THE EFFECT OF EMBEDMENT DEPTH ON THE UNDRAINED RESPONSE OF SKIRTED FOUNDATIONS TO COMBINED LOADING

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ABSTRACT

The response of skirted foundations to combined vertical (V), moment (M) and horizontal (H) undrained loading is investigated using finite element and upper bound plasticity analysis. The study has focused in particular on the effect of the foundation embedment depth on the size and shape of the combined V - M - H yield locus. The yield locus was found to be of similar shape to that of a surface foundation but the size of the yield locus was increased. Normalisation of the yield locus by the single load bearing capacities (V_0 , M_0 and H_0) allowed the yield locus to be generalised for varying soil profiles and footing embedment depths. Upper bound plasticity analysis was carried out to calculate V_0 , M_0 and H_0 for a range of footing embedment depths and undrained soil shear strength profiles and design charts are presented for their determination.

Key words: bearing capacity, caisson, plasticity theory, shallow foundation, undrained shear strength (IGC: D6/E3)

INTRODUCTION

The widespread adoption of skirted rafts and caisson foundations for offshore structures has focused attention on how the embedment of the foundation affects its capacity to withstand vertical, horizontal and moment loading. Traditionally, bearing capacity calculations have concentrated on vertical loading, using classical plasticity solutions for strip foundations at the soil surface (Prandtl, 1921; Hill, 1950). Approximate adjustment factors (e.g. Meyerhof, 1953; Hansen, 1970) have then been introduced to allow for foundation shape and embedment and for load inclination (that is, a component of horizontal load), using an equivalent foundation width that takes account of eccentric (or moment) loading.

Thus, the bearing capacity, V_0 , of the foundation is expressed as

$$\frac{V_0}{A} = s_c d_c i_c N_c s_u \quad (1)$$

where A is the (effective) area of the footing, s_u is the undrained shear strength of the soil, N_c is the bearing capacity factor for a strip footing (=5.14 for uniform, homogenous material) and s_c , d_c and i_c are the correction factors for shape, embedment depth and load inclination respectively.

As will be discussed later, for offshore foundations where the horizontal and moment loading can be significant, the bearing capacity of the foundation under the full combined loading is better treated by means of a

three-dimensional yield envelope, rather than correction factors applied to the vertical bearing capacity. However, it is still necessary to evaluate each 'pure' capacity (under separate vertical, horizontal or moment loading) in order to define the extremes of the yield envelope. The separate capacities will be affected by the embedment depth of the foundation, and also by the depth variation of shear strength.

For vertical loading, previous research (e.g. Meyerhof, 1953; Hansen, 1970) has suggested that the depth factor, d_c , may be approximated as a linear function of the normalised embedment depth, d/D (where D is the footing width or diameter) as

$$d_c = 1 + n \frac{d}{D} \quad (2)$$

where n is in the range 0.2 to 0.4. One of the aims of this paper is to verify this approach, by using finite element and upper bound plasticity analyses, and also to derive similar relationships for horizontal and moment loading.

Idealised Strength Profiles

Skirted foundations and caissons are being used increasingly in soft sediments, even where the shear strength at the seabed is very low (Tjelta and Haaland, 1993; Dyvik et al., 1993). In such soils, the increase in shear strength with depth is an important consideration, and it is often sufficient to characterise the strength profile as varying linearly with depth, according to

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Fig. 1. Geometry of the foundation

$$s_u = s_{ut} + kz \quad (3)$$

where s_{ut} is the undrained shear strength at the soil surface and k is the rate of shear strength increase with depth, z (Fig. 1).

A typical foundation may be considered as having a diameter (or width), D , and skirts that penetrate to a depth, d . The skirts confine the upper, soft soil, essentially forcing the failure mechanism down to soil beneath the foundation where, at the depth, $z=d$, the shear strength is $s_{uo} = s_{ut} + kd$.

It is convenient to present solutions for bearing capacity in terms of the shear strength at the tip of the foundation skirts, s_{uo} , and to represent the degree of non-homogeneity of strength by the dimensionless group, kD/s_{uo} . It may be noted that, for the particular case of a skirted foundation in normally consolidated soil, with $s_{ut}=0$, the quantity kD/s_{uo} is numerically equal to the ratio of diameter to skirt penetration, D/d .

YIELD FUNCTION APPROACH

Recent studies related to offshore foundation behaviour (e.g. Tan, 1990; Murff, 1994; Butterfield and Gottardi, 1995; Houlsby and Martin, 1992; Dean et al., 1992; Salençon and Pecker, 1995; Ngo-Tran, 1996) have analysed the combined loading response of shallow foundations using a plasticity framework rather than the traditional approach of Eq. (1). Isotropic yield loci in V - M - H space have been deduced which describe the load combinations at failure for a given footing penetration. These have been proposed for different foundation systems in different soil types. A yield surface is given by

$$f\left(\frac{V}{As}, \frac{M}{AsD}, \frac{H}{As}\right) = 0 \quad (4)$$

where A is the area of the foundation, D its diameter or breadth and s is a representative soil strength or effective stress. Associated plasticity is often assumed so that this yield surface also describes a plastic potential defining

the relative magnitudes of the incremental plastic displacements during elasto-plastic yielding (e.g. Murff, 1994). Associated plasticity will be observed for soils obeying normality at failure and when footing detachment from the soil surface does not occur (e.g. Bransby and Randolph, 1997a). However, when either of these conditions are violated, a separate plastic potential will be required (e.g. Tan, 1990).

To use the normality relationships, the load and displacement definitions must form work conjugate pairs so that normalised total system work, W is written as

$$\frac{W}{ADs} = \left(\frac{V}{As}\right) \frac{\delta v}{D} + \left(\frac{M}{ADs}\right) \delta\theta + \left(\frac{H}{As}\right) \frac{\delta h}{D} \quad (5)$$

where $\delta v/D$ is the normalised vertical footing displacement, $\delta h/D$ is the normalised horizontal displacement and $\delta\theta$ is the footing rotation and these displacements are measured at the same point at which the loads are taken to act.

Skirted foundations have significant uplift capacity (particularly for short-term loading) and so detachment of the footing from the soil surface is unlikely even under moment loading with low (or tensile) vertical loads. Caisson tests conducted in the UWA beam centrifuge, in low permeability calcareous sediments which are undrained during typical loading events (Watson and Randolph, 1997a), suggest that the ultimate uplift load, $V_i \approx -V_0$ and that the yield locus is symmetrical about the $V=0$ axis (Watson and Randolph, 1997b).

Several forms of yield loci have been recommended (e.g. Murff, 1994) but they are expressed in terms of V_0 , M_0 and H_0 the ultimate vertical, moment and horizontal loadings respectively. There have been few new design methods to calculate the single load bearing capacities, V_0 , M_0 and H_0 .

Tani and Craig (1995) carried out lower bound plasticity analyses and centrifuge tests to investigate the vertical capacity of skirted foundations in non-homogenous soils. They concluded that, for strip footings, the soil above the level of the skirt tips gave no contribution to

Fig. 2. The moment transformation

the vertical bearing capacity, while for circular footings its contribution was small.

Given the conclusions of Tani and Craig (1995), analyses of equivalent surface foundations (Fig. 1) were carried out by Bransby and Randolph (1998). That study combined finite element and upper bound plasticity analysis to investigate the shape of the yield locus under combinations of vertical, moment and horizontal loading for one typical soil stratigraphy (with $kD/s_{uo}=6$). By examination of the soil deformation mechanisms occurring at failure in the finite element analysis, a single generic upper bound mechanism was developed, somewhat different in shape from those adopted in previous studies (Lauritzen and Schjetne, 1976; Murff and Miller, 1977). The mechanism was found to give close agreement with the finite element analyses.

The yield locus was found to be eccentric in $M-H$ space, with the maximum moment being sustained under positive horizontal loading. This eccentricity was explained using a 'scoop' type plasticity mechanism (Fig. 2) and the kinematics of this mechanism was used to recommend a simplifying moment transformation. The moment, M^* was measured about the centroid of the scoop at a height L above the base of the footing, to give $M^* = M - LH$, which made the yield locus symmetric and allowed easier curve fitting.

A curve fit using the moment transformation and the upper bound scoop mechanism derived M_0 value was recommended by Bransby and Randolph (1998):

$$f = \left(\frac{V}{V_0}\right)^{2.5} - \left(1 - \frac{H}{H_0}\right)^{1/3} \left(1 - \frac{M^*}{M_0}\right) + \frac{1}{2} \left(\frac{M^*}{M_0}\right) \left(\frac{H}{H_0}\right)^5. \quad (6)$$

This curve fit was recommended solely for a surface footing with $kD/s_{uo}=6$ although it has been seen to be appropriate for other soil strength profiles. Analyses were also carried out for other values of kD/s_{uo} . The scoop mechanism obtained the correct M_0 value and gradient of the yield locus in $M-H$ space near M_0 for these different soil profiles but the shape of the yield locus in $M-H$ space was seen to differ slightly. Better curve fits will be obtained by changing slightly the exponents in the yield loci equations as kD/s_{uo} varies.

Although Tani and Craig (1995) suggested that the bearing capacity of a shallow foundation was independent of the soil strength above the level of the base of the footing (i.e. there were no embedment effects) for undrained conditions, this finding has not been verified using upper bound analysis or finite element analysis or for the effects of combinations of loading. This is investigated in this paper by carrying out upper bound and plasticity analyses to investigate the shape and size of the yield locus in $V-M-H$ space for embedded foundations in order to compare the results with those from the surface strip footing.

Aims

The principal aim was to investigate the yield locus for embedded footing in $V-M-H$ space in order to compare its shape and size with that obtained for a surface strip footing (Bransby and Randolph, 1998). This was to be done using finite element analysis. Also of interest was the form of soil deformation mechanisms occurring at yield and how these compared with those observed with the surface footing. Of particular interest was the magni-

tude of the ultimate loads, V_0 , M_0 and H_0 , and how these varied with embedment depth and kD/s_{uo} .

GEOMETRY UNDER CONSIDERATION

The geometry considered is that of an embedded strip footing with $d/D=1/6$ and in undrained soil with $kD/s_{uo}=6$ and with undrained shear strength at the surface, $s_{uo}=0$. This will allow comparison with results from a surface foundation with similar kD/s_{uo} (Bransby and Randolph, 1998). By modelling the caisson as an embedded footing, it is implicitly assumed that there are no deformation mechanisms which deform the soil between the skirts. For low embedment ratio skirted foundations (e.g. for gravity based platforms) internal mechanisms would dominate, but these foundations have internal stiffeners between the skirts, and for footings with larger d/D (e.g. caissons) these mechanisms do not dominate. Hence, this is a reasonable assumption. As in previous analysis, the footing will be considered to be fully attached to the soil. This will be a good approximation to the behaviour at the base of the footing where suction due to the skirts will prevent detachment on uplift, but may be less good for the footing side-walls.

During predominantly horizontal loading it is likely that detachment will occur on the active side-wall after moderate displacements. The displacement of the footing may no longer be associated with the yield locus and the yield locus shape will change. However, this will only occur after moderate displacements and may possibly not occur for monotonic loading, as observed by Watson and Randolph (1997b) in their centrifuge investigation of skirted foundations.

Additionally, the vertical side-walls may not be fully rough and so only a proportion of the local shear strength, s_u , may be mobilised, so that the limiting shear stress on the side walls will be $\tau=\alpha s_u$, where α is the shear stress ratio. However, this effect should not invalidate the normality property of the global yield locus, but merely reduce the size of the yield locus somewhat. The reduction in size of the yield locus with varying α can be investi-

gated easily by means of upper bound analyses.

FINITE ELEMENT ANALYSIS

The strip footing geometry of the present (embedded footing) analysis and the previous surface analysis (Bransby and Randolph, 1998) is shown in Fig. 3. The footing was assumed to be rigid, rough, of breadth, D , and with penetration d . It was considered to be infinitely long to enable plane strain analysis. The soil was modelled as an elastic-plastic Tresca material with an undrained shear strength profile described by $s_u=kz$. The shear strength at the base of the foundation, $s_{uo}=kd$ and so $kD/s_{uo}=D/d$. The rigidity index for the soil was taken as, $G/s_u=200$, but the findings of the analysis will be independent of soil stiffness as only ultimate behaviour is investigated. This simple soil model approximated the soil behaviour appropriately for investigation of the system undrained failure (yield) locus but would have to be defined for investigation of pre-failure displacements or drained failure.

Footing loads and displacements are as defined in Fig. 3. The reference point for moment (and vertical displacement) is in the centre of the footing at the soil-footing interface. The position of this reference point will affect the results (Martin, 1994; Butterfield and Gottardi, 1995; Bransby and Randolph, 1998).

Finite element analysis was carried out using CRISP90 (Britto and Gunn, 1990) on a 5DX66 Personal Computer and the finite element mesh consisted of 360 cubic strain triangles. A consolidation analysis was carried out with small time increments and low permeability soil so that the soil behaviour was undrained.

Yield Locus Probes

Displacement controlled events were found to be more suitable for probing the yield locus than stress controlled probes as discussed by Bransby and Randolph (1997a).

Two types of displacement probes were carried out. In the first a single probe was carried out at a constant ratio of $\delta v/D:\delta\theta:\delta h/D$ until the footing loading did not vary with increased displacement. By choosing a given ratio of

Fig. 3. Foundation geometry for finite element analysis

$\delta v/D:\delta\theta:\delta h/D$ (e.g. $(\delta h/D)/(\delta v/D)=2$, $\delta\theta=0$ as shown as OZ in Fig. 6) both the shape of the yield locus and direction of plastic incremental displacement vector at failure could be ascertained.

The second type of displacement probe follows that used by Tan (1990) during his centrifuge tests and is known as a 'side-swipe' test. It consisted of two stages. In the first stage (e.g. OA in Fig. 6) a given displacement (typically vertical) was prescribed to the footing until ultimate load was reached. Then a second displacement probe (AB in Fig. 6) was applied during which the vertical displacement increment was set to zero. The premise of the test is that because elastic stiffnesses are much greater than plastic stiffnesses, the stress path in the second stage will almost exactly follow the shape of the yield locus as negligible expansion of the yield locus will be required to balance the small elastic deformation. This means that the shape of the yield locus (on a certain plane in V - M - H space) may be determined in one test. However, because elasto-plastic yielding occurs within the yield locus as individual elements plastically yield, the side-swipe test tracks a load path marginally *inside* the yield locus; plastic displacements occurring within the global yield locus balance the elastic displacements along an internal load path to give $\delta v=0$.

PLASTICITY ANALYSIS

Upper bound plasticity calculations were carried out in order to provide independent estimates of the yield locus. The collapse mechanisms used were extensions of those used for the surface foundation and also similar to those observed from the present series of finite element analy-

ses. By the nature of the upper bound approach the calculated yield locus will lie on or outside the true locus.

The form of the deformation mechanisms were similar to the combination 'scoop' and 'wedge' mechanisms used for the surface foundation (Bransby and Randolph, 1998) except that extensions to the wedge and scoops were required to allow for the soil above the level of the footing base (Fig. 4). When combinations of these 'wedge' and 'scoop' mechanisms were made, the global upper bound mechanisms shown in Fig. 5 resulted.

For any given two load components e.g. V and M , a kinematic mechanism of one of the types shown in Fig. 5 with associated work dissipation was used to give a prediction of the unknown load, e.g. H , at failure. The procedure was repeated with different mechanisms until a minimum upper bound value of the unknown load was obtained for the specified loading (Bransby and Randolph, 1998). This was then repeated for a range of loads in order to describe the upper bound yield locus. Plasticity theorems state that normality will be obeyed on the deduced yield locus and the yield locus must be convex (Chen, 1975).

RESULTS FROM FINITE ELEMENT AND PLASTICITY ANALYSES

Yield Locus in Vertical-Horizontal (V - H) Load Space

The results from three FE displacement probes and side-swipe tests to investigate vertical-horizontal load space are shown in Fig. 6. Load path OZ is from a displacement probe with $\delta h/\delta v=2$, load path OY is for pure horizontal displacement and only the final point is shown of a probe with $\delta h/\delta v=3$. Together, the probes

Fig. 4. Component upper bound plasticity mechanisms

Fig. 5. Combined wedge-scoop mechanisms

Fig. 6. The yield locus in $V-H$ space

and sideswipes allow the shape of the yield locus for the embedded foundation to be deduced. This is shown together with the yield locus for the surface foundation on Fig. 6. Also shown are the different soil deformation mechanisms observed in the FE analysis at different

points of the yield locus and the yield locus obtained using upper bound plasticity based on the extended 'wedge-wedge' deformation mechanism.

A deep mechanism (also shown on Fig. 6) was investigated as it agreed well with the soil deformation mecha-

Fig. 7. Normalised yield locus in V - H space

nism observed in the finite element analysis. The yield locus produced by this mechanism was very slightly outside that produced by the previous mechanism for the case considered in Fig. 6. However, for $D/d=3$ (and $kD/s_{uo}=3$), the 'deep' wedge-wedge mechanism gave a better prediction of the yield locus than the first extended wedge-wedge mechanism.

The yield locus for the embedded foundation is larger than that for the surface foundation but of similar shape. For the embedded foundation, $V_0/Ds_{uo}=12.1$ for $H/Ds_{uo}=0$, which compares with $V_0/Ds_{uo}=10.7$ for the surface foundation. Lower bound plasticity solutions for rough strip surface footings have obtained $V_0/Ds_{uo}=10.49$ (Davis and Booker, 1973; Tani and Craig, 1995). The normalised vertical capacity, V_0/Ds_{uo} is increased by the embedment of the foundation for the same kD/s_{uo} , which contrasts with the conclusions of Tani and Craig (1995).

The horizontal capacity of the footing is also increased with embedment so that $H_0/Ds_{uo}=1.49$ for the embedded foundation, compared with $H_0/Ds_{uo}=1$ for the shallow foundation.

In order to compare their shapes, both yield loci are replotted as V/V_0 against H/H_0 in Fig. 7. Although the general form of the loci are the same, the yield locus for the embedded foundation lies slightly within that for the surface foundation (when normalised by V_0 and H_0) and thus lies closer to the line suggested by Murff, 1994 (Fig. 7). Also plotted are the normalised yield loci deduced by the upper bound plasticity analysis for both the surface and embedded foundation. Interestingly, both normalised upper bound yield loci are identical; the forms of the deformation mechanisms are very similar at the same load ratios.

Yield Locus in Vertical-Moment (V - M) Load Space

Results from finite element displacement probes and

upper bound plasticity analyses to investigate the yield locus in vertical-moment load space are shown in Fig. 8. As in V - H space, the yield locus for the embedded foundation is larger than for the surface footing. For the embedded foundation, $M_0/ADS_{uo}=1.56$, which compares with $M_0/ADS_{uo}=1.449$ for the surface foundation. The form of the deformation mechanisms at different points on the yield surface (Fig. 8) appears very similar to the 'scoop-wedge' mechanism used for upper bound plasticity analysis for the surface foundation. This mechanism was extended for use for the embedded foundation and the resulting upper bound yield locus is shown in Fig. 8. As for V - H space, the upper bound analysis over-predicts the size of the yield locus.

The deduced yield loci for both the finite element results and the upper bound analysis of the embedded and surface footings are replotted normalised by V_0 and M_0 in Fig. 9. The shapes of the yield loci are almost identical. Thus, the yield locus in V - M space for the embedded foundation can be produced by a simple scaling of the yield locus for the surface foundation.

Yield Locus in Moment-Horizontal (M - H) Load Space

The form of the yield locus in moment-horizontal load space is similar to that for the surface foundation (Fig. 10) and has a significant degree of eccentricity: peak moment is sustained with positive horizontal loading. The deformation mechanisms at failure are similar to those observed with the surface footing and so upper bound analyses were carried out with extended versions of the scoop mechanism.

The yield surface is larger than that from the surface footing, but its shape is similar as demonstrated by plotting the yield locus normalised by H_0 and M_0 (Fig. 11). Again, this suggests that yield loci for embedded foundations can be obtained by simple scaling of yield locus shapes for surface foundations.

Fig. 8. The yield locus in V - M space Fig. 9. The yield loci in normalised V - M space

Normalisation of Results

As the shapes of yield locus are the same for the embedded foundation and the surface foundation, it is only necessary to scale the yield locus for the surface foundation in order to obtain the yield locus for the embedded

foundation. Each axis may be scaled by the failure loads when the other two load components are zero (i.e. by H_0 , M_0 and V_0). Indeed, if the yield locus curve fit for the surface foundation is normalised in terms of V/V_0 , M/M_0 and H/H_0 (as in Eq. (6)) it is only necessary to calculate

Fig. 10. The yield locus in M - H space

V_0 , M_0 and H_0 for a foundation in order to obtain the entire yield locus. The bearing capacities, V_0 , M_0 and H_0 are easy to calculate in comparison with bearing capacities for combinations of loads, and can be expressed in a tabulated form for different geometries. For example, the graph of V_0/Ds_{uo} for differing kD/s_{uo} and embedment ratios (d/D) presented by Tani and Craig (1995) might form one such design graph.

PREDICTION OF V_0 , M_0 AND H_0 USING UPPER BOUND PLASTICITY SOLUTIONS

In order to obtain the size of the yield locus for any surface or shallow footing, bearing capacities must be obtained for solely vertical, moment and horizontal loading (i.e. V_0 , M_0 and H_0) for a range of soil strength profiles (kD/s_{uo}) and embedment ratios (d/D). Upper bound plasticity analysis of the three load cases have been carried out and the results are presented below and compared with previous recommendations (where available) and corresponding results from finite element analysis. For the case of a skirted foundation in normally consolidated soil, $kD/s_{uo} = D/d$ and so the results for these values are of particular interest.

Vertical Bearing Capacity (V_0)

Previous work investigating the effects of either non-uniform soil (Davis and Booker, 1973; Houlsby and Wroth, 1983) or embedment depth (Skempton, 1951) or

Fig. 11. The normalised M - H yield locus

both (Tani and Craig, 1995) has relied primarily on lower bound plasticity analysis. This leaves a good database of alternative findings with which to compare solutions.

Upper bound analysis of the vertical capacity of a strip footing was carried out using both the Hill and Prandtl type mechanisms. In both mechanisms, the external wedges were extended into the soil zone above the level of the footing base (Fig. 12). The Prandtl type mechanism has a single wedge of soil beneath the footing which extends the footing displacement into the soil body (Fig. 12(b)) and is more suitable for more uniform soil strength profiles ($kD/s_{uo} < 2.8$ for surface footings),

Fig. 12. Upper bound mechanism for calculation of V_0

Fig. 13. Upper bound prediction of V_0 with varying embedment depth and kD/s_{uo}

whereas the Hill type mechanism has two separate side 'wedges' (Fig. 12(a)) and is more suitable for non-uniform soil strength profiles. The vertical bearing capacity as calculated using the upper bound mechanisms is plotted against kD/s_{uo} for a range of embedment ratios in Fig. 13 (note, same form of graph as presented by Tani and Craig, 1995). On the same graph is shown the Tani and Craig solution for both the surface and shallow foundation which plot as one line. There is good agreement between the upper and lower bound solutions for the surface foundation when $kD/s_{uo} < 2$ suggesting that the exact solution is closely predicted by either solution for small values of kD/s_{uo} but less well bracketed for $kD/s_{uo} > 2$. The upper bound analyses predict that the degree of embedment affects significantly the vertical bearing capacity. This contrasts directly with the findings of Tani and Craig (1995) using lower bound analyses.

Results from the two plasticity solutions can be compared with results from finite element analyses (Fig. 13). For the case already considered when $kD/s_{uo} = D/d = 6$, finite element analysis gave $V_0/Ds_{uo} = 10.7$ for the surface foundation and $V_0/Ds_{uo} = 12.1$ for the embedded foundation. The lower bound plasticity solutions for strip footings gave $V_0/Ds_{uo} = 10.49$ (Davis and Booker, 1973; Tani and Craig, 1995) and upper bound analyses gave $V_0/Ds_{uo} = 11.45$ for the surface foundation and $V_0/Ds_{uo} = 12.54$ for the embedded foundation. This suggests that although the upper bound solution overpredicts the correct solution for both the surface and embedded foundation, it predicts approximately the 10% increase in vertical bearing capacity between the embedded and surface foundations. To examine the significance of embedment, the embedment factor, d_c (where $d_c = V_{0,embed} / V_{0,surf}$) is plotted against embedment ratio for a range of different

Fig. 14. Deduced embedment factors for varying d/D and kD/s_{uo}

Fig. 15. Upper bound mechanisms for calculation of H_0

kD/s_{uo} in Fig. 14. The relationship between d_c and d/D is non-linear and depends on kD/s_{uo} . For the case of uniform soil ($kD/s_{uo} = 0$) a linear approximation of $d_c = 1 + d/D$ is reasonably accurate. However, previous design recommendations for circular footings suggest that the factor n (Eq. (2)) should lie in the range 0.2 to 0.4, which significantly underpredicts the embedment effects observed with the upper bound analysis for strip footings in uniform soil. This may be because the foundation is circular or because the upper bound analysis is overestimating the effects of embedment. Finite element analysis of a foundation with $d/D = 0.167$ in uniform soil revealed that $V_0/Ds_{uo} = 5.88$ and thus $d_c = 1.14$, which compares well with the upper bound solution giving $d_c = 1.184$ and $V_0/Ds_{uo} = 6.09$. Further analysis is required to determine whether prediction of d_c from the upper bound analysis is generally closer to the exact solution than the lower bound solution.

On Fig. 14 is also plotted the embedment factor against embedment ratio for the soil conditions with $kD/s_{uo} = D/d$, the case of normally consolidated soil. Interestingly, the relationship is almost exactly linear and can be approximated by

$$d_c = 1 + 0.47 \left(\frac{d}{D} \right) \quad (7)$$

Finite element results with $kD/s_{uo} = D/d = 6$ and $kD/s_{uo} = D/d = 2.5$ are shown on Fig. 14. The deduced d_c values are similar to those from the upper bound analysis.

Horizontal Bearing Capacity (H_0)

For surface foundations, the solution for H_0 is trivial as only base shear occurs and so $H_0/Ds_{uo} = 1$. For embedded foundations there is both base shear and passive and

active pressures on the side walls. For fully attached side-walls, upper bound analysis suggests that two equal angle wedges will form on the side of the footing to give the upper bound mechanism pictured in Fig. 15(a). When detachment of the active side-wall is possible the mechanism will change to that shown in Fig. 15(b) with only a single wedge deformation.

By equating the work input ($H_0 \delta h$) to the work dissipation on the slip planes and due to potential energy changes, the maximum horizontal load can be expressed as

$$\frac{H_0}{Ds_{uo}} = 1 + a \left(\frac{d}{D} \right) \left(1 - \frac{kd}{2s_{uo}} \right) \left[\alpha \tan \delta + \frac{1}{\cos \delta \sin \delta} \right] - (a-2) \left(\frac{\gamma d}{2s_{uo}} \right) \left(\frac{d}{D} \right) \quad (8)$$

where $\alpha = \tau/s_u$ and $a=2$ for a two-sided mechanism (fully attached footing) and $a=1$ for a single-sided mechanism following detachment of the active face of the footing. For the boundary conditions considered in the finite element analysis, the footing is fully attached to the soil and so $a=2$ and $\alpha=1$. Therefore the equation simplifies to

$$\frac{H_0}{Ds_{uo}} = 1 + 2 \left(\frac{d}{D} \right) \left(1 - \frac{kd}{2s_{uo}} \right) \left[\tan \delta + \frac{1}{\cos \delta \sin \delta} \right] \quad (9)$$

When $\alpha=1$, for all d/D and kD/s_{uo} the minimum H_0 is obtained with $\delta=0.615$. Therefore the equation further simplifies to

$$\frac{H_0}{Ds_{uo}} = 1 + 5.66 \left(\frac{d}{D} \right) \left(1 - \frac{1}{2} \left(\frac{kd}{s_{uo}} \right) \left(\frac{d}{D} \right) \right) \quad (10)$$

Figure 16 plots this expression in the same form as in Fig. 13. Interestingly H/Ds_{uo} reduces with increasing kD/s_{uo} . This is because soil above the footing base weakens with respect to s_{uo} as kD/s_{uo} increases.

The alternative plot of horizontal capacity factor, d_h defined as $d_h = H_{0,embed}/H_{0,surf}$ (for the same kD/s_{uo}), against embedment ratio for different soil profiles is shown as Fig. 17. The equation for the lines is obtained from Eq. (10) as

$$d_h = 1 + 5.66 \left(\frac{d}{D} \right) - 2.83 \left(\frac{d}{D} \right)^2 \left(\frac{kD}{s_{uo}} \right). \quad (11)$$

The increase of horizontal capacity with embedment (for a fixed kD/s_{uo}) is clearly shown together with the non-linearity of the plots. As soil strength uniformity increases, the effect of embedment increases. For the case of normally consolidated soil, when $kD/s_{uo} = D/d$, Eq. (11) simplifies further to give

$$d_h = 1 + 2.83 \left(\frac{d}{D} \right) \quad (12)$$

as shown in Fig. 17; there is a linear dependency of horizontal capacity on embedment. This is because the additional horizontal capacity of the embedded footing is governed by the average shear strength, $s_{u,ave}$ in the soil above the footing where $s_{u,ave} = s_{uo} - 0.5(kD/s_{uo})(d/D)$.

Fig. 16. Upper bound prediction of H_0 with varying embedment depth and kD/s_{uo}

Fig. 17. Upper bound prediction of horizontal capacity factor with varying d/D and kD/s_{uo}

For the case when $kD/s_{uo} = D/d$ then $s_{u,ave} = s_{uo}/2$ (i.e. it is independent of kD/s_{uo}) and so the work dissipation due in the soil is merely a result of the length of the slip planes, which scale with d/D .

For the foundation when $kD/s_{uo} = D/d = 6$ (as investigated earlier), $H_0/Ds_{uo} = 1.471$ which agrees well with the value $H_0/Ds_{uo} = 1.49$ obtained from finite element analysis.

Moment Capacity (M_0)

Moment capacity can be predicted with the scoop mechanism shown previously (Fig. 10) and explained in more detail by Bransby and Randolph (1997b). For any given geometry and kD/s_{uo} value, the height, L , of the scoop centroid above the base of the footing is varied until a minimum M_0 is obtained.

Results from this procedure are shown in Fig. 18. For $d/D = 0.5$, M/Ds_{uo} reduces with increasing kD/s_{uo} because the effect of the weakening soil above the footing base as kD/s_{uo} increases is greater than the effect of the strengthening soil beneath it. For smaller embedment ratios than 0.33, M/Ds_{uo} increases with kD/s_{uo} as would be expected intuitively.

The graph of the moment capacity factor, d_m

($=M_{0,embed}/M_{0,surf}$) against embedment ratio for different kD/s_{uo} conditions again reveals that the effects of embedment are significant but reduce with increasing kD/s_{uo}

Fig. 18. Upper bound prediction of M_0 with varying embedment depth and kD/s_{uo}

Fig. 19. Upper bound prediction of moment capacity factor against d/D for different kD/s_{uo}

(Fig. 19). The relationship is also shown for the case of normally consolidated soil, when $kD/s_{uo}=D/d$. Although this relationship is still not linear; it may be approximated for $d/D \leq 0.5$ by

$$d_m = 1 + 0.6 \left(\frac{d}{D} \right) \quad (13)$$

Finite element results for a range of surface footings are shown on Fig. 18. There is almost exact agreement with the upper bound solutions. For the embedded foundation with $kD/s_{uo}=6$ ($d/D=0.167$) the upper bound analysis gave $M/ADS_{uo}=1.559$ while the FE analysis gave $M/ADS_{uo}=1.56$. Additionally, the failure mechanism observed in the finite element analysis for M_0 appear almost identical to the scoop mechanisms. Given the similarity of the mechanisms, it is likely that upper bound analysis will give excellent agreement with the exact solution for a wide range of kD/s_{uo} and d/D values.

COMPARISON OF DESIGN ENVELOPES

Finite Element Results

Figure 20 shows the finite element calculated yield locus for the case of the embedded foundation with $d/D=0.167$ and $kD/s_{uo}=6$ (dotted lines). On the same figure is plotted the yield locus obtained using the design method proposed above based on Eq. (6). In order to obtain the curve fit, $M_0/ADS_{uo}=1.559$ and $L/D=0.398$ were taken from the scoop mechanism, V_0/DS_{uo} was taken as the multiplier of the Tani and Craig (1995) lower bound solution and the expression for d_c so that

$$V_0/DS_{uo}=10.49(1+0.47/6)=11.31 \quad (14)$$

and $H_0=1.5$ (from the upper bound mechanism). Agreement between the two yield loci is reasonably good although V_0 is underestimated.

Finite element analyses were also carried out for soil

Fig. 20. Comparison between finite element derived yield locus and curve fit

$$f = \left(\frac{V}{V_0} \right)^{2.5} - \left(1 - \frac{H}{H_0} \right)^{\frac{1}{3}} \left(1 - \frac{M^*}{M_0} \right) + \frac{1}{2} \left(\frac{M^*}{M_0} \right) \left(\frac{H}{H_0} \right)^5$$

Fig. 21. Comparison between the finite element derived yield loci and the curve fit for embedded and surface foundations with $kD/s_{uo}=2.5$

Fig. 22. Centrifuge test results of caisson side-swipe tests in undrained calcareous silt (from Watson and Randolph, 1997b)

with $kD/s_{uo}=2.5$ for the case with $d/D=0$ and $d/D=0.4$ ($=s_{uo}/kD$). The shape of the yield locus was estimated from a series of side-swipe tests and the results from the two sets of analyses are shown on Fig. 21. On the same figure is plotted curve-fits of the yield loci from Eq. (6) with the second exponent changed from $1/3$ to 0.6 . Values of $V_0/Ds_{uo}=8.12$ (Tani and Craig, 1995), $H_0/Ds_{uo}=1$ and $M_0/ADs_{uo}=1.02$ (scoop mechanism) were used for the surface foundation. For the embedded foundation, $d_c=1.2$ (Eq. (7)) so that $V_0=9.744$, $d_r=2.132=H_0/As_{uo}$ (Eq. (13)) and $M_0/ADs_{uo}=1.271$, $L=0.276D$ from the scoop mechanism, which gave $d_m=1.245$ (cf. $d_m=1.24$ from Eq. (13)). Curve fits for both the embedded and surface footing appear quite good suggesting that the design method is suitable for a varied range of geometries and soil strength profiles.

The curve-fit for $d/D=s_{uo}/kD=0.4$ is least convinc-

ing in $M-H$ space. This is because the upper bound estimation of $M_0/ADs_{uo}(=1.271)$ using the scoop mechanism differs from the finite element measured value of $M_0/ADs_{uo}=1.13$. This, despite the fact that the failure deformation mechanism observed in the finite element analysis appears to be a scoop mechanism. It is not clear why these values differ, although this is only expected to occur for large embedment ratios.

Centrifuge Model Tests

Model tests have been carried out to determine $V-H$ yield loci for circular skirted foundations in calcareous soil in the UWA geotechnical centrifuge (Watson and Randolph, 1997a, b). Tests were carried out on with a 12 m (prototype) caisson with $d/D=0.4$ in normally consolidated calcareous silt. The silt was measured to have an almost uniform shear strength profile with $s_{ut}=0$ and

$k=1.4$ kPa/m and had permeability and compressibility such that the soil was undrained at the rate tested.

A side-swipe test to characterise the V - H locus was carried out at the installed position ($d/D=0.4$). The caisson was then penetrated vertically and a further side-swipe test was undertaken (now with $d/D=0.5$). The procedure was repeated at further depths. Thus, monotonic yield loci in V - H space were obtained for undrained conditions at varying foundation penetration depths (Fig. 22). The shape of the yield locus in V - H space at different depths appear similar in shape but the yield loci vary in magnitude. As penetration increases (and d/D increases), the geometry is such that $kD/s_{uo}=D/d$ and so the yield loci shown are for a range of d/D and kD/s_{uo} . The results suggest that the shape of yield loci are affected little by foundation shape, embedment or soil strength profile.

DESIGN METHODOLOGY

The full yield locus in vertical, moment and horizontal load space for any given strip footing geometry can be obtained using the yield locus equation obtained originally for the surface footing (Eq. (6)) and scaling with the V_0 , M_0 and H_0 values as calculated in the section above. This method gives very good approximation to the yield locus in full V - M - H load space. Although not justified theoretically, the differences between the true and curve-fitted yield locus will be negligible for design purposes. Use of the new design method will reduce the current conservatism in design, as it allows for both the increase in the size of the yield locus due to embedment (previously considered negligible by Tani and Craig, 1995) and the eccentricity of the yield locus in M - H space, which allows for the fact that larger moments can be sustained by footings undergoing positive horizontal loading (the most likely design case for offshore foundations).

This V_0 , M_0 , H_0 normalisation method may be suitable for approximation of the yield locus for more complex footing geometries (e.g. circular and rectangular footings) and soil stratigraphies, although agreement using this method with the true yield locus may be less good than for the case considered here.

CONCLUSIONS

Finite element analysis and upper bound plasticity analyses have been carried out to determine the monotonic yield locus for strip footings with shallow embedment in undrained, non uniform soil in vertical, moment and horizontal load space. Reasonable agreement was obtained between the finite element analysis and the plasticity analysis.

Contrary to the behaviour reported by Tani and Craig (1995), the embedment depth of the footing affected the bearing capacity and therefore the size of the yield locus. The size of the yield locus increased as embedment depth increased for the same shear strength at skirt tip depth level. However, the shape of the yield locus for the embed-

ded footing was seen to be very similar to that for the surface footing and so the two were almost identical when plotted in $V/V_0-M/M_0-H/H_0$ space. The similarity of the shapes of the yield loci suggested that the curve fits recommended for the surface footing analyses could be used for embedded foundations with suitable scaling dependent on V_0 , M_0 and H_0 .

Plasticity analysis was carried out to obtain upper bound solutions for the vertical (V_0), moment (M_0) and horizontal (H_0) bearing capacity of footings with varying embedment depths (d/D) and soil strength profiles (kD/s_{uo}). These results showed that embedment depth affected the bearing, moment and horizontal capacity of the footing significantly. The upper bound predictions of H_0 and M_0 agreed very well with the finite element analysis while the predictions on the increase of V_0 due to embedment was also reasonably good. Recommendations were given for these increases for soil with $kD/s_{uo}=D/d$, applicable to skirted foundations in normally consolidated soil.

A design method was recommended with consisted of scaling the yield locus from a surface footing by the V_0 , M_0 and H_0 values obtained by upper bound plasticity analysis. This method was tested by comparison with finite element results from a footing with $d/D=0.4$ and $kD/s_{uo}=2.5$. Good agreement between yield loci magnitudes and shapes suggested that the method may be generally applicable, although alternative methods may be required for foundations with larger embedment ratios.

The analyses reported here are currently being extended to circular and square foundation geometries. Initial indications are that similar shapes of yield envelope will continue to be valid, but that the extreme values of V_0 , M_0 , and H_0 will need to be adjusted.

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NOTATION

a	Number of sides to upper bound mechanism
A	Projected area of foundation
d	Embedment length of foundation
d_v, d_h, d_m	Embedment depth factor for vertical, horizontal and moment capacity respectively
D	Diameter or width of footing
G	Shear modulus of soil
H	Horizontal load
H_0	Horizontal capacity of footing with no vertical or moment loading
i_c	Load inclination factor
k	Shear strength gradient
L	Height of scoop centroid above footing base
M	Moment loading

M^*	Moment loading with moment transformation
M_0	Moment capacity of footing with no vertical or horizontal load
N_c	Vertical bearing capacity factor
s_c	Shape factor for vertical bearing capacity
s_u	Undrained shear strength of soil
s_{uo}	Undrained shear strength of soil immediately beneath foundation
s_{ut}	Undrained shear strength of soil at seabed ($z=0$)
V	Vertical load
V_0	Vertical capacity of footing in compression
V_t	Vertical capacity of footing in tension
W	External work
z	Depth beneath seabed
α	Shear stress ratio ('adhesion') on side-wall of footing
δv	Vertical displacement
δh	Horizontal footing displacement
$\delta \theta$	Rotation of footing
γ	Bulk unit weight of soil
τ	Shear stress
$\alpha, \beta, \gamma, \delta$	Geometrical variables in upper bound mechanisms

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