ and $a2$ are the material parameters. The three terms reflect the above three features respectively. If $\sigma_3 \leq 0.01$ MPa, set $\sigma_3 = 0.01$ MPa.

Damage Variable and the Evolution Rule

The development of microcracks is obviously the direct cause of the damage evolution. The volumetric strain generally influences the structure of the sample slightly in the concerned range of confining pressure, so that it is assumed that the damage is only caused by the octahedral shear strain. Furthermore, the shear stress ratio of the structural part

$$\xi = q^s/p^s \quad (13)$$

also has some influence on the damage evolution, where q^s is the octahedral shear stress and p^s is the mean stress of the structural part. The damage variable is assumed as the following equation,

$$r = 1 - \exp(-\eta_1 \bar{\epsilon}^{\eta_2} \xi) \quad (14)$$

where $\bar{\epsilon}$ is the octahedral shear strain, and η_1 and η_2 are the material parameters. Obviously, when $\bar{\epsilon} = 0$, $r = 0$, and when $\bar{\epsilon} \rightarrow \infty$, $r = 1$.

The evolution of the damage variable is,

$$dr = \eta_1(1-r)(\eta_2 \bar{\epsilon}^{\eta_2-1} \xi d\bar{\epsilon} + \bar{\epsilon}^{\eta_2} d\xi) \quad (15)$$

The Stress-Strain Relation in Incremental Form

The volumetric strain and octahedral shear strain are defined as,

$$\epsilon_v = \epsilon_{ii}, \quad \bar{\epsilon} = [(2/3)e_{ij}e_{ji}]^{1/2} \quad (16)$$

where, $e_{ij} = \epsilon_{ij} - (1/3)\delta_{ij}\epsilon_v$.

The mean stress and the octahedral shear stress are defined as,

$$p = (1/3)\sigma_{ii}, \quad q = [(3/2)s_{ij}s_{ji}]^{1/2} \quad (17)$$

where, $s_{ij} = \sigma_{ij} - \delta_{ij}p$.

From Eqs. (5), (13), (15), (16) and (17) we can get,

$$d\sigma_{ij} = (1-r)d\sigma_{ij}^s + C_2 R_{ij} L_{mn} d\sigma_{mn}^s dr + rd\sigma_{ij}^f + C_1 R_{ij} T_{kl} d\epsilon_{kl} \quad (18)$$

where R_{ij} , T_{ij} and L_{ij} are second order tensors,

$$R_{ij} = \sigma_{ij}^s - \sigma_{ij}^f, \quad T_{ij} = (2/3)e_{ij}/\bar{\epsilon},$$

$$L_{ij} = (1/p^s)^2 [(3/2)(p^s/q^s)s_{ij}^s - q^s \delta_{ij}/3]$$

C_1 and C_2 are scalar coefficients,

$$C_1 = \eta_1(1-r)\eta_2 \bar{\epsilon}^{\eta_2-1} \xi \quad C_2 = \eta_1(1-r)\bar{\epsilon}^{\eta_2}$$

Note that the constitutive relations of the structural part and the frictional resistance part are expressed as follows, respectively,

$$d\sigma_{ij}^s = D_{ijkl}^s d\epsilon_{kl}, \quad d\sigma_{ij}^f = D_{ijkl}^f d\epsilon_{kl} \quad (19)$$

where D_{ijkl}^s and D_{ijkl}^f are the fourth order stiffness tensor of the structural part and the frictional resistance part, respectively.

Substituting Eq. (19) into Eq. (18) results in,

$$d\sigma_{ij} = D_{ijkl} d\epsilon_{kl} \quad (20)$$

where D_{ijkl} is the fourth order stiffness tensor of the typical element in the sample and is expressed as,

$$D_{ijkl} = (1-r)D_{ijkl}^s + C_2 R_{ij} L_{mn} D_{mnkl}^s + rD_{ijkl}^f + C_1 R_{ij} T_{kl} \quad (21)$$

The constitutive relation of the overall material is:

$$d\sigma_{ij} = D_{ijkl} d\epsilon_{kl}, \quad dr \geq 0 \quad \text{for loading} \quad (22a)$$

$$d\sigma_{ij} = D_{ijkl}^0 d\epsilon_{kl}, \quad dr = 0 \quad \text{for unloading} \quad (22b)$$

where D_{ijkl}^0 is the initial stiffness tensor and the loading condition is as follows,

$$S^a \geq S_{\max}^a \quad \text{loading}, \quad S^a < S_{\max}^a \quad \text{unloading}, \quad (23)$$

where S^a is the mobilized shear stress level which is expressed as,

$$S^a = (\sigma_1 - \sigma_3)^a / (\sigma_1 - \sigma_3)_u^a \quad (24)$$

$S_{\max}^a$ is the maximum mobilized shear stress level, $(\sigma_1 - \sigma_3)_u^a$ is the deviatoric stress at the ultimate state, $a=s$ refers to the structural part and $a=f$ refers to the frictional resistance part.

REFLECTION OF THE MECHANICAL PROPERTIES OF SOIL-CEMENT MIXTURE

The comparison of the fitting and test curves of some group of samples is shown in Figs. 2–5, where the values of the parameters are shown in Table 1 and the mix proportions are shown in Table 2. How the damage model reflects the main mechanical properties of a soil-cement mixture are analyzed as follows, where sample J5 is given as an example.

The Stress-Strain Relation

Figures 6(a) and (b) show the stress-strain curves of the structural part and the frictional resistance part, respectively, under different confining pressure, when they are mobilized individually. Generally, in classical damage theory, the strength of the damaged part is always lower than that of the initial material. Whereas in this model, for the structural part, the value of c is generally fairly large and that of ϕ is fairly low; otherwise, for the frictional resistance part, c is generally fairly low and ϕ is fairly large. As a result, when the confining pressure is low, the strength of the structural part may be higher than that of the frictional resistance part and when the confining pressure is high, the strength of the structural part may be lower than that of the frictional resistance part.

As far as the stress increment is concerned, accompanying with the evolution of damage, the area that carries the structural stress gradually decreases and the stress on the decreasing area transforms into the frictional

Fig. 2. Fitting and test curves of $(\sigma_1 - \sigma_3) \sim \epsilon_a$ and $\epsilon_v \sim \epsilon_a$

Fig. 3. Fitting and test curves of $(\sigma_1 - \sigma_3) \sim \epsilon_a$ and $\epsilon_v \sim \epsilon_a$

Fig. 4. Fitting and test curves of $(\sigma_1 - \sigma_3) \sim \varepsilon_a$ and $\varepsilon_v \sim \varepsilon_a$ Fig. 5. Fitting and test curves of $(\sigma_1 - \sigma_3) \sim \varepsilon_a$ and $\varepsilon_v \sim \varepsilon_a$

Table 1. The parameters of the damage model of some group of samples

Sample No.	E_i (MPa)	c^s (MPa)	ϕ^s (°)	c^f (MPa)	ϕ^f (°)	μ_0	$n1$	$a1$	$a2$	$n2$	η_1	μ_2
J5	270	0.65	22.9	0.02	45.3	0.39	0.36	0.55	0.0023	4.2	350	2.2
G1	405	0.84	11.5	0.5	26.4	0.46	0.15	0.30	2.4	2.0	350	2.0
H4	680	0.46	40.7	0.2	40.1	0.465	0.10	0.60	2.0	0.457	350	2.2
J4	500	0.8	22.9	0.2	37.2	0.48	0.18	0.60	0.02	5.93	350	2.0

Table 2. The mix proportion of the samples

Unit: kg/m³

	Cement	Bentonite	Clay	Sand	Gravel	Water
J5	80	50	140	696	740	370
G1	100	6	240	488	600	410
H4	110	35	280	694	615	337
J4	80	40	160	689	817	340
EP	90	40	140	696	740	360

resistance stress, which results in the variation of the apparent stress, i.e. the term of $(\sigma_{ij}^s - \sigma_{ij}^f)dr$ in Eq. (5).

Referring to Eq. (4), from the view of constitution of apparent stress, when the deformation is small, the struc-

tural stress is dominant. Along with the increase of the deformation, the fraction of the frictional resistance stress increases to such an extent that, at the residual stage, all the load is carried by the frictional resistance part. Such stress transformation is shown in Fig. 7, and the evolution of the damage variable is shown in Fig. 8. It can be seen that, at the initial loading, the value of $r(\sigma_1 - \sigma_3)^f$ is so small that the value of E_{i0}^f has little influence on the total constitution. Furthermore, although the stress-strain relations of both the structural part and the frictional resistance part satisfy hyperbola, the total stress-strain relation does not satisfy it as a whole. If and only if $\sigma_{ij}^f = \sigma_{ij}^s$, the apparent stress-strain curve tends to hyperbola.

On the basis of the above analysis, when the confining pressure is low, along with the evolution of damage, the

Fig. 6. Stress-strain curves of the structural part and the frictional resistance part

Fig. 7. Decomposition of stress

Fig. 8. The evolution of the damage variable r

decrease of the mobilized strength of the structural part is less than the increase of that of the frictional resistance part; i.e. the value of $(\sigma_{ij}^s - \sigma_{ij}^f)dr$ is negative. When the structural force reaches its full level, i.e. $(1-r)(\sigma_1 - \sigma_3)^s$ reaches its peak value, the apparent stress strain curves begins to show strain softening. On the other hand, when the confining pressure is high, the value of $(\sigma_{ij}^s - \sigma_{ij}^f)dr$ is positive and the apparent stress strain curves will show strain hardening. Especially when the confining pressure reaches a certain large value, such as $\sigma_3 = 0.7-1.2$ MPa for sample J5, when the strength of the structural part gives its full effort, the strength of the frictional resistance part still has much to be mobilized. Therefore,

Fig. 9. The change of the Poisson's ratio and volumetric strain

along with the increase of the strain in a certain range, the apparent stress increases stably while the tangent modulus displays no obvious decrease, which agrees with the test, such as in Fig. 2 within $\varepsilon_a = 2-7\%$.

Volumetric Strain

As shown in Eq. (12), at the initial stage of loading, $r=0$ results in $\mu = \mu_0 / (\sigma_3 / p_0)^{n1}$, so μ_0 represents the initial Poisson's ratio at $\sigma_3 = p_0$ (0.1 MPa) and $n1$ reflects the change of the initial Poisson's ratio with the confining pressure. On the other hand, $a1$, $a2$ and $n2$ reflect the change of the Poisson's ratio with the confining pressure in the process of the progressive damage. Furthermore, when $r \rightarrow 1$, Eq. (12) results in $\mu \rightarrow 0.5$ due to the third term. For example, the change of the Poisson's ratio of sample J5 during the process of loading and the corresponding volumetric strain curves are shown in Fig. 9. When the confining pressure is low, such as $\sigma_3 = 0.1$ MPa, the Poisson's ratio increases markedly over a certain range of axial strain to become even larger than 0.5, then decreases and tends finally to be 0.5. It can be seen that, Eq. (12) can reflect the effect that the volumetric strain curves change from dilatation to contraction as the confining pressure increases and at the residual stage, the volumetric strain will not change anymore in spite of the confining pressure.

DETERMINATION OF PARAMETERS IN THE DAMAGE MODEL

The 12 parameters in the damage model can be identified by the results of a group of conventional triaxial tests and the procedures are described briefly as follows.

(1) E_{i0} : the initial modulus is calculated under each confining pressure and the average value is set as E_{i0} .

(2) c^f , ϕ^f : the damage variable r tends to 1 at the residual stage under each confining pressure, so that the ultimate strength is just that of the frictional resistance part. Therefore c^f and ϕ^f can be obtained from the ultimate strength.

(3) c^s , ϕ^s : in line with Eq. (4), $\sigma_{ij}^f = \sigma_{ij}^s$ results in $\sigma_{ij} = \sigma_{ij}^s$, i.e. the apparent stress-strain curve tends to hyperbola. It has been illustrated that if and only if $\sigma_{ij}^f = \sigma_{ij}^s$, the apparent stress-strain curve tends to hyperbola just as those of the structural part, and the frictional resistance part. Among all the stress-strain curves under various confining pressure, choose the one which is closest to hyperbola and set the confining pressure as σ_c . Under this condition ($\sigma_3 = \sigma_c$), the strength of the structural part is close to that of the frictional resistance part and that of the total sample, so that from Mohr-Coulomb's strength criteria we can get,

$$\sigma_c [\tan^2(45^\circ + \phi^s/2) - 1] + 2c^s \tan(45^\circ + \phi^s/2) = (\sigma_1 - \sigma_3)_{cu} \quad (25)$$

where $(\sigma_1 - \sigma_3)_{cu}$ is the ultimate deviatoric stress of the sample when $\sigma_3 = \sigma_c$. Furthermore, according to the data fitting of a lot of groups of tests with various mix proportions, c^s and q_u have such a relation,

$$c^s = C_k q_u \quad (26)$$

where $C_k = 1.67 - 27.7 / (q_u / \sigma_0 - 6)$, and q_u is the unconfined strength. From Eqs. (25) and (26), c^s and ϕ^s can be solved.

(4) $\mu_0, n1$: the initial Poisson's ratio under each confining pressure can be evaluated from the $\varepsilon_v - \varepsilon_a$ curves. At the initial stage of loading, $r=0$ and Eq. (12) results in,

$$\mu = \mu_0 / (\sigma_3 / \sigma_0)^{n1} \quad (27)$$

Plotting $\mu - \sigma_3 / \sigma_0$ data in the double logarithm coordinate system, the minus slope is $n1$ and the intercept at $\sigma_3 / \sigma_0 = 1$ is μ_0 .

(5) $a1, a2, n2$: the value of the damage variable r at any stage can be calculated by Eq. (3), which is expressed as,

$$r = [(\sigma_1 - \sigma_3)^s - (\sigma_1 - \sigma_3)] / [(\sigma_1 - \sigma_3)^s - (\sigma_1 - \sigma_3)^f] \quad (28)$$

where $(\sigma_1 - \sigma_3)^s$ and $(\sigma_1 - \sigma_3)^f$ can be obtained by virtue of the parameters already calibrated, and $(\sigma_1 - \sigma_3)$ can be obtained directly from the tested apparent stress-strain curves.

Equation (12) can be changed as,

$$a1 + a2(\sigma_3 Pa)^{n2} = r / [(\mu - 0.5r) / (1 - r) - \mu_0 / (\sigma_3 / Pa)^{n2}] \quad (29)$$

Resorting to Eq. (29) and by data fitting under various confining pressure, the parameters $a1, a2$ and $n2$ can be calibrated.

(6) η_1, η_2 : Eq. (14) can be rewritten as,

$$-\lg(1 - r) / \xi = \eta_1 \bar{\varepsilon}^{\eta_2} \quad (30)$$

where $\bar{\varepsilon}$ can be calculated by the tested $\varepsilon_v - \varepsilon_a$ curves, ξ can be calculated by the parameters determined above and the values of r calculated by Eq. (28). Plotting $-\lg(1 - r) / \xi - \bar{\varepsilon}$ data in the double logarithm coordinate system, the slope is η_2 and the intercept at $\bar{\varepsilon} = 1$ is η_1 .

VERIFICATION OF THE DAMAGE MODEL BY HOLLOW CYLINDER TESTS

Experiments

A group of conventional triaxial tests was first performed to determine the parameters of the damage model. Then two kinds of hollow cylinder tests with complicated stress states for the same mix proportion were carried out to verify the damage model. For the first type, the sample was wrapped up with membrane and there was a closed liner membrane inside the hollow. The loading process was as follows: (1) exert a certain value of pressure inside and around the sample; (2) exert pressure difference inside or around the sample so that the inner pressure is p_i and the outer pressure is p_o and the sample consolidates under such condition; (3) exert vertical loading progressively and measure the volume of the discharged water. Such a kind of test is called an "IOV test" for short.

Table 3. The cases of the IOV test and SOV test

Test No.	p_i (MPa)	p_o (MPa)	p_s (MPa)
IOV0112	0.1	1.2	
IOV0107	0.1	0.7	
IOV0704	0.7	0.4	
SOV00803	0.	0.8	0.3
SOV00403	0.	0.4	0.3

Fig. 10. The sample and the joint with the loading and draining set-up

Fig. 11. Fitting and test curves of $(\sigma_1 - \sigma_3) \sim \varepsilon_a$ and $\varepsilon_v \sim \varepsilon_a$

The other type, which is briefly called the “SOV test”, was the same as the IOV test except that the inner surface bears the water pressure directly and the loading process was as follows: (1) exert a certain value of confining pressure p_0 around the sample and the sample consolidates under such a condition; (2) exert water pressure p_i inside the hollow resulting in a steady seepage in the sample; (3) exert vertical loading progressively and measure the volume of the seepage water. Strain control is adopted for vertical loading in all the tests and the vertical loading speed is 0.1 mm/min. The cases for the IOV and SOV tests are shown in Table 3.

The sample for the conventional triaxial tests is a cylinder 300 mm in height and 150 mm in diameter; for the IOV and SOV tests there is a hole 30 mm in diameter in the center. The mix proportion is shown in Table 2. All the samples are saturated.

Two metal slabs are laid on both ends of the sample.

The face close to the sample is a plane and there are many slots on the other face. Two layers of silica oil sandwiched in three layers of plastic membrane are put between the sample and the metal slabs to weaken the end restriction. Gauze strips are stuck to the side of the sample and joined with the slots of the slabs for draining (as shown in Fig. 10).

The results of the conventional triaxial tests are shown in Fig. 11. It can be seen that it displays the common features of soil-cement mixtures.

The results of the IOV tests are shown in Figs. 12–14. In the IOV0112 test, the stress-strain curve shows strain hardening and the volumetric strain shows contraction. In the IOV0704 test, the inner pressure (0.7 MPa) is greater than the outer pressure (0.4 MPa), therefore the circular stress is the minor principal stress and the value is fairly small. Thus the sample displays brittle behavior as a whole and consequently it shows strain softening

Fig. 12. Comparison of the experimental and predicted data and the calculated σ_r and σ_θ along the radius

Fig. 13. Comparison of the experimental and predicted data and the calculated σ_r and σ_θ along the radius

and volumetric dilative tendency when the axial strain is rather large. The result of the IOV0107 test is between those of the two tests just mentioned.

The results of the SOV tests are shown in Figs. 15–16. In the SOV00803 test, the outer confining pressure is quite high and the minor principal stress is also quite high over a large range in the sample so that the stress-strain curve shows strain hardening as a whole; otherwise, in the SOV00403 test, the outer confining pressure is fairly low and the minor principal stress is also fairly low in the sample so that the stress-strain curve shows a general brittle behavior.

FEM Analysis of the Hollow Cylinder Tests

All the tests were carried out under axial symmetric condition. Circular elements were used, of which only principal stress and principal strain were needed so that calculation was greatly simplified. Furthermore, only a

layer of elements was used in the calculation because the stress and strain does not change along the vertical direction. The cross section of the element is an isosceles right triangle. The discretized mesh is shown in Fig. 17.

Under stable seepage, the seepage force j at any point in the radial direction can be expressed as (Sun, 1985),

$$j = p_s / [\bar{r} \ln(b/a)] \quad (31)$$

and the nodal force p_{ir} is,

$$p_{ir} = (\pi/4) p_s d^2 / \ln(b/a) \quad (32)$$

where $\bar{r}$ is the radius at the node, b and a are the radius of the sample and the hollow, respectively, and d is the length of the right side of the elements.

Comparison between the Experiment and the Calculation

The parameters of the damage model determined by

Fig. 14. Comparison of the experimental and predicted data and the calculated σ_r and σ_θ along the radius

Fig. 15. Comparison of the experimental and predicted data and the calculated σ_r and σ_θ along the radius

the triaxial tests are shown in Table 4 and the fitting curves denoted by solid lines in Fig. 11 are very close to the test dots.

Comparison between the results of experiment and calculation for the IOV tests are shown in Figs. 12–14. The minor principal stress distributes nonuniformly along the radial direction so that the distribution of the axial stress is also nonuniform along the radial direction and consequently a complicated stress state arises in the sample. For the volumetric strain, the prediction agrees very well with the tests. For the stress-strain curves, the predicted

Fig. 16. Comparison of the experimental and predicted data and the calculated σ_r and σ_θ along the radius

Fig. 17. The discretized mesh

results and that of the test are close to each other at both ends, but in the middle part, the tested curves are higher than the predicted ones. This may result from the stress concentration, whereas the effect of the intermediate principal stress is not included in the damage model.

Comparison between the stress-strain relations of experiment and calculation for the SOV tests are shown in Figs. 15–16, where only the predicted volumetric strain are shown because the volume change is not measured in the experiment. It can be seen that the prediction of the stress-strain relation shows good correlation with the observed response.

As a whole, the damage model provides satisfactory prediction for laboratory tests with relatively complicated stress states.

CONCLUSIONS

(1) In the frame of the continuous damage theory and based on experimental observation, a damage model for soil-cement mixtures is constituted using the unified form with one set of parameters.

(2) The suggested model can reflect the main mechanical properties of soil-cement mixtures, especially the following features: as the confining pressure increases, the stress-strain relations change from strain-softening to strain-hardening while the volumetric strain changes from dilatation to contraction; when the confining pressure is rather high, the tangent modulus in the middle of the stress-strain curves of some samples has no obvious change over a wide range.

(3) How the damage model for soil-cement mixture can reflect its main mechanical properties has been analyzed.

(4) The procedures to determine the parameters of the damage model were described. All the parameters can be calibrated by a group of conventional triaxial tests.

(5) Hollow cylinder tests with complicated stress state were conducted to verify the damage model. The results predicted by the damage model agree well with the tests which indicates that the suggested damage model is applicable in complicated stress states.

Table 4. The parameters of the damage model of the sample

E_i (MPa)	c^s (MPa)	ϕ^s (°)	c^f (MPa)	ϕ^f (°)	μ_0	$n1$	$a1$	$a2$	$n2$	η_1	η_2
280	0.55	25.8	0.045	45	0.43	0.6	0.551	0.0023	4.2	350	2.2

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