A NUMERICAL MODEL FOR SEEPAGE THROUGH UNSATURATED SOIL

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ABSTRACT

A numerical model is proposed to simulate the seepage through unsaturated soil based on some mechanical and probabilistic considerations on the particle scale. In the model the probability density functions are introduced to estimate the state of voids in soil. The hydraulic properties of unsaturated soil; i.e. the relations among the water content, pF -value and permeability coefficient are theoretically derived by using the model, and numerically calculated by using a personal computer. The numerical result is compared with a pF -test and unsaturated permeability test on Toyoura sand. In the proposed model the hydraulic properties of unsaturated soil can be derived by only the grain size distribution.

Key words: numerical model, pF -value, probability theory, seepage, unsaturated permeability coefficient, water content (IGC: E7)

INTRODUCTION

Above the ground water level soil is generally in an unsaturated condition. Even below the ground water level some soil is occasionally in this condition, e.g. submarine soil (Poulos, 1988). The hydraulic properties of unsaturated soil, i.e., the relations among the water content, pF -value and permeability coefficient, play key roles in conventional seepage analysis (e.g. Nishigaki, 1983). Nakano (1971) investigated unsaturated ground water flow and proposed a model based on microscopic considerations. Van Genuchten, M. Th. (1980) proposed a close-form equation for the hydraulic properties of unsaturated soil, and Takeshita and Kohno (1993) studied a method to estimate the parameters in Van Genuchten's equation.

It is qualitatively well known that slope failure due to heavy rainfall is caused by a decrease in the pF -value with the increase in the water content in unsaturated soil. In this paper the previous numerical model (Kitamura and Uto, 1991) is refined to simulate and analyze the saturated-unsaturated mechanical behavior of soil as the first stage for a quantitative analysis of slope failure due to heavy rainfall.

MODELING OF VOID

Figure 1(a) is a typical picture showing the arrange-

Fig. 1. Modeling of particles and void in soil mass: (a) A few soil particles in container; (b) Model of soil mass by pipe

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ment of a few soil particles in a soil mass, which is popularly observed by optical microscope. The voids and soil particles in Fig. 1(a) can be modeled by a pipe and other parts as shown in Fig. 1(b), where a few soil particles and voids in the container correspond to the impermeable part (shadow part in Fig. 1(b)) and permeable part (part of pipe in Fig. 1(b)) respectively. The diameter and inclination angle of the pipe are defined as D and θ .

The diameter D and inclination angle θ can be regarded as random variables because the shape and size of soil particles are random, and so the structure of their assembly is irregular. Consequently probability density functions for D and θ can be introduced to estimate the distribution of voids in a soil mass. The void ratio, water content and degree of saturation of soil mass are derived by using the probability density functions of D and θ as follows.

$$e = \int_0^\infty \int_{-(\pi/2)}^{\pi/2} \frac{Vp}{Ve - Vp} Pd(D) \cdot Pc(\theta) \cdot d\theta dD \quad (1)$$

$$e(d) = \int_0^d \int_{-(\pi/2)}^{\pi/2} \frac{Vp}{Ve - Vp} Pd(D) \cdot Pc(\theta) \cdot d\theta dD \quad (2)$$

$$w = \frac{e(d) \rho_w}{\rho_s} \quad (3)$$

$$S_r = \frac{e(d)}{e} \quad (4)$$

where

- e : void ratio,
- $e(d)$: void ratio where pipes ranged from 0 to ' d ' in diameter are filled with water,
- ρ_s : density of soil particle,
- ρ_w : density of water,
- w : water content,
- S_r : degree of saturation,
- Ve : volume of container in Fig. 1(b),
- Vp : volume of pipe in Fig. 1(b),
- $Pd(D)$: probability density function of D
- $Pc(\theta)$: probability density function of θ .

The volume of laminar flow in the pipe per unit time is expressed by using Poisseuille's law as follows.

$$Qp = \frac{\gamma_w}{8\mu} \cdot \left(\frac{D}{2} \right)^2 \cdot i \cdot \frac{\pi}{4} \cdot D^2 \quad (5)$$

where

- Qp : volume of water in pipe,
- μ : viscous coefficient of fluid in pipe,
- γ_w : unit weight of fluid,
- D : diameter of pipe,
- i : hydraulic gradient.

In Fig. 1(b) the hydraulic gradient for the laminar flow in the pipe is defined by the following equation.

$$i = \frac{\Delta u / \gamma_w}{DH / \sin \theta} \quad (6)$$

where

- i : hydraulic gradient,
- Δu : pressure head,
- DH : height of container in Fig. 1(b),
- θ : inclination angle in Fig. 1(b).

Substituting Eq. (6) into Eq. (5), the following equation is derived.

$$Qp = \frac{D^4 \cdot \sin \theta \cdot \Delta u \cdot \pi}{128 \cdot \mu \cdot DH} \quad (7)$$

On the other hand the volume of flow through the container in Fig. 1(a) is expressed by the following equation in which Darcy's law is applied.

$$Qp = k \cdot \bar{i} \cdot A = k \cdot \bar{i} \cdot D \cdot \left[\frac{D}{\sin \theta} + \frac{DH}{\tan \theta} \right] \quad (8)$$

where

- A : cross section area of container in Fig. 1(b),
- k : permeability coefficient,
- $\bar{i}$: mean hydraulic gradient of soil mass in Fig. 1(a).

The mean hydraulic gradient can be expressed for the soil mass in Fig. 1(a) as follows.

$$\bar{i} = \frac{\Delta u / \gamma_w}{DH} \quad (9)$$

From Eqs. (7), (8) and (9) the permeability coefficient is derived as follows.

$$k = \frac{\gamma_w \cdot D^3 \cdot \pi \cdot \sin \theta}{128 \cdot \mu \cdot \left[\frac{D}{\sin \theta} + \frac{DH}{\tan \theta} \right]} \quad (10)$$

Introducing the probability density functions of D and θ to take account of the randomness of voids in the soil mass, the permeability coefficient is expressed by the following equation.

$$k = \int_0^d \int_{-(\pi/2)}^{\pi/2} \frac{\gamma_w \cdot D^3 \cdot \pi \cdot \sin \theta}{128 \cdot \mu \cdot \left[\frac{D}{\sin \theta} + \frac{DH}{\tan \theta} \right]} Pd(D) \cdot Pc(\theta) \cdot d\theta dD \quad (11)$$

where

- k : unsaturated permeability coefficient for $d \neq \infty$, saturated permeability coefficient for $d = \infty$.

Figure 2 shows the water head due to the surface tension between the pipe and water. This situation can directly be applied to obtain the pF -value in the proposed model. The following equation is derived for pF -value.

$$pF = \log_{10} (h_c) = \log_{10} \left(\frac{4 \cdot T_s \cdot \cos \alpha \cdot}{\gamma_w \cdot d} \right) \quad (12)$$

where

- h (cm): water head in Fig. 2,
- T_s : surface tension,
- α : contact angle in Fig. 2,
- d : maximum diameter of pipe filled with water in Fig. 1(b) for a given water content.

Fig. 2. Water head due to surface tension in narrow pipe

The hysteresis loop in the wetting and drying processes can be expressed by using d and $d + \Delta d$ in Eq. (12) where Δd is considered to depend on the properties of shape and distribution of voids in the soil mass.

NUMERICAL EXPERIMENT

The values of model parameters are needed to calculate the hydraulic properties of unsaturated soil by means of the proposed model explained in the previous section. In this section a method to estimate the model parameters is discussed and then a numerical experiment is carried out.

It is found out from the grain size analysis that the size of soil particle is approximately expressed by the logarithmic normal distribution. The distribution of void in the soil mass was measured with a mercury intrusion porosimetry apparatus (Sato et al., 1992; Yamaguchi and Ikenaga, 1993). From these experimental results the logarithmic normal distribution can be assumed as $Pd(D)$ for sands soil as follows.

$$Pd(D) = \frac{1}{\sqrt{2\pi} \cdot \zeta_v \cdot D} 10^m \quad (13)$$

where

$$m = \left[-\frac{(\log D - \lambda_v)^2}{2\zeta_v^2} \right]$$

log: common logarithm

λ_v : mean value of logarithmic normal distribution,

ζ_v : standard deviation of logarithmic normal distribution.

Two parameters are needed to determine the shape of the logarithmic normal distribution curve: the mean value λ_v and standard deviation ζ_v . Kitamura and Fukuhara (1994) examined the effect of these parameters

on water retention curves for sandy soils. The permeability coefficient of sand was related to D_{10} by Hazen and to D_{20} by Creager experimentally. In the proposed model the mean value of $Pd(D)$ is assumed to be ranged between $1/10 \sim 1$ of D_{50} , depending on the kind of soil and void ratio, i.e., $\lambda_v = \log_{10}((1/10 \sim 1) \times D_{50})$. Furthermore, assuming that the coefficient of variation for the diameter D is the same as that of grain size, the value of standard deviation of $Pd(D)$ can be obtained by the procedure shown in Fig. 3. Thus, the probability density function $Pd(D)$ can be described by the grain size distribution.

The contact angle at the contact point of adjacent particles was measured by Oda (1972) and the shape of distribution was approximately regarded as a triangle or pentagon. A triangular shape (Kitamura, 1981) and a pentagonal shape (Kitamura, 1985) were assumed when using Kitamura's model to calculate the stress-strain relation of sandy soil. The distribution of inclination angle θ in Fig. 1(b) is considered to be related to that of contact angle β , i.e., the conjugate relation. Figures 4(a) and (b) are the conjugate probability density functions $P(\beta)$ and $Pc(\theta)$. In the numerical experiment, the parameter of $Pc(\theta)$, ζ_c in Fig. 4(b), is determined so that the ratio of the minimum height to the maximum of $Pc(\theta)$ might be

 Fig. 3. Procedure to obtain standard deviation of $Pd(D)$

 Fig. 4(a) Probability density function $P(\beta)$ of contact angle; (b) Probability density function $Pc(\theta)$ of inclination

1:3. The anisotropic permeability can be estimated by shifting the probability density function of θ , $P_c(\theta)$, i.e., transforming the coordinates in Fig. 4(b).

As an example a numerical experiment is carried out by supposing Toyoura sand. Table 1 shows the values of

Table 1. Values of model parameters used in numerical experiment

DH in Fig. 1	0.01601 (cm)
λ_v in Eq. (13)	-1.79551 (cm)
ζ_v in Eq. (13)	0.11358 (cm)
ζ_c in Fig. 4(b)	1.59×10^{-1}
μ in Eq. (11)	1.139×10^{-3} (Pa·s)
T_s in Eq. (12)	0.07355 (N/m)
α in Eq. (12)	0.0 (°)
ρ_s in Eq. (3)	2.65 (g/cm ³)
ρ_w in Eq. (3)	1.0 (g/cm ³)

Fig. 5. Numerical result of hydraulic properties

the model parameters used. DH in Fig. 1 is assumed to be the same as $D50$ and obtained from the grain size distribution. λ_v and ζ_v are decided by the procedure shown in Fig. 3. In the numerical experiment $\lambda_v = \log_{10} D50$ is tentatively adopted. ζ_c is obtained by assuming the ratio of the minimum height to the maximum of $P_c(\theta)$ in Fig. 4(b). The parameters, μ , T_s and ρ_w , are decided by common scientific tables. The parameter, ρ_s , is obtained by the soil particle density test. Figures 5 are a numerical results which show the relation between the water content and pF -value and permeability coefficient, respectively. In Fig. 5 the pF -test (drying process) and unsaturated permeability test results on Toyoura sand are also shown, which were carried out by Abe (1994). It is found from Fig. 5 that the model can qualitatively express the hydraulic properties of unsaturated sandy soil although the numerical plots have not been able to accurately follow the soil test results at the present stage.

CONCLUSIONS

A numerical model for seepage behaviour through unsaturated soil is proposed based on mechanical and probabilistic considerations on the soil particle scale. It is found out from the comparison of numerical and soil test results that the model shows promise in expressing the hydraulic properties of unsaturated sandy soil although more pF -tests, unsaturated permeability tests and other soil tests should be carried out to prove its validity with numerical experiments in the near future. More accurate values of model parameters can be expected to be obtained by developing both the measuring apparatus and techniques of size and shape of voids in soil, although these parameters are assumed for the state of soil mass at the present stage.

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