UNDRAINED PLANE STRAIN SHEAR TESTS ON SATURATED SAND USING A HOLLOW CYLINDER TORSIONAL SHEAR APPARATUS

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ABSTRACT

A hollow cylinder torsional shear apparatus was modified and improved so that automatic stress-strain control and measurements were possible. Using this apparatus, a series of undrained plane strain shear tests was conducted on sand, and stress-strain behavior, such as rotation of principal stresses, shifting of the magnitude of intermediate principal stresses, and variation of normal stresses and strains, were closely examined. An attempt was also made to compare the plane strain shear behavior with the undrained shear behavior of sand in simple shear conditions.

Key words: consolidated undrained shear, liquefaction, plane strain, sand, stress path (IGC: D6)

INTRODUCTION

Monotonic undrained shear tests have been performed on sands by many researchers for the evaluation of flow failure due to liquefaction. As the undrained response of soil is strongly affected by the stress-strain conditions during deformation, shear testing conditions in the laboratory should be carefully selected.

The plane strain condition, defined by $\varepsilon_2=0$, is one of the deformation modes of a soil element that is most likely to be encountered in the field. For simulating this condition, plain strain compression and extension tests with rigid boundaries in the direction of ε_2 have been widely conducted. In these tests, the directions of the principal stresses are fixed to be vertical and horizontal so the effect of principal stress axis rotation during plain strain shear deformation cannot be studied. The friction between the sample and the rigid boundaries is another problem. Furthermore, it is difficult to apply a given initial intermediate principal stress on the sample due to the fixed rigid confining plate system. To overcome these limitations, plane strain shear tests on hollow cylindrical specimens with flexible boundaries in the ε_2 direction were conducted in this study. The technique for controlling the plain strain conditions was similar to the drained simple shear tests by Shibuya and Hight (1987) and Pradhan et al. (1988).

APPARATUS

An automatic stress-strain measurement-control sys-

Table 1. Initial conditions of the plane strain shear tests

Test no.	Void ratio after consolidation e	Relative density D_r (%)	Initial consolidation stress ratio $K_c = \sigma'_h / \sigma'_v$
11	0.872	35	0.4
5	0.874	34	0.6
6	0.878	33	0.8
10	0.871	35	1.0
7	0.868	36	1.2

tem was introduced to a hollow cylinder torsional shear testing device at Istanbul Technical University in Turkey. In this apparatus, axial load, F_v , and torsional moment, T , on the top of the sample were measured by means of a torque-load transducer in the triaxial cell excluding the friction between the axial rod and the triaxial cell. Outer cell pressure, p_o , inner cell pressure, p_i , and pore water pressure u , were measured by pressure transducers. Axial load, F_v , and torsional moment, T were acting on the sample through rigid porous stones on the top and bottom of the sample, while cell pressures, p_o and p_i , were applied to the sample laterally through flexible membranes (Fig. 1).

Vertical axial displacement, ΔH , and rotational angle of the cap, $\Delta \theta$, were measured by a LVDT and a potentiometer, respectively. The volume changes of specimen, ΔV_s , and inner cell, ΔV_i , were monitored by differential pressure transducers which measure the water head in burrettes connected to the water-saturated specimen and the inner cell, respectively.

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Fig. 1. The definition of the external forces and the stress components on hollow cylindrical specimen

These nine values of measurement, F_v , T , p_o , p_i , u , ΔH , $\Delta\theta$, ΔV_s and ΔV_i , were automatically acquired by a personal computer through amplifiers and an AD-converter, and then displayed on a screen in real time and recorded in digital form.

At the same time, outer cell pressure, p_o , and inner cell pressure, p_i , were automatically adjusted by electric-pneumatic transducers (EPTs) controlled by the same personal computer through a DA-converter. Axial load, F_v , was generated by means of an air cylinder, which was also automatically controlled by EPT in the same manner as the cell pressures. F_v , p_o and p_i could be precisely adjusted to the target values by the feedback system. On the other hand, the torque, T , was applied to the sample in the manner of displacement control at a constant speed. Back pressure, u , was manually controlled during consolidation.

STRESS-STRAIN CALCULATIONS AND CONTROL OF PLANE STRAIN

The stress components on the sample (Fig. 1) were calculated by the following formulae, with reference to Hight et al. (1983), Pradhan et al. (1988) and Pradel et al. (1990).

$$\sigma_z = (F_v + \pi(p_o r_o^2 - p_i r_i^2)) \frac{1}{A_s} \quad (1)$$

$$\sigma_{z\theta} = \frac{1}{2} \left\{ \frac{3T}{2\pi(r_o^3 - r_i^3)} + \frac{T}{\pi(r_o^2 + r_i^2)(r_o - r_i)} \right\} \quad (2)$$

$$\sigma_r = \frac{p_o r_o + p_i r_i}{r_o + r_i} \quad (3)$$

$$\sigma_\theta = \frac{p_o r_o - p_i r_i}{r_o - r_i} \quad (4)$$

F_v , T , p_o and p_i are corrected for membrane forces assuming that the membrane is an elastic material with a Young's modulus $E=1400$ kPa and a Poisson's ratio $\nu=0.5$, except p_o and p_i in Eq. (1) which are acting on the cap directly. Additionally, r_o and r_i are the current outer and inner radii of the specimen while A_s is the horizontal cross sectional area of the sample.

Fig. 2. Particle size distribution of the tested material

The strain components were calculated using equations;

$$\varepsilon_z = \frac{\Delta H}{H_0} \quad (5)$$

$$\gamma_{z\theta} = 2\varepsilon_{z\theta} = \frac{2\Delta\theta(r_o^3 - r_i^3)}{3H(r_o^2 - r_i^2)} \quad (6)$$

$$\varepsilon_r = -\frac{\Delta r_o - \Delta r_i}{r_o - r_i} \quad (7)$$

$$\varepsilon_\theta = -\frac{\Delta r_o + \Delta r_i}{r_o + r_i} \quad (8)$$

where H_0 is the initial height of the specimen, Δr_o and Δr_i are the changes of the outer and inner radii of the specimen calculated from ΔH , ΔV_s and ΔV_i assuming that the specimen maintained right cylindrical configuration. These stress and strain components were also displayed on the screen in real time as well as the measured external forces and displacements.

During the plane strain shearing, the radial strain was kept at zero ($\varepsilon_r = \varepsilon_\theta = 0$), the total vertical stress was kept constant ($\sigma_z = \text{const.}$), and torsional shear was applied at a constant speed ($d\gamma_{z\theta}/dt = \text{const.}$). For the stress conditions in the circumferential direction,

$$K_\theta = \sigma'_\theta / \sigma'_z = \text{const.} \quad (9)$$

was selected among others. Equations (4) and (9) yield the relationship between p_o and p_i which is denoted as;

$$p_o = \left(1 - \frac{r_i}{r_o}\right) (K_\theta \sigma'_z + u) + \frac{r_i}{r_o} p_i \quad (10)$$

When the radial thickness of the sample tended to expand and ε_r became negative, cell pressures, p_o and p_i in Eq. (10), were increased to make ε_r zero. On the other hand, if the sample thickness contracted, cell pressures were decreased so that the plane strain condition in r -direction was kept precise.

SAMPLE PREPARATION AND TEST PROCEDURE

The original material used in this study was Kartal sand that was taken from a sea coast area in Istanbul, Turkey. This sand was sieved to get the tested material, which has a mean particle diameter $D_{50}=0.25$ mm, a

Fig. 3. Undrained shear behavior of Kartal sand in plane strain condition

minimum void ratio $e_{\min}=0.685$ and a maximum void ratio $e_{\max}=0.971$. Grain size distribution of the material is shown in Fig. 2. The hollow cylindrical specimen used in this study had a height of 195 mm, an outer diameter of 100 mm, and an inner diameter of 60 mm. The air pluviation method was used for constitution of the samples. The samples were prepared to obtain relative densities in the range of $D_r=33\% \sim 36\%$ after consolidation.

The sample was saturated to a Skempton's B value greater than 0.98 under isotropic effective stress of $p'=20$ kPa. After shifting of the effective stress ratio, $K_c=\sigma'_h/\sigma'_z$ where $\sigma'_h=\sigma'_r=\sigma'_\theta$, from 1.0 to each target value, the effective stress was increased up to $\sigma'_z=100$ kPa for all of the samples with a constant K_c value. Five sets of plane strain shear tests were conducted with various initial consolidation stress ratios, $K_c=\sigma'_h/\sigma'_z$ from 0.4 to 1.2. Initial torsion stress, $\sigma_{z\theta}$, was zero. After waiting for about one hour for consolidation, the samples were sheared under undrained plain strain conditions keeping the effective circumferential stress ratio ($K_\theta=\sigma'_\theta/\sigma'_z$) constant and equal to the K_c value.

TEST RESULTS AND DISCUSSIONS

The Accuracy of Stress and Strain Control

The results of the undrained plain strain shear tests are summarized in Fig. 3. From Fig. 3(e), it can be seen that the plain strain condition was maintained fairly well with ε_2 in the order of $\pm 0.01\%$ in spite of the fact that the sample dimension in ε_2 direction was only 2 cm. In the calculation of strains, the effect of membrane penetration was not taken into account. Yoshimine et al. (1998) suggested that the error of ε_2 due to membrane penetration can be up to 0.03% with similar test conditions and materials to those used in this study.

Figure 3(g) shows the variation of $K_\theta=\sigma'_\theta/\sigma'_z$ that should be kept constant. Although several small drops in K_θ occurred during shearing, it was kept almost constant throughout the shearing procedure. The drops in K_θ may be attributed to instability of the vertical stress control due to the friction of the axial rod.

The Direction of σ_1 and the Magnitude of σ_2

Yamada and Ishihara (1981), Symes et al. (1985), Uthayakumar (1996) and Yoshimine et al. (1998) have pointed out that the direction of the principal stresses and the magnitude of the intermediate principal stresses are key factors in the undrained behavior of sand. In this context, the variation of these parameters during undrained plane strain shear was examined. The α -value, which is the angle of the maximum principal stress from the vertical (Fig. 1), and b -value, which is the intermediate principal stress coefficient, were calculated using the following formulae;

$$\alpha = \frac{1}{2} \tan^{-1} \left(\frac{2\sigma_{z\theta}}{\sigma_\theta - \sigma_z} \right) \quad (11)$$

$$b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} = \frac{1}{2} \left(\frac{\sigma_r - (\sigma_z + \sigma_\theta)/2}{\sqrt{(\sigma_z - \sigma_\theta)^2/4 + \sigma_{z\theta}^2}} + 1 \right). \quad (12)$$

Figure 3(c) shows the variation of the angle α . At the initial state, σ_1 was vertical and $\alpha=0^\circ$ if $K_c (=K_\theta) < 1.0$, whereas σ_1 was horizontal and $\alpha=90^\circ$ if $K_c > 1.0$. In both cases the direction of principal stresses rotated very quickly when torsional shear was applied, and α became almost constant at $\gamma_{z\theta}=2\%$ and larger deformations. In the case of $K_c=1.0$, α was almost constant and equal to 45° throughout the shearing process. The residual α value was different depending on K_θ . Larger K_θ values resulted in larger residual α values.

The variation of the intermediate principal stress coefficient b is plotted in Fig. 3(d). Here b was equal to 0 if $K_c < 1.0$, and $b=1$ if $K_c > 1.0$ at the initial state. When shearing started, b quickly increased or decreased towards approximately 0.25 at large deformation, irrespective of the K_θ value.

Horizontal and Vertical Normal Strains

In the plane strain condition, $\varepsilon_r=\varepsilon_2$ was fixed to be zero while the other strain ε_θ and the vertical normal strain ε_z were observed to change depending on K_θ . ε_θ was always equal to $-\varepsilon_z$ because of the undrained conditions. Figure 3(f) shows the variation of $\varepsilon_\theta=-\varepsilon_z$ during plane strain shearing. It can be seen that the increment of the normal strains was almost in proportion to the increment of the shear strains $\gamma_{z\theta}$. When K_θ was smaller, the samples dilated in the horizontal direction (θ -direction) and contracted in the vertical (z -direction). The opposite tendency was seen for larger K_θ values. K_θ equal to 0.8 or slightly higher may result in zero normal strains, which is the same as simple shear conditions.

Normal Stress in Plane Strain Direction

The effective normal stress ratio in the plane strain direction defined as $K_r=\sigma'_r/\sigma'_z$, is plotted in Fig. 3(h). Although K_r increased or decreased during the first stage of shearing, it became almost constant at large deformations. The residual K_r value was larger for larger K_θ values.

Comparison with Simple Shear Tests

Yoshimine et al. (1998) conducted a series of undrained simple shear tests using the same type of torsional shear apparatus at the University of Tokyo in Japan. They used Toyoura sand which has $D_{50}=0.17$ mm, $e_{\min}=0.597$ and $e_{\max}=0.977$. The results of these simple shear tests are rearranged and displayed in Figs. 4 and 5. Test results with initial isotropic consolidation are shown in Fig. 4 whereas Fig. 5 indicates the case of $K_c=\sigma'_h/\sigma'_z=0.5$ at initial conditions before shearing. In both cases, the stress parameters were approximately constant at large deformations beyond the phase transformation state, in which α was between 40° and 45° , b was fixed to around 0.25, K_θ became 0.8 to 0.9, and K_r settled to nearly 0.6. These observations in the simple shear tests coincide well with the fact that, when $K_\theta=0.8$ was selected in plane strain shear tests, normal strains ε_θ and ε_z were very near to zero (Fig. 3(f)) with the residual values of $\alpha=40^\circ$, $b=0.25$ and $K_r=0.6$. (Fig. 3(c), (d) and (h)).

Fig. 4. Undrained shear behavior of Toyoura sand in simple shear condition ($K_c = \sigma'_h / \sigma'_z = 1.0$ at initial state)

Fig. 5. Undrained shear behavior of Toyoura sand in simple shear condition ($K_c = \sigma'_h / \sigma'_z = 0.5$ at initial state)

It should be noted that these stress parameters, namely, α , b , K_r and K_θ are not independent but related to each other by the relationship;

$$(1 - K_\theta)(1 - 2b) = (1 + K_\theta - 2K_r) \cos 2\alpha, \quad (13)$$

derived from Eqs. (11) and (12).

CONCLUDING REMARKS

An existing hollow cylinder torsional shear apparatus was modified and improved so that automatic stress-strain control and measurement were possible. Using this apparatus, a series of undrained plane strain shear tests on saturated sand was conducted. During the shearing process, it was observed that the plane strain condition in the radial direction of the sample was achieved with good accuracy.

In each plane strain shear test, the effective normal stress ratio in the direction of torsion to the vertical, $K_\theta = \sigma'_\theta / \sigma'_z$, was fixed to various values from 0.4 to 1.2. It was found that the stress parameters such as the direction of σ_1 from the vertical, α , the intermediate principal stress coefficient, b , and the effective normal stress ratio in the direction of plane strain to the vertical, $K_r = \sigma'_r / \sigma'_z$, tended to be nearly constant at large deformations. Larger K_θ values resulted in larger residual α and K_r values whereas b converged towards 0.25 irrespective of the K_θ value.

If we look at phenomena in the field such as a spreading of an embankment, the material at the toe of the slope may be compressed horizontally with larger K_θ and positive ε_θ values. On the contrary, the soil at the top of the embankment may be subjected to horizontal extension with smaller K_θ and negative ε_θ values. Consideration of these stress and strain conditions may also be important in the behavior of liquefied back fill soil behind retaining wall structures which deform towards the sea or river during earthquakes. Although the K_θ value was kept constant during shear deformation in the laboratory testing, it may change in those real cases depending on the boundary conditions of the structures and the behavior

of the soil itself.

When K_θ was equal to 0.8, strain conditions became close to simple shear with the residual stress parameters of $\alpha = 40^\circ$, $b = 0.25$ and $K_r = 0.6$. This fact was shown to be in harmony with the results of undrained simple shear tests, though the theoretical background of these residual values is not clear.

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