EFFECTS OF PRINCIPAL STRESS DIRECTION AND INTERMEDIATE PRINCIPAL STRESS ON UNDRAINED SHEAR BEHAVIOR OF SAND

MITSUTOSHI YOSHIMINE\textsuperscript{1)}, KENJI ISHIHARA\textsuperscript{2)} and WILLIAM VARGAS\textsuperscript{3,4)}

ABSTRACT

Undrained monotonic loading triaxial compression and extension tests were conducted on Toyoura sand. It was found that the shear behavior was more contractive and softer in triaxial extension than in triaxial compression. This difference suggests that the stress conditions, such as the direction of the principal stress and the magnitude of the intermediate principal stress have some effects on the undrained behavior of sand.

To clarify these effects, a series of tests was performed by means of an automated hollow cylindrical torsional shear apparatus. Dry-deposited Toyoura sand was used in these tests. The angle of the maximum principal stress from the vertical to the bedding plane, \( \alpha \), and the intermediate principal stress coefficient, \( b \), were fixed in each test. The intermediate principal stress was fixed to horizontal. For any density, tests with a larger \( \alpha \)-value, namely, a larger inclination of \( \sigma_1 \) from the vertical, and a larger intermediate principal stress coefficient \( b \) were shown to generate greater excess pore water pressure.

In addition, the undrained simple shear behavior of sand under initial isotropic and anisotropic stress conditions was studied. It was made clear that triaxial compression (\( \alpha = 0^\circ, b = 0 \)) gives the highest resistance with lowest contractancy, while triaxial extension (\( \alpha = 90^\circ, b = 1 \)) gives the opposite extreme in the assessment of flow failure. Simple shear mode of deformation was shown to exhibit an intermediate stress-strain behavior between triaxial compression and extension, which is closer to most of the field conditions.

Key words: anisotropy, consolidated undrained shear, dilatancy, liquefaction, sand, stress path (IGC: D6)

INTRODUCTION

Flow failure of alluvial sandy ground induces catastrophic damage such as spreading of embankments and permanent lateral displacement of ground. Since flow deformation is rather one-directional and mainly controlled by the static gravity force, monotonic undrained shear tests are suitable for evaluation of the flow characteristics of sand. As flow failure is a phenomenon resulting from the steady-state deformation, it is important to study post-peak collapse tendency, development of pore-water pressure, and the residual strength of the soil mass during undrained shearing.

Although triaxial compression tests have been generally used for undrained monotonic loading in large deformation by many researchers such as Castro (1969), Poulos (1981), Been and Jefferyes (1985), Konrad (1990), Ishihara (1993), and Verdugo and Ishihara (1996), recently it has been pointed out that the undrained shear behavior of sand in triaxial extension is far more contractive than the behavior in triaxial compression.

A typical result of undrained monotonic triaxial compression and extension tests on saturated Toyoura sand is displayed in Fig. 1. The samples were reconstituted in their loosest state by the dry deposition method (Ishihara, 1993) and sheared from initial isotropic consolidation stress levels of 50 kPa to 500 kPa. The triaxial test device and the test procedure are the same as described by Verdugo and Ishihara (1996). From this test result, it can be noticed that a big difference exists in the shear behavior of sand between triaxial compression and extension, even though the density is almost the same for each initial consolidation stress \( \sigma_1 \). When the sample was sheared undrained under triaxial compression conditions, excess pore water pressure developed up to around 50% of the initial effective confining stress. Although a slight drop of shear stress was observed after the peak, hardening was the predominant behavior in large deformation. On the other hand, when the sample was sheared in triaxial extension mode, the shear stress dropped down...

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Fig. 1. Triaxial compression and extension tests on Toyoura sand

to zero, associated with 100% of excess pore-water pressure development. The same trend or difference in behavior has been reported by other researchers such as Tsutouka and Ishihara (1973), Hanzawa (1980), Miura and Toki (1982) and Vaid et al. (1990).

It can be said that the triaxial compression and triaxial extension tests exhibit two extreme stress conditions in terms of the following two points;
1. the direction of the principal stress relative to the layered structure of soil
2. the magnitude of the intermediate principal stress.

In triaxial compression, the angle of the maximum principal stress to the normal of the bedding plane of the sample, is equal to 0° and the intermediate principal stress coefficient \( b = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3) = 0 \), whereas in triaxial extension, \( \alpha = 90° \) and \( b = 1 \). The large discrepancy between the test results in triaxial compression and triaxial extension suggests that, in general, the stress conditions as above would have profound effects on the undrained shear behavior of sand.

Yamada and Ishihara (1981) investigated the effects of three dimensional stress conditions on the undrained monotonic shear behavior of sand by means of a true triaxial apparatus. They found that when the vertical principal stress was smaller relative to the other two principal stresses, the undrained shear behavior of sand was more contractive. Symes et al. (1985) observed that a larger \( \alpha \)-value and a larger \( b \)-value resulted in more contractive undrained behavior of sand, using a hollow cylinder torsion shear apparatus. Recently Nakata et al. (1995), Vaid et al. (1996a, 1996b) and Uthayakumar (1996) have also reported similar results for hollow cylinder torsion shear tests. However, in the experiments listed above, the combinations of test conditions, such as direction of principal stresses, magnitude of intermediate principal stress and density of the material, were limited and mutual effects of these conditions on the undrained behavior of sand have not been well clarified, especially when the material is looser and exhibits large softening behavior.

In the following sections, the effects of the direction of the principal stress and the magnitude of the intermediate principal stress will be closely examined by means of hollow cylinder torsion shear tests under a wide range of combinations of stress conditions and density of the material.

**APPARATUS AND SAMPLE PREPARATION METHOD**

The hollow cylindrical specimen used in this study has a height of 195 mm, an outer diameter of 100 mm, and an inner diameter of 60 mm. The hollow cylinder torsion shear apparatus employed by Ishihara and Towhata (1983) and Pradel et al. (1990) was improved with automatic stress and strain control system by means of a personal computer. This system allows the individual automatic control of axial force \( F_a \), torque \( T \), inner cell pressure \( p_i \) and outer cell pressure \( p_o \) with automatic measurement and recording of those external forces, as well as axial displacement \( \Delta H \), torsional angle \( \Delta \theta \), inner cell volume change \( \Delta V_i \) and specimen volume change \( \Delta V_s \).

Inner and outer cell pressures were controlled by means of electric-pneumatic transducer (EPT). Axial force and torque were applied in the manner of displacement control using the loading system developed by Tsutouka et al. (1994). This loading system is designed so that loading and unloading can be started and stopped without any noticeable time lag, under the constant displacement rate of the loading piston. In the case of the torque, the linear motion of the loading piston is converted to torsion by means of a wire-pulley system. Precise load control is possible thanks to the very small backlash of the motion of the loading piston, which is less than 1 \( \mu \)m. The configuration of the hollow cylinder torsion shear apparatus including loading, measurement and control system is described in Fig. 2.

The tested material was Toyoura sand, which has a mean particle diameter \( D_{50} = 0.17 \) mm, a minimum void ratio \( e_{\min} = 0.597 \) and a maximum void ratio \( e_{\max} = 0.977 \). The dry deposition method was used for the constitution of the samples. Dry sand was deposited in 8 layers. Firstly, a hollow cylindrical paper funnel was inserted between the inner and outer molds and filled with dry sand. Then, the funnel was lifted up slowly and the dry sand was very loosely deposited in layers of around 25 mm in...
thickness. Tapping was applied to the outer mold after each layer was deposited to bring the sample to the expected uniform density. This sample preparation procedure may be very similar to the “dry tapping method” described by Miura and Toki (1982). They suggested that while pluviation methods create the most significant anisotropy and rod plunging makes an almost isotropic fabric, the tapping method gives intermediate results in terms of the constitution of sand specimens.

After saturation, with a Skempton’s B value greater than 0.96, the sample was isotropically consolidated to an initial confining stress of $p' = 100$ kPa for most of the tests in this study. Several simple shear tests were performed with the anisotropic initial consolidation state. The void ratio of the sample was measured with an accuracy of 0.001 by the same method described by Verdugo and Ishihara (1996).

STRESS AND STRAIN CONTROL IN HOLLOW CYLINDRICAL SPECIMEN

Calculation of Stresses and Strains in Hollow Cylindrical Specimen

In the hollow cylinder torsion apparatus, four stress components on the specimen, $\sigma_1$, $\sigma_2$, $\sigma_3$, $\sigma_0$ (Fig. 3) can be adjusted by controlling the outer cell pressure $p_0$, the inner cell pressure $p_i$, the vertical axial force $F_v$, and the torque $T$. The averaged stress components in hollow cylindrical specimens were calculated, with reference to the work by Pradhan et al. (1988) and Pradel et al. (1990), as follows:

\[
\sigma_z = (F_v + \pi (p_0 r_0^2 - p_i r_i^2)) \frac{1}{A_s}
\]

\[
\sigma_{\theta\theta} = 2 \frac{T}{2\pi r_0 (r_0^2 - r_i^2) + \pi (r_0^2 + r_i^2) (r_0 - r_i)}
\]

\[
\sigma_r = \frac{p_0 r_0 + p_i r_i}{r_0 + r_i}
\]

\[
\sigma_0 = \frac{p_0 r_0 - p_i r_i}{r_0 - r_i}
\]

$F_v$, $T$, $p_0$, and $p_i$ are corrected for membrane force assuming that the membrane is an elastic material with a Young’s modulus $E = 1400$ kPa and a Poisson’s ratio $\nu = 0.5$, except $p_0$ and $p_i$ in Eq. (1). $r_0$ and $r_i$ are the current outer and inner radii of the specimen, while $A_s$ and $A_t$ are the horizontal cross section area of the sample and the axial rod of the cell. Although $F_v$ and $T$ were measured outside of the cell, the friction of the rod was kept negligibly small.

The intermediate principal stress $\sigma_2$ is always fixed to the horizontal direction and equal to the radial normal stress $\sigma_0$. The maximum and minimum principal stresses are mobilized on the $z$-$\theta$ plane. The angle of the maximum principal stress from the vertical, $\alpha$, and the intermediate principal stress coefficient, $b$, were calculated by the following equations.

\[
\alpha = \frac{1}{2} \tan^{-1} \left( \frac{2\sigma_0}{\sigma_0 - \sigma_z} \right)
\]

\[
b = \frac{\sigma_2 - \sigma_0}{\sigma_0 - \sigma_1} = \frac{1}{2} \left( \frac{\sigma_0 - (\sigma_1 + \sigma_2)/2}{\sqrt{(\sigma_1 - \sigma_0)^2 + 4\sigma_0^2}} + 1 \right)
\]

Strain components, defined in the same coordinate system as stress components, were calculated also in the same manner as described in Pradel et al. (1990).

\[
\varepsilon_z = -\frac{\Delta H}{H_0}
\]

\[
\gamma_{\theta\theta} = 2\varepsilon_\theta = \frac{2\Delta \theta (r_0^2 - r_i^2)}{3H (r_0^2 - r_i^2)}
\]

\[
\varepsilon_r = \frac{\Delta r_0 - \Delta r_i}{r_0 + r_i}
\]

\[
\varepsilon_\theta = \frac{\Delta r_0 + \Delta r_i}{r_0 + r_i}
\]

where $H_0$ is the initial height of the specimen, and $\Delta r_0$ and $\Delta r_i$ are the changes of the outer and inner radii of the specimen calculated from $\Delta H$, $\Delta V_0$, and $\Delta V_i$, assuming that the specimen maintained right cylindrical configuration.

The Effects of Membrane Penetration

In the calculation of stresses and strains, the effects of the membrane penetration was not taken into account. Tokimatsu and Nakamura (1986) examined the mem-
brane penetration using a solid cylindrical sample of Toyoura sand which had a height of 150 mm and a diameter of 75 mm. When the effective confining stress was reduced from 98.0 kPa to 9.8 kPa, they observed about \(-0.03\%\) of volumetric strain due to the membrane penetration. If this observation is applied to the sample dimensions of the present study assuming that the membrane penetration per unit area was the same, then \(\Delta e_v = -0.034\%\) and \(\Delta e_r = -0.023\%\) are calculated as additional strains due to the membrane penetration when the effective inner and outer cell pressures are reduced to be 10% of the initial value. These strain decrements are equivalent to the changing of the void ratio as much as \(-0.001\) under undrained conditions. From the considerations described above, it can be said that the effects of the membrane penetration are negligible in the present study even if large excess pore water pressure is developed during testing.

**Stress Control in Shear Tests with Fixed \(\alpha\) and \(b\)-values**

In all of the tests, the outer cell pressure \(p_o\) was kept constant. Hence, the magnitude of total mean principal stress changed during shearing, but it did not affect the effective stress path as the process was undrained and the sample was highly saturated.

In the case of tests with constant \(\alpha\) and \(b\)-values, inner cell pressure \(p_i\) and vertical stress \(\sigma_z\) or shear stress \(\sigma_{\theta\theta}\) were controlled to achieve the desired stress conditions.

When \(\alpha \neq 0^\circ\) and \(\alpha \neq 90^\circ\), monotonic torsional shear was applied to the sample at a constant strain rate \(\dot{e}_\theta = \text{d}e_\theta/\text{d}t\) of around 0.1%/min and the shear stress \(\sigma_{\theta\theta}\) was observed. Equations (3) to (6) yield

\[
\sigma_z = p_o + \left(2A - B \frac{r_o + r_i}{2r_o}\right)\sigma_{\theta\theta}
\]

\[
p_i = p_o + B \frac{r_o^2 - r_i^2}{2r_o r_i} \sigma_{\theta\theta}
\]

where

\[
A = \frac{1}{\tan 2\alpha}
\]

\[
B = (2b - 1)\sqrt{A^2 + 1} + A = \frac{b - \sin^2 \alpha}{\sin \alpha \cos \alpha}
\]

Then, using these equations, \(\sigma_z\) and \(p_i\) were controlled in the tests so that \(\alpha\) and \(b\) were kept constant.

If \(\alpha = 0^\circ\) or \(\alpha = 90^\circ\), torque was controlled to be kept zero, as Eq. (5) requires. In the case of \(\alpha = 0^\circ\), monotonic axial compression was applied to the sample at a strain rate around \(\dot{e}_\theta = 0.1\%/\text{min}\) and the axial stress \(\sigma_z\) was observed, whereas the inner cell pressure \(p_i\) was controlled using the following equation

\[
p_i = \frac{(2r_o r_i - b r_o (r_o + r_i))}{2r_o r_i - b r_o (r_o + r_i)} p_o + B (r_o^2 - r_i^2) \sigma_z
\]

which is derived from the Eqs. (3), (4) and (6) under the condition of \(\sigma_z > \sigma_r\). Setting \(b = 0\) gives \(p_i = p_o\) in which the stress condition is the same as the conventional triaxial compression.

In the case of \(\alpha = 90^\circ\), monotonic axial extension was applied to the sample at a strain rate around \(\dot{e}_\theta = -0.1\%/\text{min}\) and the axial stress \(\sigma_z\) was observed, whereas inner cell pressure \(p_i\) was controlled using the equation

\[
p_i = \frac{(b - 1)r_o^2 + (b + 1)r_i (r_o r_i)}{(b - 1)r_i^2 + (b + 1)r_o r_i} p_o + (1 - b)(r_o^2 - r_i^2) \sigma_z
\]

which is derived under the condition of \(\sigma_z < \sigma_r\). Setting \(b = 1\) gives \(p_i = p_o\) in which the stress condition is the same as the conventional triaxial extension.

**Strain Control in Simple Shear Tests**

The undrained simple shear conditions were controlled in the same manner as described in Pradhan et al. (1988). The height of the hollow cylindrical specimen was fixed, and both the specimen and the inner cell were kept undrained while torque was applied with strain control. The outer cell pressure \(p_o\) was kept constant. All of the three normal strain components are zero in this condition; that is to say, the dimensions of the sample do not change and only torsional deformation is possible.

**Stress Non-Uniformity in the Specimen**

It is known that stress and strain non-uniformities are created in hollow cylindrical specimens due to the differential pressure between inner and outer cell, the differential torsional displacement in the radial direction of the specimen, and the rigid confinement of the upper and lower boundaries of the specimen. Among these, the non-uniformity of \(\sigma\), may be denoted as

\[
\frac{p_o - p_i}{\sigma_r} = \frac{b - \cos^2 \alpha}{2 \sin \alpha \cos \alpha} \left\{ \left( \frac{r_o}{r_i} \right) - \left( \frac{r_i}{r_o} \right) \right\} \sigma_{\theta\theta}
\]

from Eqs. (12) and (14), where \(\sigma_r\), is the averaged radial stress calculated by Eq. (3). According to this equation, a large absolute value of \((b - \cos^2 \alpha)\) and a large shear stress ratio \(\sigma_{\theta\theta}/\sigma_r\) result in large stress non-uniformity. To minimize the stress non-uniformity in this study, the stress conditions were limited to the combination of the

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**Fig. 4. Stress non-uniformity in hollow cylindrical**
angle $\alpha$ and $b$-value denoted as solid circles in Fig. 4. Fixing the inner to outer radius ratio to $r_i/r_o=0.6$ which was used in this study, the stress non-uniformity at the mobilized internal friction angle of $\phi_{mob}=40^\circ$ is calculated by Eq. (17) and displayed by contour lines in Fig. 4. According to this figure, the non-uniformity of $\sigma_i$ at $\phi_{mob}=40^\circ$ is less than 30% except in the stress conditions given by the pairs $(\alpha=30^\circ, b=0), (\alpha=45^\circ, b=0)$ and $(\alpha=60^\circ, b=0.25)$ in this study. Although Height et al. (1983) and Vaid et al. (1990) calculated more detailed distributions of stress or stress ratio in hollow cylinder specimens using isotropic linear elasticity or hardening elastoplasticity theories, the quantitative effect of the stress and strain non-uniformities on the averaged stress-strain behavior is not well known, especially when the softening of the material is large, as it is in present study.

BEHAVIOR OF SAND UNDER FIXED $\alpha$ AND $b$-VALUE CONDITIONS

Sixty-one undrained shear tests with constant $\alpha$ and $b$-values were conducted. Stress condition, void ratio after consolidation ($e$), minimum effective mean stress at phase transformation ($p'_{fr}$) and maximum excess pore water pressure ratio ($u_r$) employed for each test are summarized in Table 1. The maximum excess pore water pressure ratio $u_r$ is defined as

$$u_r = \left(1 - \frac{p'_{fr}}{p'_c}\right) \times 100\% \quad (18)$$

The initial consolidation stress was $p'_c=100$ kPa for all of the tests.

Figure 5 is a typical result of a test series in which the $b$-value was fixed to 0.5, whereas the direction of the principal stress was varied from $\alpha=15^\circ$ to $75^\circ$. The density of the samples was in a narrow range, from $D_s=39\%$ to $41\%$. One may notice the strong influence of the direction of the principal stress on the undrained behavior of sand from this figure. When the inclination of the maximum principal stress $\alpha$ became larger, the behavior became clearly softer and more contractant. In the case of $\alpha=15^\circ$, only 20% of excess pore water pressure developed and the behavior showed only hardening. On the other hand, in the case of $\alpha=75^\circ$, pore water pressure developed up to nearly 90% and the behavior showed strong strain-softening.

Figure 6 shows the typical result of a test series in which the direction of the principal stress was fixed to $\alpha=45^\circ$ while the magnitude of the intermediate principal stress varied from $b=0$ to 1.0. The density of the samples was in a narrow range, from $D_s=31\%$ to $34\%$. One may notice the clear influence of the intermediate principal stress on the undrained shear behavior of sand. When the magnitude of the intermediate principal stress became

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<th>$p'_{fr}$ (kPa)</th>
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larger, the sand tended to exhibit a behavior which was considerably softer. When the $b$-value was 0, an excess pore water pressure of 65% was developed, but the condition of $b=1.0$ resulted in development of excess pore water pressure of as much as 97%.

To better visualize the effects of the stress conditions on the undrained shear behavior, test results are plotted in $\alpha - b - u_r$ space as shown in Fig. 7. From these figures, we can clearly see that larger maximum excess pore water pressure is developed under the conditions of larger inclination of the maximum principal stress and larger intermediate principal stress coefficient.

If the density of the sample is around $D_r = 25\%$ and the stress condition is $\alpha = 0$ and $b = 0$, which is the same condition of conventional triaxial compression tests, only about 50% of the excess pore water pressure may be developed, whereas in a wide range of combinations of larger $\alpha$ and larger $b$ with the same initial condition, shearing results in complete liquefaction associated with 100% excess pore water pressure. This result is fairly coincident with the conventional triaxial tests shown in Fig. 1.

If the density of the sample is around $D_r = 33\%$ and the stress condition is $\alpha = 0^\circ$ and $b = 0$, the amount of excess pore water pressure development will be only about 10% of the initial consolidation stress and the expected behavior is considerably dilative and stiff. In contrast to this, if the stress condition is near the triaxial extension mode with $\alpha = 90^\circ$ and $b = 1$, fully contractive behavior is expected with 100% excess pore water pressure development.

In the case of $D_r = 40\%$, almost no excess pore water pressure development is expected under triaxial compression mode. When it is sheared, the sample becomes harder and harder, as if it was a rigid material. On the other hand, when the same material is sheared at the same density under triaxial extension mode, excess pore water pressure will develop up to more than 90% of the initial effective confining stress and the sample will behave like a liquid.

**BEHAVIOR OF SAND UNDER SIMPLE SHEAR CONDITIONS**

Undrained simple shear tests starting from isotropic consolidation and anisotropic consolidation states were conducted with various densities of Toyoura sand.

Figure 8 shows the simple shear test results with initial isotropic consolidation stress of 100 kPa. To see the effects of the direction of principal stress and the magni-
Fig. 7. Maximum excess pore water pressure during undrained shear of Toyoura sand under various stress conditions

tude of the intermediate principal stress, the variations of \( \alpha \) and \( b \)-values during simple shear are plotted in Figs. 8(c) and (d). It may be seen that the maximum principal stress was directed to \( 40^\circ \sim 45^\circ \) from the vertical throughout the shearing process. It is also seen that the \( b \)-value was around 0.5 at the beginning of the simple shear and it gradually decreased to 0.25, then it was kept almost constant after the phase transformation, which is the point of minimum effective mean stress.

Figure 9 shows the simple shear test results starting from anisotropic initial consolidation. The initial lateral effective stress was \( p_l = \sigma_z = \sigma_z = 100 \text{ kPa} \) and the vertical effective stress was \( p_c = \sigma_c = 200 \text{ kPa} \). Thus, the confining stress ratio was \( K_c = p_l / p_c = 0.5 \) and the initial mean effective stress was \( p_{eq} = 133 \text{ kPa} \). The variations of \( \alpha \) and \( b \)-values during shear are plotted in Figs. 9(c) and (d). In this case, the initial state was the same as the triaxial compression condition which is denoted by \( \alpha = 0^\circ \) and \( b = 0 \) (point A in Fig. 9). When shearing started, the direction of the maximum principal stress rapidly rotated and the intermediate principal stress coefficient increased towards \( \alpha = 40^\circ \) and \( b = 0.25 \) (Point B to C). Thereafter, this stress condition was kept unchanged in the large deformation state following the phase transformation (Point C to D).

From these observations, it can be said that the stress conditions during undrained simple shear of sand at large deformation are \( \alpha = 40^\circ \) to \( 45^\circ \) and \( b = 0.25 \) regardless of the density of the sand or the initial consolidation stress ratio.

**DISCUSSIONS**

Generally speaking, a soil structure formed under gravity, both in the field and in the laboratory, has an anisotropic nature, such that the material response is stiffer in the vertical compression and softer in the horizontal compression. In other words, when a soil sample is compressed in the direction perpendicular to the bedding plane, the behavior will be more dilative, and
Fig. 8. Simple shear tests with initial isotropic consolidation state

Fig. 9. Simple shear tests with initial anisotropic consolidation state
when compressed parallel to the bedding plane, the behavior will be more contractive. The effect of the direction of the maximum principal stress observed in this study may be attributed to the anisotropic nature of the sample and its degree may depend on material or sample preparation method.

On the other hand, the mechanism as to the effects of the $b$-value is not fully understood. From Eqs. (11) and (14), one may notice that decreasing the $b$-value makes the vertical normal stress $\sigma_z$ larger. This fact indicates that at least a part of the effect of the $b$-value on the undrained behavior of sand may also be attributed to the anisotropy of the sample when the direction of the intermediate principal stress is horizontal.

To summarize the effect of the stress conditions on the undrained behavior of sand, effective mean stress at phase transformation ($p_{tr}$) during triaxial compression ($\alpha=0^\circ$, $b=0$), triaxial extension ($\alpha=90^\circ$, $b=1.0$), and simple shear is plotted on $e$ (void ratio) $-p_{tr}$ plane in Fig. 10. Additionally, conventional triaxial compression and extension tests were conducted in a wider range of density; the results are plotted together in Fig. 10. Note that the initial consolidation stress is $p_c=100$ kPa for all the data points in this figure. The phase transformation line for simple shear exists around the intermediate between the lines for the triaxial compression and extension. If all of the test results with various combinations of $\alpha$ and $b$-values are plotted on this figure, the phase transformation lines for triaxial compression and triaxial extension form the upper and lower boundary of the transformation points, respectively.

The phase transformation points for shear tests with fixed stress condition of $\alpha=45^\circ$ and $b=0.25$ are also shown in Fig. 10. The phase transformation line for this stress condition locates very near to the line for simple shear in which the condition of $\alpha=40^\circ$ to $45^\circ$ and $b=0.25$ was achieved at large deformation as mentioned before. This coincidence implies that $\alpha$ and $b$-values are the key factors controlling the undrained behavior.

**CONCLUSIONS**

The comparison of the results of undrained triaxial compression and extension tests show that the shear behavior is more contractive and softer in triaxial extension than the behavior in triaxial compression mode. This difference indicates that the direction of the principal stress and the magnitude of the intermediate principal stress have profound influences on the undrained behavior of sands.

To clarify these effects, a series of undrained shear tests on Toyoura sand was performed by means of an automatically controllable hollow cylinder torsion shear apparatus. In these tests, the angle of the maximum principal stress to the normal of the bedding plane $\alpha$ and intermediate principal stress coefficient $b$ were kept constant during the shear stress application. The direction of the intermediate principal stress was fixed to horizontal. Samples with a relative density of $D_r=25\%$, $33\%$ and $40\%$ were tested under 23 combinations of $\alpha$ and $b$-values. For any density, a larger $\alpha$-value (a larger inclination of $\sigma_1$ from the vertical) and a larger intermediate principal stress coefficient $b$ generated greater excess pore water pressure. This means that triaxial compression ($\alpha=0^\circ$, $b=0$) gives the highest resistance and lowest contractancy while triaxial extension ($\alpha=90^\circ$, $b=1.0$) gives the opposite extreme.

As simple shear is the most common mode of shear deformation encountered in the field, the undrained simple shear behavior of sand was studied. In the initial stage of shearing, rotation of principal stresses and shifting of $b$-value were observed to take place until the state of phase transformation was reached. At this stage the condition of $\alpha=40^\circ-45^\circ$ and $b=0.25$ was achieved irrespective of the initial consolidation stress ratio. Shear resistance during simple shear at large deformation was shown to be nearly the same as the shear resistance encountered when the shear test was performed with the values of $\alpha=45^\circ$ and $b=0.25$ fixed from the beginning.

These test results show that careful consideration of field conditions is highly necessary in the assessment of the undrained flow failure of ground and earth structures.

**REFERENCES**