PREDICTION OF ULTIMATE BEARING CAPACITY OF SURFACE FOOTINGS WITH REGARD TO SIZE EFFECTS

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ABSTRACT

A simplified prediction method based on Terzaghi's bearing capacity formula for surface footing, with regard to size effects, is proposed. Clarification of size effects, which is indispensable for appropriate design of large foundations, may permit use of the results of small scaled loading tests and prevent increases in the construction costs of foundations. Size effects may be mainly attributed to stress-level effects on the shear strength of ground materials; stress-level effects are taken into account in the proposed method.

The proposed method incorporates a method for determination of shear strength constants c and ϕ that should be applied to the bearing capacity formula. A nonlinear Finite Element Method which incorporates the confining stress dependent shear strength is used to calculate stress distribution in foundation grounds beneath the footings. It is shown that the effective mean principal stress σ_m tends to range from $2 \gamma B$ to $10 \gamma B$ for strip footings and from $1 \gamma B$ to $15 \gamma B$ for circular footings; within this region the parameters c and ϕ are determined from the straight fitting line.

The proposed method is verified with the results of some series of model surface footing loading tests on cohesionless and cohesive grounds reported by the authors and other researchers. This verification confirms the applicability of the proposed method to strip, circular and square footings on cohesive and cohesionless foundation grounds with sufficient accuracy for practical purposes.

Key words: bearing capacity, finite element method, shallow foundation, shear strength, (stress-level effect) (IGC: E3/E13)

INTRODUCTION

Terzaghi's bearing capacity formula is widely used in current design codes (Terzaghi and Peck, 1967):

$$q_u = S_c c N_c + S_\gamma \frac{1}{2} \gamma B N_\gamma + S_q \gamma D_f N_q \quad (1)$$

where q_u is ultimate bearing capacity and c is cohesion; B and D_f are breadth and embedded depth of the footing, respectively. Parameters S_c , S_γ and S_q are shape factors, and N_c , N_γ and N_q are bearing capacity factors; $S_c=1.3$, $S_\gamma=0.6$ and $S_q=1.0$ for circular footings, and $S_c=1.3$, $S_\gamma=0.8$ and $S_q=1.0$ for square footings. According to Terzaghi and Peck (1967), Prandtl and Reissner derived the exact mathematical expressions for bearing capacity factors N_q and N_c :

$$N_q = \exp(\pi \tan \phi) \tan^2 \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \quad (2)$$

$$N_c = (N_q - 1) \cot \phi \quad (3)$$

For N_γ several formulae have been proposed by many researchers, as cited by Zadoroga (1994). Vesić (1973) recommended using the bearing capacity factor proposed by Caquot and Kérisel (1953) which can be approximated by

$$N_\gamma = 2(N_q + 1) \tan \phi \quad (4)$$

In this study Eqs. (3), (4) are employed for the calculation of the ultimate bearing capacity of surface footings.

Bearing capacity factors N_c , N_γ and N_q are functions of angle of internal friction ϕ and are independent of footing breadth B in the formulae as shown by Eqs. (2)–(4); however, the bearing capacity factor N_γ calculated from the loading tests of footings on sand grounds reduces with increasing footing size (Golder, 1941; De Beer, 1963, 1965; Okahara et al., 1988; Kusakabe et al., 1991, 1992; Ueno et al., 1994). This phenomenon is referred to

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Manuscript was received for review on August 9, 1997.

Written discussions on this paper should be submitted before April 1, 1999 to the Japanese Geotechnical Society, Sugayama Bldg. 4F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101-0063, Japan. Upon request the closing date may be extended one month.

as the size effects of footing breadth on the bearing capacity factors. To compensate for the reduction of N_γ , exceedingly large safety factors are usually used. While Terzaghi's bearing capacity formula has contributed remarkably to the practical design of foundations, the lack of consideration of scale effects has prevented an appropriate design and led to inflated construction costs. It is, therefore, important to clarify the mechanism of size effects and develop a rational calculation method for the bearing capacity of footings.

Tatsuoka et al. (1991) investigated the size effects of the bearing capacity of spread footing experimentally and analytically; they found that the size effects are caused by confining stress level effects on shear strength, progressive failure of foundation grounds and scale effects of footing breadth with respect to the particle size of ground materials. It should be noted that the scale effect cannot be detected in centrifuge loading tests, and Ovesen (1979) commented that the scale effect is emphasized only in small scaled footings whose footing breadth B is less than 30 times the grain size of foundation ground material. Tatsuoka et al. (1991) succeeded in simulating the size dependent behaviors of spread footings by means of an advanced FEM. However, simplified methods which incorporate the conventional bearing capacity formula are needed for practical design purposes.

There are simplified methods taking the stress level effects into account, in which the stress-level effect is regarded as a main cause of the size effects. Meyerhof (1950) and De Beer (1970) proposed that the angel of internal friction ϕ to be applied to the bearing capacity formula, must be selected in accordance with the stress level beneath the footing. Meyerhof (1950) suggested that the mean normal stress on the failure planes σ_0 is about a tenth of ultimate bearing capacity q_u , and De Beer (1970) presented the formula for averaged σ_0 as

$$\sigma_0 = \frac{q_u + 3\gamma D_f}{4} (1 - \sin \phi) \quad (5)$$

Hettler and Gudehus (1989) supported Meyerhof's suggestion, and Perkins (1995) applied it to a highly frictional material. Though this type of methods seems to be promising for practical design, it is applicable only to cohesionless soils and not to materials with cohesion, such as cemented sand and gravel, because internal friction is determined from secant line to curved failure envelop, and the term for cohesion c in the bearing capacity formula is neglected in their methods. Also an iterative process is needed for determining the angle of internal friction ϕ corresponding to the stress range of interest, because the stress level is given as a function of ultimate bearing capacity q_u . Kutter et al. (1988) introduced a method for determining shear strength of the Mohr-Coulomb type with regard to apparent cohesion c even for cohesionless soils, and used the same constants for footings with different sizes.

The authors propose a method for evaluating size effects on the bearing capacity of surface foundations, in

which the stress level effect is considered as suggested by Meyerhof (1950) and others. To find the stress level beneath the footing, a nonlinear FEM devised by the authors, which takes the confining stress dependency of the shear strength of ground materials into account, as applied in plane strain condition for strip footings and axial symmetrical condition for circular footings. In the proposed method the stress range concerned is given as a function of footing breadth B and unit weight of ground soil γ , but not of ultimate bearing capacity q_u ; the strength constants c and ϕ that are given from a straight fitting line of curved failure envelope are applied to the bearing capacity formula (Eq. (1)). Validations of the proposed method with both numerical analysis and experiments are presented in this paper.

STRESS DEPENDENCY OF SHEAR STRENGTH AND SIZE EFFECTS

Figures 1(a)–(c) show the results of a series of conven-

Fig. 1. Mohr's stress circles at failure and failure envelopes obtained from conventional drained triaxial compression tests on: (a) Toyoura sand after Miura (1981); (b) Shirohata scoria after Kusakabe et al. (1992); (c) Mori volcanic soil after Takeda (1992)

Fig. 2. Logarithmic relationship between $\tan \phi$ determined from secant lines and mean principal stress σ'_m for granular materials

tional triaxial compression tests on three different granular materials. While the Mohr-Coulomb failure criterion assumes straight failure envelopes as shown with dashed lines in these figures, actual failure envelopes for the materials are upper convex curves. While Toyoura sand in Fig. 1(a) is cohesionless, Shirohata scoria and Mori volcanic ash soil in Figs. 1(b), (c) have notable cohesion in undisturbed state; all the tested materials show marked nonlinearity of envelope. Shown in Fig. 2 is the logarithmic relationship of $\tan \phi$ calculated from secant lines as explained in Fig. 3(a) with mean principal stress $\sigma_m = (\sigma_1 + \sigma_3)/2$. Maeda, K. and Miura (1998) investigated the confining stress dependency of mechanical properties of sands extensively; deformation-failure behaviors of about 80 granular materials were observed in conventional drained triaxial compression tests under variable effective confining stresses. Additionally they showed that all the tested cohesionless materials except glass beads exhibit a notable confining stress dependency of ϕ and the linear relationships can be found in $\log \tan \phi$ vs. $\log \sigma'_m$ plots, as for Toyoura sand shown in Fig. 2.

For cohesive soils the confining stress dependent failure criterion must be extended as shown in Figs. 3(b), (c):

$$\tan \phi^* = \tan \phi_a^* \left(\frac{\sigma_a}{\sigma_m^*} \right)^\alpha \quad (6)$$

$$\sigma_m^* = \sigma_m + \sigma_t$$

where σ_t is tensile strength, and σ_a is reference atmospheric stress equal to 98 kPa. Parameters α and ϕ_a^* were introduced to express the assumed linearity in $\log \tan \phi^*$ vs. $\log \sigma_m^*$ plots, shown in Fig. 3(c); according to Maeda, K. and Miura (1998), α for cohesionless granular materials seems to range from 0.02 to 0.12. This extended failure criterion for the confining stress dependency was incorporated into the *Multi-Directional Sliding Model* developed by Miura and Finn (1988) for predicting three dimensional deformation-failure behaviors of soils.

Figure 4 shows the logarithmic relationships between normalized ultimate bearing capacity $2q_u/\gamma B$ and normalized footing breadth $\gamma B/\sigma_a$ of surface footings,

Fig. 3. Failure criterion with confining stress dependency of shear strength: (a) Nonlinear failure envelope for cohesionless soils; (b) Extended nonlinear failure envelope for cohesive soils; (c) Logarithmic linear relationship between $\tan \phi^*$ and extended mean normal stress σ_m^*

which were obtained from some test series conducted recently by many researchers including the authors. Further information about the test results quoted in Fig. 4 is given in Table 1. According to the bearing capacity formula, for surface footings on cohesionless foundation ground, $2q_u/\gamma B$ should be equal to bearing capacity factor N_γ ; the test series shown in Fig. 4 are, except for the SQ series, all for cohesionless grounds. The normalization for the breadth of footing $\gamma B/\sigma_a$ is effective in demonstrating the size effects on bearing capacity, based

Fig. 4. Relationship between ultimate bearing capacity and footing breadth quoted from the test results by various researchers; see Table 1

on the results from the model tests including centrifuge tests, where γ increases with centrifugal acceleration. While N_γ is expected to be constant independently of the variation of B from the original meaning of Terzaghi's bearing capacity formula (Eq. (1)), the plots of $\log(2q_u/\gamma B)$ against $\log(\gamma B/\sigma_a)$ with negative correlation indicate a notable reduction in N_γ with increasing B ; this reduction is referred to as size effects (see e.g., De Beer, 1963, 1965).

The above mentioned confining stress dependency of the shear strength of ground material is one of the most important factors in controlling the size effect of surface footings; even for cohesionless materials, ϕ is not constant and reduces with an increase in confining stress. If a footing becomes larger, the location of the failure surface becomes deeper; the depth is approximately proportional to the breadth of footing B . Additionally, in an identical foundation ground made of non-cohesive soil, bearing capacity q_u could be approximately in proportion to the value of γB , as long as the intensity of the confining stress dependency of shear strength is low. Therefore, it can be expected that the stress range to be interested in the foundation ground beneath the footing should be in proportion to the value of γB . Higher stress leads to a lower angle of internal friction ϕ , which yields a smaller bearing capacity factor N_γ . Consequently, N_γ

does decrease with increasing γB ; this is a major mechanism of size effects related to stress level effect on shear strength.

NUMERICAL ANALYSIS METHOD-Finite Element Method

In order to derive a prediction method for ultimate bearing capacity, idealized numerical model grounds were prepared and analyzed by means of a Finite Element Method (FEM) developed by the authors. The FEM can take account of the confining stress dependency of shear strength specified by Eq. (6), and the constitutive model used was a bi-linear model, where if the maximum shear stress $\tau_m(=(\sigma_1 - \sigma_3)/2)$ exceeds the failure value $\tau_{mf}(=\sigma_m \sin \phi^*)$ obtained from Eq. (6) in an element, maximum shear stress τ_m and shear modulus G are reduced to the failure value τ_{mf} and one ten-thousandth of the initial value, respectively, as shown in Figs. 5(a), (b). To dissipate the excess shear stress over the failure criterion, the dynamic time historical calculation schema proposed by Zienkiewicz and Corneau (1974) was incorporated, where the inertial force and quasi viscous stress by Rayleigh's method are combined to stabilize calculation processes. The time step used was equal to 0.02 sec, and the iterative calculation step was continued until the excess shear stress becomes less than a hundredth of the critical failure value in all the finite elements. Further information about this dynamic nonlinear FEM is provided in Ueno (1991).

Figure 6 shows the shape of a numerical model foundation ground, whose dimensions are $4.5B$ in width and $4B$ in depth; the proportion was kept identical through a series of analyses. Both plane strain condition for strip footings and axial symmetric condition for circular footings were accounted for through the series of analysis. The properties of footings and model grounds are tabulated in Table 2. The numerical values of the moduli of elasticity including Young's modulus, Poisson's ratio and shear modulus, were employed as typical values, because it was certified in the preliminary investigation by Ueno (1991) that the selection of these values does influence the load-displacement curves, but not the ultimate bearing capacity and ultimate stress distribution in failure zone. Shown in Figs. 7(a), (b) are typical calculation results for strip and circular footings by the FEM employed in this study: $\phi^*=40$ deg, $\sigma_t=17$ kPa ($c=19.6$

Table 1. Testing conditions of loading tests of footings quoted in present paper

Series	Footing	Method	Soil	D_r (%)	Citation
TC	Circular	Centrifuge	Toyoura sand	70%	Ueno et al. (1994)
TS-1	Strip	Centrifuge	Toyoura sand	88%	Okahara et al. (1994)
TS-2				74%	
TS-3				58%	
IC	Circular	Centrifuge	Inagi sand	80%	Kusakabe et al. (1991)
SQ	Square	in-situ	Shirahata scoria	—	Kusakabe et al. (1992)
BS	Circular	Centrifuge	Monterey 0/30 sand	93–95%	Kutter et al. (1988)

Fig. 5. Explanation of excessive shear stress: (a) Definition of excessive shear stress with respect to failure criterion; (b) Correction of excess shear stress in the FEM

kPa) and $\alpha=0$. These figures show that the ultimate bearing capacity can be determined easily, due to the efficiency of the above mentioned algorithms for the dissipation of stress fraction exceeding failure condition and the stabilization of the calculation after the failure of the foundation ground. Typical failure patterns in, foundation ground for strip and circular footings are shown in Figs. 8(a), (b); $\phi^*=40$ deg, $\sigma_t=17$ kPa ($c=19.6$ kPa)

Fig. 6. Geometry of numerical model ground and footing used for the calculation by FEM

Table 2. Conditions of performed parametric FEM analysis

Footing shape	strip and circle
Footing breadth, B (m),	0.2-20
Unit weight, γ (kN/m ³)	19.6
Coef. of earth pressure at rest, K_0	0.5
Young's modulus	$E=E_a\sigma_a\left(\frac{\sigma_m}{\sigma_a}\right)^m$ $E_a=1000$ $m=0.5$
Referential stress, σ_a (kPa)	98
Poisson's ratio, ν	0.3, 0.4
Shear modulus	$G=\frac{E}{2(1+\nu)}$
Shear strength	$\tan \phi^*=\tan \phi_a^*\left(\frac{\sigma_a}{\sigma_m^*}\right)^\alpha$
Friction angle, ϕ_a^* (deg.)	0-45
Tensile strength, σ_t (kPa)	0-34
Stress dependency, α	0-0.3

and $\alpha=0$. In these figures the elements whose maximum shear stress τ_m is greater than $0.98\tau_{mf}$ are shaded.

Figures 9(a), (b) show bearing capacity factors N_c and N_γ of the numerical model ground obtained by FEM an-

Fig. 7. Typical calculation results by FEM: (a) For strip footing; (b) For circular footings

Fig. 8. Failure patterns in foundation grounds ($\gamma=19.6 \text{ kN/m}^3$, $\phi_a^*=40 \text{ deg}$, $\sigma_t=17 \text{ kPa}$ ($c=19.6 \text{ kPa}$) and $\alpha=0$): (a) For strip footing; (b) For circular footing

Fig. 9. Comparisons of bearing capacity factors calculated by FEM and calculated from Eq. (3), (4): (a) For N_c ; (b) For N_γ

alysis for the case of no stress dependency of shear strength ($\alpha=0$). The factors N_c and N_γ were determined from the linear relationship between q_u and $\gamma B/2$, where the slope and the intercept correspond to N_γ and cN_c , respectively. This linear relationship assumed in Eq. (1) was also recognized in the parametric study with the FEM and other numerical methods; see Ueno (1991) and Pope (1975). Bearing capacity factors given by Eqs. (3), (4) are also plotted in Figs. 9(a), (b). The tendency is recognized in these figures for strip footing N_c and N_γ by FEM to be smaller than those by Eqs. (3), (4) in the region of the large angle of internal friction ϕ . It is expected from preliminary examinations that finer meshing and a finer time step will lessen the gaps between the values obtained by the FEM and Eqs. (3) and (4) (Ueno, 1991); nevertheless, the authors compromised on the mesh model illustrated in Fig. 6 and the above mentioned calculation scheme to economize on effort for the calculations. The perceptible gaps between the FEM and Eqs. (3), (4) and the comparison between the values of strip and circular footings are beyond the scope of this paper.

PROPOSED PREDICTION METHOD OF ULTIMATE BEARING CAPACITY

Stress Range in the Ground beneath Footing

The distributions of stress in the failure zone of the foundation ground are shown in Figs. 10(a)–(d); the area of elements judged to have failed with $\tau_m > 0.98\tau_{mf}$ was arranged for normalized effective mean principal stress $\sigma_m/\gamma B$ in these histograms; see also Figs. 8(a), (b). While only limited histograms are presented in this paper, the following useful aspects were found:

- The stress distributions are represented almost uniquely with normalized effective mean principal stress $\sigma_m/\gamma B$, despite the difference in footing breadth B and the strength parameters α , ϕ_a^* and σ_t as shown in Figs. 10(a)–(h).
- Though the distribution pattern is different for strip and circular footings, it is clear that the stress distribution of the failure zone can be uniquely presented by $\sigma_m/\gamma B$ for both types of footings.
- The range of $\sigma_m/\gamma B$ of interest is derived from all

Fig. 10. Histograms for distribution of effective mean principal stress in failed elements: a) For strip footing with variable B ; b) For circular footing with variable B ; c) For strip footing with variable α ; d) For circular footing with variable α ; e) For strip footing with variable ϕ_a^* ; f) For circular footing with variable ϕ_a^* ; g) For strip footing with variable σ_i ; h) For circular footing with variable σ_i .

the calculation results by the FEM including those not shown in Figs. 10(a)–(h); $\sigma_m/\gamma B$ is assumed to range from 2 to 10 for strip footing and from 1 to 15 for circular footing.

In this process the mean normal stress σ_m normalized by γB was selected as the best parameter for the unique representation of the stress range independently of the footing breadth B (Fig. 10(a), (b)), the stress dependency of shear strength (Fig. 10(c), (d)), secant internal friction angle ϕ_a^* (Fig. 10(e), (f)) and existence of cohesion σ_t (Fig. 10(g), (h)). If the normal stress on the failure plane σ_0 is used as suggested by Meyerhof (1950) and others, the stress distribution cannot be explained uniquely and is largely affected by cohesion c . Minor and major principal stresses, σ_1 and σ_3 , could also not be used for the same reason.

Proposal of Simplified Prediction Method

First appropriate strength constants c and ϕ for particular foundations which should be applied to Terzaghi's bearing capacity formulae (Eq. (1)), are determined. As shown by the calculation results from the FEM, the stress range of interest for determining c and ϕ can be represented as

$$a\gamma B \leq \sigma'_m \leq b\gamma B \quad (7)$$

where parameters a and b for strip and circular footings are listed in Table 3. A conceptual illustration for the relationship between footing breadth and confining stress range is shown in Fig. 11; soil elements beneath foundations are subjected to the confining stress whose magnitude depends on the breadth or diameter of footings. In this figure the breadth of Foundation-2 B_2 is three times larger than that of Foundation-1 $B_1 (=B_2/3)$, and the stress range for Foundation-2 would become higher and wider than that for Foundation-1. Consequently, ϕ_2 is smaller than ϕ_1 , while c_2 is larger than c_1 .

The procedure proposed for the prediction of ultimate bearing capacity is as follows:

- Prepare the test results of shear strength obtained by laboratory element tests such as triaxial compression tests for the stress range from $\sigma_m = a\gamma B$ to $b\gamma B$. If proper data is not available, the shear strength for the range can be estimated by interpolation or extrapolation with the aide of a confining stress dependent failure criterion by Figs. 3(a)–(c) and Eq. (6).
- Obtain the appropriate shear strength parameters c and ϕ on $\tau_m - \sigma_m$ plane as shown in Fig. 12(a). If two failure shear stresses τ_a and τ_b corresponding to

Fig. 11. Conceptual illustration of size effect on bearing capacity related to confining stress dependency of shear strength of ground

Fig. 12. Illustration of determination method of c and ϕ : (a) in $\tau_m - \sigma_m$ plane; (b) in $\tau - \sigma$ plane

$\sigma_m = a\gamma B$ and $b\gamma B$ are determined, c and ϕ can be given by

$$\begin{aligned} \sin \phi = \tan \psi &= \frac{\tau_a - \tau_b}{(1 - b)\gamma B} \\ c &= \frac{\eta}{\cos \phi} = \frac{\tau_a - a\gamma B \sin \phi}{\cos \phi} \end{aligned} \quad (8)$$

- Calculate ultimate bearing capacity q_u by substituting c and ϕ obtained above into the bearing capacity formulae Eqs. (1), (3) and (4).

Table 3. Stress ranges of interest for determining shear strength constants c and ϕ , for strip footing and circular footing

Footing	$\sigma_m/\gamma B$	
	a	b
Strip footing	2	10
Circular footing	1	15

VERIFICATION OF THE PROPOSED PREDICTION METHOD

The proposed prediction method for the bearing cap-

acity of surface footing is verified in this section; the numerical simulation results by FEM and the experimental results shown in Fig. 4 and Table 1 are used for the verification.

Verification with Numerical Simulation Results

By means of the FEM introduced in the preceding section, the behaviors of strip and circular footings were simulated; the bearing capacity of the footings on the ground with different confining stress dependent shear strength were calculated. The conditions of the calculations are listed in Table 2.

Figures 13(a)–(d) and 14(a)–(d) show comparisons between the normalized ultimate bearing capacity $2q_u/\gamma B$ predicted by the proposed method and that calculated by the FEM analysis. The sensitivity of the calculated values of ultimate bearing capacity were examined with respect to the parameter a and b for strip footings in Figs. 13(a)–(d). These comparisons presented in Figs. 14(a)–(d) show that the predicted and calculated values agree with sufficient accuracy for engineering practice under various situations for strip and circular footings. The proposed method can predict the trends of the reduction in normal-

ized bearing capacity $2q_u/\gamma B$ with increasing footing breadth B for non-cohesive soils with confining stress dependent shear strength as shown in Fig. 14(a)–(d), and it seems also to be applicable to cohesive soils with tensile strength as shown in Fig. 14(e), (f). While only limited cases are presented in Figs. 14(a)–(f), the good agreement between the predicted and calculated values can be recognized for circular footings as well as strip footings.

Verification with Experimental Results

The proposed method is verified with experimental results reported by researchers, shown in Fig. 4 and Table 1. Shear strength constants c and ϕ were obtained from the experimental results by laboratory element tests such as drained triaxial compression tests. Due to some scatters included in the data from the element tests, the test results were first fitted to the confining stress dependent failure criteria proposed by the authors (Eq. (6)). Then c and ϕ for the stress range of interest from $\sigma_m = a\gamma B$ to $b\gamma B$ were determined and substituted into the bearing capacity formula Eq. (1). The surcharge term in the formula vanishes, because all the footings were on the surface. The values of bearing capacity factors N_c and N_γ

Fig. 13. Sensitivity of calculated ultimate bearing capacities for strip footing with respect to parameters a and b : (a) $\alpha=0.1$ with variable a ; (b) $\alpha=0.1$ with variable b ; (c) $\alpha=0.3$ with variable a ; (d) $\alpha=0.3$ with variable b

were evaluated by Eqs. (3), (4), except for Series BS. For Series BS, Hansen's formula of N_γ was used instead of Eq. (4), in order to make a comparison with the predic-

tion by Kutter et al. (1988). The following five paragraphs are descriptions of each of these verifications.

[Series TC] Series TC is a part of the results of cen-

Fig. 14. Comparisons between FEM analysis and proposed method: (a) For strip footing with variable α ; (b) For circular footing with variable α ; (c) For strip footing with variable ϕ_a^* ; (d) For circular footing with variable ϕ_a^* ; (e) For strip footing with variable σ_t ; (f) For circular footing with variable σ_t .

trifuge model loading tests conducted by one of the authors on a surface circular footing having a diameter of 3 cm, under accelerations ranging 98 m/sec² (10 g) to 980 m/sec² (100 g). Details of this series of centrifuge model tests are described in Ueno et al. (1994). The sand used in Series TC was air-dried Toyoura sand, which possesses no cohesion. The strength characteristics of Toyoura sand were evaluated by conventional drained triaxial compression tests (Miura, 1981), where the confining stress dependency of shear strength was accounted for. The results are shown in Figs. 15 and 16. With the assumption of independence between confining stress dependency and density dependency, the shear strength constants were defined as a function of effective mean normal stress σ_m^* and relative density D_r :

$$\tan \phi^* = \tan \phi_a^* \left(\frac{\sigma_a}{\sigma_m} \right)^{0.05}, \quad \alpha = 0.05, \quad \sigma_t = 0 \text{ kPa}$$

$$\tan \phi_a^* = 0.425 D_r + 0.500 \quad (9)$$

The results of the prediction from this shear strength are shown in Fig. 17 and Table 4. While the proposed prediction method slightly overestimates ultimate bearing capacity q_u compared with experimental results in this case, the tendency of decrease in $2q_u/\gamma B$ with an increase

Fig. 15. Determination of shear strength of ground materials from laboratory element tests with the aid of proposed confining stress dependent failure criterion

Fig. 16. Dependency of shear strength of Toyoura sand on density

in B by prediction and experiments are in good agreement.

[Series TS] Series TS is a series of centrifuge model loading tests on surface strip footings by Okahara et al. (1988). The sand used was Toyoura sand, so that the expression of shear strength used is identical with that for Series TC. In this series, the model grounds had three different relative densities D_r of 58%, 74% and 88%. Therefore, it can be expected that these test results will verify the applicability of the proposed method in a wider range of conditions, including that of Series TC.

The comparison between predicted and measured

Fig. 17. Comparison of predicted and measured ultimate bearing capacity for Series TC: Circular footings in centrifuge model tests

Table 4. Prediction of ultimate bearing capacity for 4 series of experiments

Series	B (m)	c (kPa)	ϕ (deg.)	Bearing factors		q_u (MPa)
				N_c	N_γ	
TC $\gamma = 15.7 \text{ kN/m}^3$	0.03	0.03	46.4	160.	358.	0.061
	0.30	0.33	43.1	106.	190.	0.336
	3.00	3.28	39.8	74.0	106.	1.95
TS-1 $\gamma = 15.7 \text{ kN/m}^3$	0.03	0.05	47.3	181.	428.	0.109
	0.30	0.46	44.0	118.	224.	0.582
	3.00	4.59	40.7	81.2	124.	3.28
TS-2 $\gamma = 15.2 \text{ kN/m}^3$	0.03	0.04	45.2	138.	284.	0.071
	0.30	0.45	41.9	93.0	154.	0.391
	3.00	4.41	38.7	65.6	87.3	2.28
TS-3 $\gamma = 14.7 \text{ kN/m}^3$	0.03	0.04	42.7	101.	176.	0.043
	0.30	0.43	39.4	70.8	98.9	0.249
	3.00	4.21	36.2	51.6	58.3	1.50
IC $\gamma = 16.5 \text{ kN/m}^3$	0.03	0.03	44.5	126.	247.	0.043
	0.30	0.26	42.0	113.	156.	0.278
	3.00	2.63	39.5	71.6	101.	1.84
SQ $\gamma = 7.35 \text{ kN/m}^3$	0.30	118.	38.2	62.8	81.1	13.4
	0.40	118.	38.2	62.7	80.8	13.4
	0.70	118.	38.2	62.3	80.0	13.3
	1.30	117.	38.1	61.7	78.7	12.2

$2q_u/\gamma B$ is shown in Fig. 18. Fairly good agreement between them can be recognized; in the worst case of $D_r=58\%$ the prediction method tends to overestimate q_u compared with the measured values. From the comparisons in Series TC and Series TS, it can be said that the proposed method predicts the ultimate bearing capacity of circular and strip footings with regard to the size effects.

[Series IC] Series IC, conducted by Kusakabe et al. (1991), is a series of centrifuge loading tests on surface circular footings on air dried Inagi sand possessing no cohesion. The confining stress dependent shear strength of Inagi sand was examined using the results of conventional triaxial compression tests conducted by Kusakabe et al. (1991). The test results are shown in Fig. 15, where the least square method was applied to find the best fit.

$$\tan \phi^* = 0.897 \left(\frac{\sigma_a}{\sigma_m^*} \right)^{0.038},$$

$$\tan \phi_a^* = 0.897, \quad \alpha = 0.038, \quad \sigma_t = 0 \text{ kPa} \quad (10)$$

The normalized bearing capacity $2q_u/\gamma B$ by the prediction method and the experiment are compared in Fig. 19. In this series the prediction method tends to underestimate q_u compared with the measured values, while in series TC and TS the prediction method tends to overestimate; nevertheless, the tendency of the reduction in $2q_u/\gamma B$ with increasing footing breadth B is in good agree-

Fig. 18. Comparison of predicted and measured ultimate bearing capacity for Series TS-1, 2, 3: Strip footings in centrifuge model tests

Fig. 19. Comparison of predicted and measured ultimate bearing capacity for Series IC: Circular footings in centrifuge model tests

ment between predicted and measured values.

[Series SQ] Series SQ was conducted as part of a series of large-scaled in situ loading tests by Kusakabe et al. (1992); square footings with different breadths of $B=0.3$ m to 1.3 m on a natural deposit were loaded in a pneumatic caisson. The submerged stratum on which the loading tests were conducted consists of Shirahata scoria, which is a kind of uniform volcanic granular soil. Because the Shirahata scoria is a diluvial deposit and cemented, it has notable tensile strength of $\sigma_t=147$ kPa:

$$\tan \phi^* = 0.816 \left(\frac{\sigma_a}{\sigma_m^*} \right)^{0.023},$$

$$\tan \phi_a^* = 0.816, \quad \alpha = 0.023, \quad \sigma_t = 147 \text{ kPa} \quad (11)$$

The comparison between predicted and measured ultimate bearing capacities is presented in Fig. 20. Series SQ is only the case where the footings were placed on the foundation ground with cohesion; as shown in this figure applicability of the proposed prediction method to cohesive ground as well as non-cohesive foundation grounds can be confirmed.

[Series BS] In Series BS, centrifuge model tests were conducted by Kutter et al. (1988) on surface circular footings on dense Monterey 0/30 sand. Shear strength properties of Monterey 0/30 sand were introduced by Kutter et al. (1988). Failure criterion of this sand was determined with the least square method as shown in Fig. 15:

$$\tan \phi^* = 1.177 \left(\frac{\sigma_a}{\sigma_m^*} \right)^{0.089},$$

$$\tan \phi_a^* = 1.177, \quad \alpha = 0.089, \quad \sigma_t = 0 \text{ kPa} \quad (12)$$

Kutter et al. (1988) demonstrated the accuracy of their prediction method compared with the methods proposed by Meyerhof (1950) and De Beer (1963). Table 5 shows the results of the authors' prediction together with those cited from Kutter et al. (1988). It is seen that the authors' method gives excellent predictions for Series BS compared with the prediction by others.

Fig. 20. Comparison of predicted and measured ultimate bearing capacity for Series SQ: square footings in in-situ loading tests

Table 5. Comparison of ultimate bearing capacity predictions: Authors' method and other methods (Kutter et al. (1986))

	$B = 96 \text{ cm}$			$B = 191 \text{ cm}$		
	$q_u \text{ (MN/m}^2\text{)}$	$c \text{ (kPa)}$	$\phi \text{ (deg.)}$	$q_u \text{ (MN/m}^2\text{)}$	$c \text{ (kPa)}$	$\phi \text{ (deg.)}$
Measured [†]	2.01, 2.06	—	—	3.05, 3.09	—	—
Proposed	2.01	2.32	46.4	2.92	4.67	44.6
Meyerhof [†]	1.50	0.00	47.3	2.14	0.00	45.6
De Beer [†]	2.11	0.00	48.9	2.90	0.00	47.1
Kutter [†]	2.32	13.01	43.1	2.99	13.01	43.1

Note: [†] cited from Kutter et al. (1988)

CONCLUSION

The authors developed a new simple prediction method of ultimate bearing capacities of surface footings with regard to size effects. Both strip and circular footings were considered in this paper.

The concept of the prediction method proposed is quite simple: a rational method for determining shear strength constants c and ϕ in accordance with the confining stress range corresponding to footing breadth. In this process the confining stress dependent shear strength is taken into account.

The nonlinear Finite Element Method which features a confining stress dependent failure criterion was used to derive the appropriate stress range of interest. Consequently, it was shown that the confining stress beneath the footing at failure ranges from $\sigma_m = 2 \gamma B$ to $10 \gamma B$ for strip footings and from $\sigma_m = 1 \gamma B$, to $15 \gamma B$ for circular footings.

The advantageous features of the proposed method are:

- The stress range to be interested in for the determination of shear strength can be determined directly from footing breadth and unit weight of the foundation ground, without iteration process. Therefore, soil testing for determining shear strength can be easily planned; the proposed confining stress dependent failure criterion is available for examining shear strength properties.
- The proposed method is applicable to strip and circular footings.
- Foundation grounds with notable cohesion as well as those without cohesion can be dealt with uniquely; the confining stress dependent failure criterion is incorporated, which is applicable to both cohesionless and cohesive materials.

The applicability of the proposed method was examined with the results of FEM simulation and the experimental results reported by some researchers, which include centrifuge loading tests and an in-situ loading test on the strip, circular and square footings on the cohesive and cohesionless foundation grounds. From the verification of the proposed method, it was confirmed that the method proposed by the authors gives admissible prediction of ultimate bearing capacities with attention to the size effects for various types of surface footings and foundation grounds.

ACKNOWLEDGEMENT

The authors are grateful to Prof. O. Kusakabe of Tokyo Institute of Technology, and Mr. M. Ohuchi of Shiraishi Co. Ltd., for providing experimental data and valuable advice through the course of this study. The authors would like to express their gratitude to Prof. S. Toki of Hokkaido University for his continuous encouragement.

NOTATION

- a, b = constants for lower and upper boundaries of the interest stress
 E_a = a normalized Young's modulus under the reference stress σ_a ; $E = E_a \sigma_a (\sigma'_m / \sigma_a)^m$
 m = a parameter describing stress dependency of deformation characteristics
 q_u = ultimate bearing capacity
 S_c, S_q, S_γ = shape factors
 α = a parameter representing stress dependency of shear strength
 γ_d = dry unit weight
 ϕ^* = extended secant internal friction angle
 ϕ_a^* = extended secant internal friction angle under $\sigma_m^* = \sigma_a$
 η = shear stress intercept of failure line in $\sigma_m - \tau_m$ plane
 ψ = gradient of failure line in $\sigma_m - \tau_m$ plane
 σ_0 = effective mean normal stress
 σ_m^* = extended mean principal stress $\sigma_m^* = \sigma_m + \sigma_t$
 σ_a = reference atmospheric stress = 98 kPa
 σ'_m = effective mean stress $\sigma'_m = (\sigma'_1 + \sigma'_3)/2$
 σ'_{mf} = effective mean principle stress at failure
 σ_r = radial stress
 τ_a = maximum shear stress at $\sigma_m = a \gamma B$
 τ_b = maximum shear stress at $\sigma_m = b \gamma B$
 σ_t = tensile strength for failure criterion

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