CHARACTERISTICS OF THE TURNING MOTION OF A TRACKED VEHICLE UNDER TRACTION ON A LOOSE SANDY SOIL —INNER/OUTER TRACK DURING DRIVING ACTION—

TATSURO Muroi, TRAN DANG Thaiii and KOUICHI Kohnoiii

ABSTRACT

The turnability characteristics of a flexible tracked vehicle under traction on a loose sandy flat surface are investigated theoretically and experimentally for both the inner and the outer track while being driven. Based on soil mechanics relationships between the surface and the flexible track belt, the turnabilities of the inner and the outer track are considered in the interaction of the surface shear resistance as a function of lateral and longitudinal slip velocity and the amount of slippage. Using terrain-track system constants, the relationships of the amount of depression, the thrust, the compaction resistance of the inner and the outer track, the effective tractive effort, the turning moment of a given tracked vehicle, and the turning radius are predicted for several kinds of steering ratio. From the results, it is determined that the vehicle speed decreases with the increment of turning radius along with the increment of resultant effective tractive effort, and the slip ratio of the outer track is always larger than that of the inner track. The amounts of depression of the front idler of the inner and the outer track decrease with slip ratio while those of the rear sprocket increase. The turning resistance moment of the inner track is always larger than that of the outer track for each surface shear resistance.

Key words: construction, machine, sand, settlement, shear strength, (slip ratio), (steering ratio), (tracked vehicle), (tractive effort), (turning motion) (IGC: E11/K4)

INTRODUCTION

Over the past decade, the authors have been involved in analysing the automatic control system of the tractive and braking performance of a flexible tracked vehicle e.g. bulldozer moving straight forward on a plane or sloped weak ground during both driving and braking action (Muro, 1989a, 1989b, 1991, 1992a, 1992b, 1993a, 1993b). In the present study, the turnability properties of a flexible tracked vehicle under traction on a loose sandy flat surface have been investigated.

The objective of this paper is to propose a simulation analytical method which will be verified experimentally, for a flexible tracked vehicle under traction in a turning motion, for both the inner and the outer track while being driven.

Compared with the turnability properties on a hard surface, it is very important to clarify the interaction problem between a soft surface and the flexible track belt of a turning tracked vehicle (Murakami et al., 1993). The flexibility of the track belt has been precisely predicted as a function of the track tension, loading, unloading and reloading properties of the terrain, and the normal pressure distribution. Also, the turnabilities of the inner and the outer track belt have been rigorously examined as a function of distribution of the lateral slip velocity, the lateral amount of slippage, and the lateral shear resistance distribution. This can be calculated from the plate traction, lack of traction and reciprocal traction properties of the terrain.

The terrain-track system constants for the longitudinal and lateral direction (Muro et al., 1996) were determined from a plate loading and depression test and a plate traction and slip depression test. A simulation analysis was carried out to estimate the amount of depression, thrust, compaction resistance of the inner and the outer track, the effective tractive effort, the turning moment of the tracked vehicle and the turning radius etc. for several kinds of steering ratios. Because of a larger track length, a larger turning resistance occurs and this leads to the need for thrust many times larger than that of a tracked vehicle moving straight forward (Ehlert et al., 1990). Consequently, the estimation of turning resistance and the establishment of this simulation analytical method, which is able to calculate the effective tractive effort under turning motion should be very useful for the improvement of
design of this kind of tracked vehicle.

SIMULATION ANALITICAL METHOD

Terrain-track System Constants

As a test surface, a soil bin of size 270 cm × 270 cm × 30 cm was filled up to 25 cm depth with air dried sandy soil of average grain size of 0.32 mm. The coefficient of uniformity and curvature of the grain size distribution was 5.07 and 0.85 respectively. The specific gravity was 2.60, the dry density was 1.55 g/cm³, and the relative density was 42.3% with a maximum density of 1.82 g/cm³ and a minimum of 1.40 g/cm³. The model track plate was made of steel of dimensions 300 mm × 100 mm × 5 mm and attached to L shaped steel angles of the grousers height H = 17 mm and the grousers pitch G_p = 51 mm. The track plate loading test, the traction test and the slip depression test were carried out in the soil bin to comply with simulation analysis.

From the track plate loading, unloading and reloading test, the terrain track system constants k_1, k_2 and n_1, n_2 were determined as follows:

\[ p = k_1 s_0^n \]  
(1)

\[ p = k_1 s_p^n - k_2 (s_p - s_0)^{n_2} \]  
(2)

where \( p \) (kPa) is the contact pressure, \( s_0 \) (cm) is an arbitrary amount of static depression and \( s_p \) (cm) is the amount of depression at the beginning of the unloading state. Here, the amount of depression was measured as the height difference between the surface and the bottom plane of the track plate. As grousers penetration was only by the self weight of the track plate into the loose sandy soil, the effect of grousers penetration on the contact pressure of the track plate could be ignored.

From the track plate traction test, the constants \( m_0, m_1 \), and \( a, k_3, n_3, m'_3, m'_4 \) and \( a' \) could be determined as follows:

\[ T = (m_2 + m_4)p \left( 1 - \exp \left( -a' \right) \right) \]  
(3.1)

where \( T \) (kPa) is the shear resistance of soil i.e. the traction force per unit contact area, \( f \) (cm) is the amount of slippage, and \( m_2, m_4 \) and \( a' \) are the constants concerned with the cohesion, the angle of internal shear resistance and the modulus of deformation of soil respectively.

Lack of traction \[ \tau = \tau_p - k_3 (j_p - j)^{n_3} \]  
(3.2)

where \( \tau_p, j_p \) is the shear resistance of soil and the amount of slippage respectively at the beginning of the unloading state.

Reciprocal traction \[ \tau = -(m'_3 + m'_4)p \times \left[ 1 - \exp \left\{ -a' (j - j) \right\} \right] \]  
(3.3)

where \( j_0 \) is the amount of slippage at the beginning of the reverse reloading state.

From the slip sinkage test, the relationship between the amount of slip depression \( s_c \) (cm), contact pressure \( p \) (kPa) and amount of slip slippage \( j_0 \) could be represented by the constants \( c_0, c_1 \) and \( c_2 \) as follows:

Table 1. Terrain-track system constants

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 = 0.546 ) N/cm²⁺¹</td>
<td>( k_2 = 3.688 ) N/cm²⁺¹</td>
</tr>
<tr>
<td>( n_1 = 2.203 )</td>
<td>( n_2 = 0.515 )</td>
</tr>
<tr>
<td>Plate traction and slip depression test in longitudinal direction</td>
<td></td>
</tr>
<tr>
<td>( m_{lat} = 0 ) kPa</td>
<td>( m_{lat} = 0.738 ) kPa</td>
</tr>
<tr>
<td>( a_{lat} = 0.0941 ) cm</td>
<td>( a_{lat} = 0.1411 ) cm</td>
</tr>
<tr>
<td>( c_{lat} = 2.66 \times 10^{-2} ) cm²⁺¹⁻² ( N/)cm²⁺¹</td>
<td>( c_{lat} = 0.749 )</td>
</tr>
</tbody>
</table>

\[ s_0 = c_0 p^2 j_0^3 \]  
(4)

where the amount of slip depression was measured as the increment of depression at the rear edge of the track plate during the traction test.

The terrain-track system constants are summarized in Table 1.

Vehicle Specifications

Figures 1(a), (b) and (c) show the front and side view, and the plan figure of a tracked vehicle with its dimensions and the several forces acting on it, when the vehicle is running on a weak flat surface in a turning motion. \( D \) is the contact length of track belt, \( B \) is the track width, \( H \) is the grouser height and \( G_p \) is the grouser pitch; \( C \) is the central distance between inner and outer track, \( r_1 \) is the radius of the front idler, and \( r_2 \) is the radius of rear sprocket. Next \( s_{lin}, s_{fo} \) and \( s_{lin}, s_{fo} \) indicate the amount of depression of the front idler and rear sprocket on the inner and the outer track, respectively, while \( W_t \) is the total weight, and \( W_1, W_2 \) are the distributed vehicle weight to the inner and the outer track belt. Finally \( T_{lat} \) is the lateral effective tractive effort and \( T_{lin} \) is the additional lateral force acting on the point \( F \) which is transmitted from the second connecting vehicle, depending on the direction of the draw-bar. From Fig. 1(a), the value of \( W_1 \) and \( W_2 \) could be calculated as follows:

\[ W_1 = W_{lin} \{ 0.5 - h_1 (tan \theta_{lat})/C \} \]
+ \( (T_{lin} - T_{lat}) h_1 / C + (tan \theta_{lin})/2 \)
- \( (T_{lin}/2 - T_{lat}) tan \theta_{lin} \)  
(5)

\[ W_2 = W_{lin} \{ 0.5 + h_1 (tan \theta_{lin})/C \} \]
- \( (T_{lin} - T_{lat}) h_2 / C - (tan \theta_{lin})/2 \)
- \( (T_{lin}/2 - T_{lat}) tan \theta_{lin} \)  
(6)

where \( \theta_{lat} \) is the angle of lateral inclination of the vehicle.

\[ \theta_{lin} = \sin^{-1} \left( (s_{lin} - s_{fo}) / C \right) \]

\[ s_{lin} = s_0 + (s_1 - s_0)(0.5 + e) \]

\[ s_{fo} = s_0 + (s_0 - s_0)(0.5 + e) \]

In addition, \( h_1 \) is the height of the center of gravity \( G \) of the vehicle measured from the bottom track belt, \( h_2 \) is the
could be assumed to act on the same position $G_i$, $G_o$ for the inner and the outer track as for position $G$. As shown in Fig. 1(b), $T_{li}$, $T_{lo}$ are the driving force transmitted from the torque $Q_i$ and $Q_o$ of the rear sprocket (Kogure et al., 1993), which is applied on the track belt at the bottom-dead-center of the rear sprocket for the inner and the outer track belt. $T_{2i}$, $T_{2o}$ are the locomotion resistance i.e. the compaction resistance acting in front of the inner and the outer track belt at the depth $z_i$ and $z_o$, which can be calculated from each rut depth, $s_i$ and $s_o$ (Muro, 1989a).

$T_{3i}$, $T_{3o}$ represent the thrust developed along the track belt at the interface between surface and the grouser of the inner and the outer track belt, which can be calculated as the integral of shear resistance of soil (Muro, 1989a). Usually the driving force $T_{3(o)}$ could be assumed to be the same as the thrust $T_{3(i)}$, which depends on the shearing characteristics of the surface.

$T_{4i\text{ani}}, T_{4o\text{ono}}$ are the effective reactive force acting on the inner and the outer track in the longitudinal direction, which could be calculated from the force balance (Muro, 1989b) as shown in the following equation:

$$T_{4i\text{ani}(o)} = T_{3i\text{ani}(o)} / \cos \theta_{i(o)} - W_{i(o)} \tan \theta_{i(o)} - T_{2i\text{ani}(o)}$$  \hspace{1cm} (7)

where $\theta_{i(o)}$ is the angle of longitudinal inclination of the inner and the outer track.

The longitudinal effective reactive force $T_{4i\text{ani}}$ acting on the longitudinal direction of the vehicle is given as the sum of each effective reactive force $T_{4i\text{ani}}$ and $T_{4o\text{ono}}$, as follows:

$$T_{4i\text{ani}} = T_{4i\text{ani}} + T_{4o\text{ono}}$$  \hspace{1cm} (8.1)

The lateral effective reactive force $T_{4i\text{lat}}$ acting on the lateral direction of the vehicle is given as the integral of the lateral shear resistance $\tau_{l\text{ani}(o)}(X)$ developed along the inner and outer track belt as follows:

$$T_{4i\text{lat}} = T_{4i\text{lat}} + T_{4i\text{lat}} = B \int_{-\infty}^{\infty} \left[ \tau_{l\text{ani}(o)}(X) \right] dX$$  \hspace{1cm} (8.2)

where $X$ is the distance from the front of the track belt. Here, the centrifugal force is not considered because of its negligibly small value due to the low vehicle speed. The angle $\delta$ of the resultant effective reactive effort $T_{4i\text{lat}}$ is determined as

$$\delta = \tan^{-1} \left( \frac{(T_{l} - T_{4i\text{lat}})}{T_{4i\text{ani}}} \right).$$  \hspace{1cm} (8.3)

$P_{pi}, P_{po}$ are the resultant normal ground reaction applied on the inner and the outer track, whose amount of eccentricity are $e_iD$ and $e_oD$ respectively. $H_i$, $H_o$ are the initial track tension for the inner and the outer track belt. As shown in Fig. 1(c), $F_i$ is the acting point of the resultant effective reactive effort $T_i$, composed of a longitudinal effort $T_{4i\text{ani}}$, a lateral effort $T_{4i\text{lat}}$ of the vehicle, and an additional lateral force $T_{1i}$, where height $h_i$ is the distance measured from the bottom-dead-center of the rear sprocket and $l_i$ is the distance from the center line of the vehicle. Additionally, $T_{4i\text{ani}}, T_{4i\text{lat}}$ and $T_{4o\text{ono}}, T_{4o\text{ono}}$ could be assumed to act on the same position $F_i$ and $F_o$ for the inner and the outer track as the position $F$ on the longitudi-
The point \( P \) is the turning center of the tracked vehicle from the center point \( O \), i.e., \( R \) is the turning radius of the vehicle and \( Y \) is the deviation from the lateral center line of the vehicle. \( M_i, M_o \) are the turning resistance moment acting around points \( P_i \) and \( P_o \) of the inner and the outer track, whose deviation are \( Y_i = e_i D \) and \( Y_o = e_o D \) from center points \( O_i \) and \( O_o \) of the inner and the outer track respectively.

The longitudinal effective tractive effort \( T_{\text{dلون}} \) can be also derived from the following moment balance equation as follows:

\[
RT_{\text{dلون}} + (l_o - Y)(T_{L} - T_{\text{dلون}}) \\
= (R - C/2)(T_{o} \cos \theta_{o} - W_i \tan \theta_{o} - T_{s}) \\
+ (R + C/2)(T_{o} \cos \theta_{o} - W_o \tan \theta_{o} - T_{s}) - M_i - M_o, \\
= (R - C/2)T_{\text{dلون}} + (R + C/2)T_{\text{dلون}} - M_i - M_o \\
Y = (e_i + e_o)D/2.
\] (9.1)

Here, the thrusts \( T_{i} \), \( T_{o} \) and the compaction resistances \( T_{2i} \), \( T_{2o} \) were assumed to be determinable as the same values which were calculated in the longitudinal direction of each track belt during straight forward motion.

Substituting Eq. (8.1) into the above equation, then

\[
T_{\text{dلون}} - T_{\text{dلون}} = 2(M_i + M_o + (l_o - Y)(T_{L} - T_{\text{dлон}})) / C \\
\] (10)

is obtained.

When \( \delta \) is zero, the difference between \( T_{\text{dلون}} \) and \( T_{\text{dлон}} \) can be calculated as \( 2(M_i + M_o) / C \) using Eq. (10) at \( T_{i} = T_{\text{dлон}} \). When the additional lateral force \( T_i \) becomes large, the difference between \( T_{\text{dлон}} \) and \( T_{\text{dлон}} \) is increased.

The resultant effective tractive effort \( T_{\text{dلون}} \) can be calculated as follows:

\[
T_{\text{dлон}} = (T_{\text{dлон}} - T_{\text{dлон}})^{1/2} \\
\] (11)

\( \beta \) is the slip angle of the center of gravity of the vehicle and \( \beta_i, \beta_o \) are the slip angles of the inner and outer track respectively, where

\[
\beta = \tan^{-1} \left( \frac{Y}{(R - C/2)} \right) \\
\beta_i = \tan^{-1} \left( \frac{Y}{(R + C/2)} \right) \\
\beta_o = \tan^{-1} \left( \frac{Y - eD}{R} \right).
\] (12.1, 12.2, 12.3)

The compaction resistance \( T_{2i} \) for a single track belt moving straight forward is given as follows (Muro, 1995):

\[
T_{2i} = B \int_{0}^{\tau_{2i}} k_{1} s^{1/2} \, ds \\
\] (13)

Table 2 shows the vehicle dimensions and the specifications used for the simulation analysis.

**Table 2. Dimensions and specifications of flexible tracked vehicle**

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle weight</td>
<td>617 N</td>
</tr>
<tr>
<td>Width of track</td>
<td>12.60 cm</td>
</tr>
<tr>
<td>Contact length of track</td>
<td>33.00 cm</td>
</tr>
<tr>
<td>Central distance between inner and outer track</td>
<td>23.00 cm</td>
</tr>
<tr>
<td>Mean contact pressure</td>
<td>9.34 kPa</td>
</tr>
<tr>
<td>Radius of front idler</td>
<td>5.4 cm</td>
</tr>
<tr>
<td>Radius of rear sprocket</td>
<td>5.4 cm</td>
</tr>
<tr>
<td>Grouser height</td>
<td>1.7 cm</td>
</tr>
<tr>
<td>Grouser pitch</td>
<td>2.6 cm</td>
</tr>
<tr>
<td>Number of track roller</td>
<td>3</td>
</tr>
<tr>
<td>Radius of track roller</td>
<td>1.9 cm</td>
</tr>
<tr>
<td>Eccentricity of center of gravity of vehicle</td>
<td>0.008</td>
</tr>
<tr>
<td>Height of center of gravity</td>
<td>25 cm</td>
</tr>
<tr>
<td>Distance between central axis of vehicle and application point of resultant effective tractive effort</td>
<td>15.0 cm</td>
</tr>
<tr>
<td>Initial track belt tension</td>
<td>147 N</td>
</tr>
<tr>
<td>Circumferential speed of rear sprocket</td>
<td>6.2 cm/s</td>
</tr>
<tr>
<td>Outer track</td>
<td>5.4 cm</td>
</tr>
<tr>
<td>Steering ratio</td>
<td>1.00 - 2.25</td>
</tr>
</tbody>
</table>

\[
\dot{\theta}_{i}(X) = \int_{0}^{t} (r_i \omega_{i} - \dot{V}_{i}(X)) \, dt = \theta_{i}(X) \\
\] (15)

where \( t \) is the moving time of the track belt, i.e., \( X/X_{i}(X) \) \( \omega_{i}(X) \) from the front to point \( X \), and \( \theta_{i}(X) \) is the slip ratio of the inner and the outer track as mentioned later.

The longitudinal shear resistance of surface under the flexible inner and outer track belt \( T_{\text{dلون}}(X) \) at points \( N_i \) and \( N_o \) was calculated as (Muro, 1991):

\[
\tau_{i}(X) = (m_{\text{e1}} + m_{\text{ton}} \rho_{i}(X)) \times \left[ 1 - \exp \left( -\alpha_{i} X \right) \right].
\] (16)

For loading state:

\[
p_{i}(X) = k_1 (s_i(X))^{n_i}
\]

For unloading state:

\[
p_{i}(X) = k_1 (s_{i}(X))^{n_i} - k_2 (s_{i}(X) - s_{o}(X))^{n_2}
\]

where \( m_{\text{e1}}, m_{\text{ton}}, \) and \( \alpha_{i} \) are the terrain track system constants in the longitudinal direction of the track plate.

Then, the main thrust of the inner and outer track \( T_{2i} \) were calculated as follows:

\[
T_{2i} = \int_{0}^{\tau_{2i}} k_{1} s^{1/2} \, ds \\
\] (17)

**Steering Ratio**

When the circumferential speed of the rear sprocket of the inner track and the outer track is set at \( r_i \omega_i \) and \( r_o \omega_o \), respectively the steering ratio \( \varepsilon \) is defined as follows:

\[
\varepsilon = r_i \omega_i / r_o \omega_o \\
\] (18)

On the other hand, the other steering ratio \( \varepsilon' \) can be defined by the following equation, when the speed of the inner and the outer track is set at \( V_i \) and \( V_o \) at slip angle \( \beta_i \),
and $\beta_0$ respectively:

$$e' = V_o \cos \beta_o / V_i \cos \beta_i$$  (19)

In this case, the slip ratios of inner and outer track $i_i, i_o$ are expressed as,

$$i_i = 1 - V_i \cos \beta_i / r_{ao} \omega_o$$  (20)

$$i_o = 1 - V_o \cos \beta_o / r_{ao} \omega_o$$  (21)

when both the track belts are being driven.

Substituting the above equations into Eq. (18), the next relationship could be derived.

$$e' = (1 - i_i) / (1 - i_o)$$  (22)

For the turning speed of the tracked vehicle $V$ at the center of gravity $G$, the running speed of the inner and the outer track $V_i$ and $V_o$ are given as follows;

$$V = \omega \sqrt{R^2 + (Y-eD)^2}$$  (23)

$$V_i = r_{ao} \omega_o (1 - i_i) / \cos \beta_i = \omega \sqrt{(R-C/2)^2 + Y^2}$$  (24)

$$V_o = r_{ao} \omega_o (1 - i_o) / \cos \beta_o = \omega \sqrt{(R+C/2)^2 + Y^2}$$  (25)

$$\omega = r_{ao} \omega_o (1 - i_i) / (R-C/2) = r_{ao} \omega_o (1 - i_o) / (R+C/2)$$  (26)

where $\omega$ is the steering angular velocity of the tracked vehicle.

Eliminating the steering angular velocity $\omega$, the turning radius of the tracked vehicle $R$ could be determined as follows:

$$R = \frac{C (r_{ao} \omega_o (1 - i_o) + r_{ao} \omega_o (1 - i_i))}{2(r_{ao} \omega_o (1 - i_i) - r_{ao} \omega_o (1 - i_o))}$$  (27)

### Amount of Slippage in Turning Motion

The calculation of the lateral slip velocity between track and surface and the amount of lateral slippage of soil under the track belt in turning motion is needed to calculate the turning resistance moment of the inner and outer track. Figure 2 shows the resultant slip velocity $V_{\text{slip}}$, where components $r_{\text{slip}} \omega_{\text{slip}}$ are in the longitudinal direction and $\omega \mathbf{P}N_i$ and $\omega \mathbf{P}N_o$ are the tangential direction. So, the lateral slip velocity $V_{\text{slip}}(X)$ of the inner and the outer track at arbitrary points $N_i$ and $N_o$ is given as the lateral component of the resultant slip velocity:

$$V_{\text{slip}}(X) = \omega \sin \alpha \sqrt{\left(R \pm C/2\right)^2 + \left(D/2 - X + Y\right)^2}$$

$$= \omega \left(\frac{D - X + Y}{\sqrt{(R \pm C/2)^2 + Y^2}}\right)$$  (28)

which is shown in Fig. 1(c).

Then, the lateral amount of slippage $j_{\text{slip}}(X)$ can be calculated as follows:

$$j_{\text{slip}}(X) = \int_0^X V_{\text{slip}}(X) \, dt$$

$$= \frac{V_{\text{slip}}(X)}{\sqrt{(R \pm C/2)^2 + Y^2}} \int_0^X \left(\frac{D - X + Y}{\sqrt{(R \pm C/2)^2 + Y^2}}\right) \, dX$$

$$= \frac{1 - i_{\text{slip}}}{r_{\text{slip}} \omega_{\text{slip}}} \sqrt{(R \pm C/2)^2 + Y^2} \left(\frac{D + Y - X/2}{(D/2 + Y - X/2)X}\right)$$  (29)

where $i$ is the time of travel of the grouser from the front part of the track belt to arbitrary points $N_i$ and $N_o$ under the inner and outer track and can be given as $X/r_{\text{slip}} \omega_{\text{slip}}$.

So, it is clear that the lateral amount of slippage $j_{\text{slip}}(X)$ becomes zero at $X = 0$ and $X = D + 2Y$, and it takes a maximum value at $X = D/2 + Y$ for both the inner and the outer track.

### Turning Resistance Moment

The lateral shear resistance $\tau_{\text{slip}}(X)$ developed along the track belt at points $N_i$ and $N_o$ under the inner and outer track can be calculated as in the following equation:

$$\tau_{\text{slip}}(X) = \begin{cases} d_{\text{slip}}(X) / dX \geq 0 \text{ and } 0 \leq j_{\text{slip}}(X) < j_l, \\ \tau_{\text{slip}}(X) = \left(m_{\text{slip}} + m_{\text{flat}} p_{\text{slip}}(X)\right) \\
 \times [1 - \exp \left(-a_{\text{slip}} j_{\text{slip}}(X)\right)] \end{cases}$$  (30.1)

$$\tau_{\text{slip}}(X) = \begin{cases} d_{\text{slip}}(X) / dX < 0 \text{ and } j_{\text{slip}}(X) \geq j_l, \\ \tau_{\text{slip}}(X) = \left(m_{\text{slip}} + m_{\text{flat}} p_{\text{slip}}(X)\right) \\
 \times [1 - \exp \left(-a_{\text{slip}} j_{\text{slip}}(X)\right)] \end{cases}$$  (30.2)

$$\tau_{\text{slip}}(X) = \begin{cases} d_{\text{slip}}(X) / dX < 0 \text{ and } j_{\text{slip}}(X) < j_l, \\ \tau_{\text{slip}}(X) = \left(m_{\text{slip}} + m_{\text{flat}} p_{\text{slip}}(X)\right) \\
 \times [1 - \exp \left(-a_{\text{slip}} j_{\text{slip}}(X)\right)] \end{cases}$$  (30.3)

where $j_l = j_{\text{slip}}(D/2 + Y)$, $\tau_l = \tau_{\text{slip}}(D/2 + Y)$, $j_l = j_{\text{slip}}(D/2 + Y)$, $p_{\text{slip}}(X)$ is the normal contact pressure distribution under the inner and outer track belt. $m_{\text{slip}}, m_{\text{flat}}$ and $a_{\text{slip}}, k_3, n_3, m_{\text{flat}}$, $m_{\text{flat}}$ and $a_{\text{slip}}$ are the terrain-track system constants in the lateral direction of the track plate, as shown in Table 1 which were determined, including the buldozing resistance of the track model plate in the track plate traction test. Also, the reaction of the additional lateral force $T_i$ is automatically included in this

![Fig. 2](image-url)  
**Fig. 2.** Lateral amount of slippage $j_{\text{slip}}$, $j_{\text{slip}}$ and lateral shear resistance $\tau_{\text{slip}}, \tau_{\text{slip}}$ for inner and outer track
distribution of the lateral shear resistance.

Then the turning resistance moment (Pott et al., 1994) $M_t$ and $M_r$ exerted around the turning points $P$ and $P_o$ of the inner and outer track can be calculated as follows:

$$M_{t(0)} = B \int_{0}^{\rho} r_{(0)j}(X)(D/2 - X + Y) \, dX$$  \hfill (31)

The total turning resistance moment $M$ is given as in the following equation.

$$M = M_t + M_r$$  \hfill (32)

The energy equilibrium equation for the straight forward motion of a tracked vehicle has already been presented (Muro, 1995). For turning motion, the input energy $E_{i(0)}$ supplied by the driving torque can be given as the summation of the depression deformation energy $E_{s(0)}$ required to make a rut under the track belt, the slippage energy $E_{s(0)}$ required to develop a thrust along the bottom track belt, the effective tractive effort energy $E_{t(0)}$ and the turning moment energy $E_{t(0)}$ for the inner track and the outer track, as follows:

$$E_{i(0)} = E_{s(0)} + E_{s(0)} + E_{t(0)} + E_{t(0)}$$  \hfill (33)

where

$$E_{i(0)} = T_{i(0)} \gamma_{i(0)}, \cos \beta_{i(0)} / (1 - i_{i(0)})$$

$$E_{s(0)} = T_{s(0)} \gamma_{s(0)} \cos \beta_{s(0)}$$

$$E_{s(0)} = T_{s(0)} \gamma_{s(0)} / (1 - i_{s(0)}) - 1 / \cos \theta_{s(0)} \nu_{s(0)} \cos \beta_{s(0)}$$

$$E_{s(0)} = T_{s(0)} \gamma_{s(0)} \cos \beta_{s(0)}$$

$$E_{s(0)} = T_{s(0)} \gamma_{s(0)} \cos \beta_{s(0)}$$

$$E_{s(0)} = T_{s(0)} \gamma_{s(0)} \cos \beta_{s(0)}$$

$$E_{s(0)} = T_{s(0)} \gamma_{s(0)} \cos \beta_{s(0)}$$

Then the total input energy $E_t$ of the vehicle and the total output energy of the compaction energy $E_s$, the slippage energy $E_s$, the effective tractive effort energy $E_t$ and the turning moment energy $E_t$ of the vehicle can be given as in the equations:

$$E_t = E_s + E_t + E_s + E_s$$  \hfill (34)

$$E_t = E_i + E_s + E_s$$  \hfill (35)

$$E_t = E_s + E_s$$  \hfill (36)

$$E_t = E_s + E_s$$  \hfill (37)

$$E_t = E_s + E_s$$  \hfill (38)

$$E_t = E_s + E_s$$  \hfill (39)

This energy equilibrium equation can be proved theoretically by using the force and moment balance equations as mentioned above. The optimum resultant effective tractive effort $T_{i, opt}$ is defined as the resultant effective tractive effort at the optimum combination of slip ratio $i_{opt}$ of the inner track and $i_{opt}$ of the outer track which takes the maximum value of the effective tractive effort energy $E_{max}$. The tractive efficiency of power $E_d$ is given as follows:

$$E_d = E_t / E_i$$  \hfill (40)

Flow Chart

The input information for the simulation analytical method includes the data of the dimensions of the flexible tracked vehicle of weight $W$, the contact length $D$ of the inner and the outer track, the track width $B$ of the inner and the outer track, the central distance $C$ between the inner and the outer track, the radius $r_1$ of the front idler, the radius $r_2$ of the rear sprocket, the radius of the track roller $r_m$, the number of track rollers $N$, the grousers height $H$, the eccentricity of the center of gravity $e$ of the vehicle, the height $h_0$ of the center of gravity of vehicle, the distance $l_0$ between the central axis of the vehicle and the application point, the height $h_0$ of the application point of the total effective tractive effort, and the initial track belt tension $H_{b0}$ as shown in Fig. 3. And then the terrain-track system constants $k_1$, $k_2$, and $n_1$, $n_2$ from the plate loading and unloading test, $m_{lons}, m_{lons}$, $m_{lons}, m_{lons}, m_{lons}, m_{lons}$ from plate traction test, and $C_{lons}, C_{lons}, C_{lons}, C_{lons}, C_{lons}, C_{lons}$ from plate slip sinkage test should be read as the initial input data.

At rest, the amount of static depression $s_0 = s_0$ and $s_0 = s_0$, the linear distribution of static depression $s(X)$ of the inner track which equals $s_0(X)$ of outer track at the distance $X$ from the contact point of the front idler on the main part of track belt. The angle of longitudinal inclination of the inner and the outer track $\theta_0 = \theta_0$, the contact pressure $p_n = p_n$ at the front idler and $p_n = p_n$ at the rear sprocket. The nonlinear normal pressure distribution $p(X)$ of the inner track which equals $p_n(X)$ of the outer track, and the eccentricity $e_0$ of the resultant force $P_{n0} = P_{n0}$ for the assumed rigid track belt should be repeatedly calculated until the distribution of static depression $s_0(X) = s_0(X)$ is determined.

For the given angular velocity of rear sprocket $\omega_0$, the inner track and $\omega_0$ at the outer track, and the steering ratio $e$, both the tractive performances of the inner and the outer track during driving action can be calculated for each combination of slip ratio $i_0$ and $i_0$. First of all, the tractive performance of the inner track is calculated for the slip ratio $i_0$ of the inner track, assuming that the distributed vehicle weight $W_1$ and $W_1$, equal half of the vehicle weight $W_2$ respectively. For the above calculated eccentricity $e_0$, the resultant normal force $P_{n0}$, the contact pressure $p_n$, $p_n$ and $p_n(X)$, the total amount of depression $s_0$, $s_0$ and $s(X)$ including the amount of slip depression, and the angle $\theta_0$ can be repeatedly recalculated depending on the three kinds of flow system divided by the value of eccentricity $e_0$, until the distribution of the total amount of depression $s_0(X)$ is determined. In order to transform the above results calculated for the assumed rigid track belt to the actual flexible track belt, the normal contact pressure distribution and the distribution of the total amount of depression should be changed to $p_n$, and $p_n(X)$ and $s_n$, $s_n$, and $s_n(X)$ for the flexible track belt considering the initial track belt tension $H_{b0}$ as mentioned in a previous paper (Muro, 1991). Then the driving force $T_{ni}$, the compaction resistance $T_{ni}$, the thrust $T_{ni}$ from Eq. (17), the effective tractive force $T_{n0}$ from Eq. (7), the angle $\theta_0$, the eccentricity $e_0$ of the normal resultant
Fig. 3. Flow chart of numerical simulation for steering performance of flexible tracked vehicle

force $P_\text{om}$, the turning resistance moment $M_i$, from Eq. (31), the running speed $V_i$ from Eq. (24), and the longitudinal effective tractive effort of the vehicle $T_{\text{slon}}$ from Eq. (8.1) can be repeatedly recalculated until the thrust $T_\text{sl}$ is determined.

Next, the tractive performance of the outer track is calculated for the slip ratio $i_\text{c}$ of the outer track assuming that the distributed vehicle weight $W_\text{d}$ equals $W/2$. For

| STOP |
| END |
the above calculated eccentricity \( e_\circ = e_\circ \) of the resultant force \( P_{po} \), the contact pressure \( p_{po}, p_{po} \) and \( p_0(X) \), the total amount of depression \( s_{\theta o}, s_{\theta o} \) and \( s_\circ(X) \) including the amount of slip depression, and the angle \( \theta_0 \) can be repeatedly recalculated depending on the three kinds of flow system divided by the value of eccentricity \( e_\circ \), until the distribution of the total amount of depression \( s_\circ(X) \) is determined. In order to transform the above results calculated for the assumed rigid track belt to the actual flexible track belt, the normal contact pressure distribution and the distribution of the total amount of depression should be changed to \( p_{po}, p_{po} \) and \( p_0(X) \) and \( s_{\theta o}, s_{\theta o} \) and \( s_\circ(X) \) for the flexible track belt considering the initial track belt tension \( H_0 \). Then the driving force \( T_{10} \), the compaction resistance \( T_{20} \), the thrust \( T_{30} \) from Eq. (17), the effective active force \( T_{40} \) from Eq. (7), the angle \( \theta_0 \), the eccentricity \( e_\circ \) of the normal resultant force \( P_{po} \), the turning resistance moment \( M_0 \), Eq. (31), the running speed \( V_0 \) from Eq. (25), and the longitudinal effective active effort of the vehicle \( T_{10} \) from Eq. (8.1) can be repeatedly recalculated until the thrust \( T_{30} \) is determined.

Thereafter, the actual slip ratio \( i_0 \) of the outer track for the given slip ratio \( i_0 \) of the inner track should be calculated repeatedly by using the two division method until the longitudinal effective active effort of the vehicle \( T_{10} \) from Eq. (10) is determined. After that, the real distribution of vehicle weight to the inner and the outer track \( W_i \) and \( W_o \) should be recalculated repeatedly from Eqs. (5) and (6) until the real angle of lateral inclination of the vehicle \( \theta_0 \) is determined.

Then the performance of the vehicle i.e., the resultant effective active effort \( T_{10} \) from Eq. (11) composed of the longitudinal effective active effort \( T_{10} \) from Eq. (8.1) and the lateral effective active effort \( T_{10} \) from Eq. (8.2), and the angle \( \delta \) of the vehicle, the position of the turning center of the turning radius of the vehicle \( R \) from Eq. (27) and \( Y \) from Eq. (9.2), the running speed \( V \) and the angle \( \beta \), the steering ratio \( e \) from Eq. (18) and \( e' \) from Eq. (19), the angle of lateral inclination of the vehicle \( \theta_0 \), the total turning resistance moment \( M \) from Eq. (32), and total amount of the input energy \( E_i \), the depression deformation energy \( E_i \), the slippage energy \( E_i \), the effective active effort energy \( E_{40} \), can be determined in detail. Finally, the optimum effective active effort \( T_{10} \) and the optimum combination of slip ratio \( i_0 \) of the inner track and \( i_0 \) of the outer track, can be determined from Eq. (39).

### Numerical Results

The simulation was carried out with a small tracked vehicle model of weight \( W = 617.4 \) N running on the soil bin as above. The flexible tracked vehicle model had the same 21 track plates as mentioned previously together with 3 road rollers of radius 19 mm, and track length \( D = 330 \) mm, track width \( B = 100 \) mm, track gauge \( C = 230 \) mm, the mean contact pressure \( p_0 = 9.36 \) kPa, eccentricity \( e = 0.008 \) and height \( h_2 = 150 \) mm of the center of gravity. The radius of the front idler was \( r_i = 54 \) mm and the rear sprocket was \( r_s = 54 \) mm. The distance between the central axis of the vehicle and point \( F \), the acting effective active effort, was \( l_2 = 260 \) mm, the height of point \( F \), \( h_2 = 150 \) mm, and the initial track tension \( H_0 = 147 \) N. For an articulated tracked vehicle (Watanabe et al., 1986), two identical flexible tracked vehicle models connected by a draw bar were considered for this simulation. The first vehicle towed the second one. The first one was the numerical object while the second one was the means to draw the first one at various levels of effective active effort. For the steady state of turning motion, the first vehicle was set to run with five steering ratios \( e = 1.25, 1.50, 1.75, 2.00, \) and 2.25. For each steering ratio, more than eight combinations of rotary speeds of the rear sprocket for the inner and the outer track were set, while the second vehicle was set at the constant combination of the lowest rotary speed of rear sprocket for the inner and the outer track. Table 2 shows the dimensions and specifications of the flexible tracked vehicle.

As shown in Fig. 4, the steering locus of the former test tracked vehicle could be assumed as the same as the locus of the latter connected tracked vehicle. The angle \( \delta \) of the resultant effective active effort \( T_i \) was calculated using the dimensions of \( L_i = 30 \) cm, \( L_M = 25 \) cm and \( L_k = 19 \) cm denoted in this figure as a function of the turning radius.
Fig. 5(a). Relationship between turning speed $V$ of tracked vehicle and turning radius $R$ for five steering ratios $\varepsilon$

Fig. 5(b). Relationship between turning speed $V_o$ of outer track and turning radius $R$ of tracked vehicle for five steering ratios $\varepsilon$

Fig. 5(c). Relationship between turning speed $V_i$ of inner track and turning radius $R$ of tracked vehicle for five steering ratios $\varepsilon$

Fig. 5(d). Relationship between steering ratio $\varepsilon'$ and turning radius $R$ of tracked vehicle for five steering ratios $\varepsilon$

Fig. 5(e). Relationship between slip ratio of outer track $i_o$ and inner track $i_i$ for five of steering ratios $\varepsilon$

Fig. 5(f). Relationships among slip ratios of inner and outer track $i_o$, $i_i$ and turning radius $R$ for five steering ratios $\varepsilon$
of the tracked vehicle $R$:
\[
\delta = \pi - \tan^{-1}\left(\frac{R}{L_f} - \cos^{-1}\left(\frac{(L_3 + L_4 - L_5)}{(2L_3(R^2 + L_3))^{1/2}}\right)\right)
\]
\[
= \pi - \tan^{-1}\left(\frac{R}{30} - \cos^{-1}\left(\frac{23.28}{(R^2 + 900)^{1/2}}\right)\right)
\] (41)

**Vehicle Speed and Slip Ratio**

Figures 5(a), (b) and (c) show the numerical relationship between the turning speed of the tracked vehicle $V$ and the turning radius $R$ for five steering ratios $\varepsilon = 1.25$, 1.50, 1.75, 2.00 and 2.25. For each steering ratio $\varepsilon$, the vehicle speed $V$ tended to decrease with the increment of the radius $R$ which was accompanied by the decrement of the steering angular velocity of the tracked vehicle $\omega$ as shown in Eq. (26). In general, the running speed of the outer track $V_o$ was always larger than that of the inner track $V_i$ and both the running speeds decreased with the increments of the turning radius $R$, the steering ratio $\varepsilon$ and the slip ratios $i_i$ and $i_o$. As shown in Fig. 5(d), the steering ratio $\varepsilon'$ which was defined as the ratio of the run-

---

**Fig. 6(a).** Relationship among thrust $T_{3n}$, compaction resistance $T_n$, longitudinal effective tractive effort $T_{ane}$, lateral effective tractive effort $T_{ane}$ and slip ratio $i_i$ for inner track (Steering ratio $\varepsilon = 1.50$)

**Fig. 6(b).** Relationship among thrust $T_{3n}$, compaction resistance $T_n$, longitudinal effective tractive effort $T_{ane}$, lateral effective tractive effort $T_{ane}$ and slip ratio $i_o$ for outer track (Steering ratio $\varepsilon = 1.50$)

**Fig. 6(c).** Relationship between angle $\delta$ of resultant effective tractive effort and turning radius $R$ for five steering ratios $\varepsilon$

**Fig. 6(d).** Relationship between longitudinal effective tractive effort $T_{ane}$ and turning radius $R$ for five steering ratios $\varepsilon$
ning speed of the inner and the outer track decreased hyperbolically with the increment of the turning radius \( R \). Figure 5(e) shows the numerical relationship between the slip ratios of the inner and the outer track, \( i_i \) and \( i_o \), for five steering ratios \( \epsilon \). The slip ratio of the outer track \( i_o \) was always larger than that of the inner track \( i_i \) for the left turning motion. From the numerical simulation results, the next regression analytical equation was derived as follows:

\[
i_o = 39.5(\epsilon - 1)^{1.13} + 2.43 e^{0.846 \epsilon - 0.491} \quad (R = 0.995) \quad (42)
\]

Figure 5(f) shows the numerical relationship for the slip ratios of the inner and the outer track \( i_i \) and \( i_o \), and the turning radius \( R \) for five steering ratios \( \epsilon \). From the numerical simulation results, the next regression analytical equation was obtained:

\[
i_i = 4.48 \{ R - 1.24(\epsilon - 1)^{-0.227} \}^{0.707} e^{0.438} + 1 \quad (R = 0.968) \quad (43)
\]

\[
i_o = 9.67 (R - 1.74)^{0.615} e^{-1.08} + 45.9(\epsilon - 1)^{1.04} \quad (R = 0.966) \quad (44)
\]

Both the slip ratios of the inner and outer track \( i_i \) and \( i_o \) increased with the radius \( R \), and \( i_o \) increased greatly with increments of the steering ratio \( \epsilon \).

**Effective Tractive Effort**

As an example, Figs. 6(a) and (b) show the numerical relationships of the thrust \( T_{360} \), the compaction resistance \( T_{360} \) and the longitudinal effective tractive effort \( T_{450} \), the lateral effective tractive effort \( T_{450} \) and the slip ratio of the inner and outer track \( i_i \) and \( i_o \), respectively, for the steering ratio of \( \epsilon = 1.50 \). In general, \( T_{360} \) was always larger than \( T_{450} \) due to the occurrence of \( T_{360} - T_{450} \) decreased slightly with the increment of slip ratio \( i_o \). For the inner track, with increments of \( i_i \), \( T_{360} \) and \( T_{450} \) increased parabolically and \( T_{360} \) increased slightly within the slip ratio of 25%. For the outer track, \( T_{360} \) increased greatly with increments of \( i_o \) and \( T_{450} \) had a maximum value of 0.125 kN at \( i_o = 42.1% \) because of the parabolically increasing compaction resistance \( T_{450} \).

Figure 6(c) shows the numerical relationships between the longitudinal effective tractive effort of the tracked vehicle \( T_{450} \) and the turning radius \( R \) for five steering ratios \( \epsilon \). In general, \( T_{450} \) increased exponentially with increments of the turning radius \( R \) and it increased with the decrement of the steering ratio \( \epsilon \) at the larger turning radii \( R \). Figure 6(d) shows the numerical relationships of the lateral effective tractive effort \( T_{450} \), the additional lateral force \( T_{450} \) and the turning radius \( R \) for five steering ratios \( \epsilon \). Both \( T_{450} \) and \( T_{450} \) decreased with increments of \( R \), and \( T_{450} \) was always larger than \( T_{450} \) for all the steering ratios.

Figure 6(e) shows the numerical relationship between the angle \( \delta \) of the resultant effective tractive effort and the turning radius \( R \) for five steering ratios \( \epsilon \). The angle \( \delta \) decreased hyperbolically with increments of \( R \) independently of the steering ratio as shown in Eq. (41).

Figure 7 shows the numerical relationships between the resultant effective tractive effort \( T_{450} \) of the tracked vehicle and the slip ratio of the outer track \( i_o \) for the straight forward motion of \( \epsilon = 1.00 \) and the turning motion of five steering ratios \( \epsilon \). As shown in this figure, the resultant effective tractive efforts \( T_{450} \) for the steering ratio \( \epsilon = 1.25, 1.50, 1.75, 2.00 \) and 2.25 had no peak values except \( \epsilon = 1.00 \), because the longitudinal effective tractive effort \( T_{450} \) of the outer track was larger than that of the inner track \( T_{450} \) due to the occurrence of turning resistance moment as shown in Eq. (10), i.e. the resultant effective tractive effort had a maximum value when the longitudinal effective tractive effort of the outer track took a peak value in the conventional Hump type curves. That is, \( T_{450} \) reached the maximum value which could be developed on the weak terrain when the combination of the slip ratios of the inner and the outer track reached the final stage of combination of \( i_i \) and \( i_o \), so the tracked vehicle could not turn any more because the difference between \( T_{450} \) and \( T_{450} \), i.e. the turning resistance moment, did not increase any more with increments of the slip ra-
In general, $T_{ar}$ decreased with increments of the steering ratio $\varepsilon$ for the same slip ratio of outer track $i_o$, also the slip ratio of the outer track $i_o$ for the same $T_{ar}$ became larger with increments of $\varepsilon$.

Figure 8 shows the numerical relationships between the resultant effective tractive effort $T_{ar}$ and the turning radius $R$ for the five steering ratios $\varepsilon$. $T_{ar}$ increased exponentially with increments of turning radius $R$ and it decreased with increments of the steering ratio $\varepsilon$ at larger turning radii $R$.

Amount of Depression and Slip Ratio

Figure 9(a) shows the numerical relationships of the amount of depression of the front idler $s_{fn}$, the rear sprocket $s_{rn}$, and the slip ratio $i_i$ of inner track for five steering ratios $\varepsilon$. $s_{fn}$ decreased with increments of $i_i$, while $s_{rn}$ increased with increments of $i_i$. These relationships were independent of the steering ratio $\varepsilon$. Figure 9(b) shows the relationships of the amount of depression of the front idler $s_{fn}$, the rear sprocket $s_{rn}$, and the slip ratio $i_o$ of the outer track for five steering ratios $\varepsilon$. $s_{fn}$ also decreased with increments of $i_o$, while $s_{rn}$ increased with increments of $i_o$. These relationships were also independent of the steering ratio $\varepsilon$. In these figures, it was clear that the relationships between amount of depression of the front idler, rear sprocket of the inner and the outer track and the slip ratio showed the same tendency, and $s_{rn}$ and $s_{fn}$ were always larger than $s_{fn}$ and $s_{rn}$ because of the increasing eccentricity of the resultant normal surface reaction with increments of slip ratio respectively, as shown.
in the next figures.

Figures 10(a) and (b) show the numerical relationship between the eccentricity \( e_i \) and \( e_o \) of the resultant normal surface reaction \( P_{in} \) and \( P_{out} \) of the inner and the outer track and the slip ratio \( i_i \) and \( i_o \) for five steering ratios \( \varepsilon \), respectively. Both \( e_i \) and \( e_o \) showed almost the same Hump type relations in which they increased parabolically with increments of the slip ratio \( i_i \) and \( i_o \) and then decreased after taking peak values, and they were independent of the steering ratio \( \varepsilon \).

Figures 11(a) and (b) show the numerical relationship between the angle of longitudinal inclination \( \theta_i \) and \( \theta_o \) of the inner and outer track and the slip ratio \( i_i \) and \( i_o \) for five kinds of steering ratio \( \varepsilon \), respectively. Both \( \theta_i \) and \( \theta_o \) also showed the same tendency and they increased parabolically with increments of the slip ratio \( i_i \) and \( i_o \) independent of the steering ratio \( \varepsilon \).

Figure 12 shows the numerical relationship between the angle of lateral inclination of the tracked vehicle \( \theta_{lat} \) and the turning radius \( R \) for five steering ratios \( \varepsilon \). As a result, it was clear that \( \theta_{lat} \) increased with increments of turning radius \( R \) and steering ratio \( \varepsilon \) and it tended to have a minimum value at a certain turning radius for smaller steering ratios.

**Turning Resistance Moment**

The turning resistance moment of the inner track was always larger than that of the outer track corresponding with each lateral shear resistance. Figure 13(a) shows the numerical relationship between the turning resistance moment \( M_i \) and the slip ratio \( i_i \) of the inner track for five steering ratios \( \varepsilon \). Figure 13(b) shows the numerical relationship between the turning resistance moment \( M_o \) and the slip ratio \( i_o \) of the outer track for the five steering ratios \( \varepsilon \). As shown in these figures, \( M_i \) and \( M_o \) decreased hyperbolically with increments of each slip ratio. The total turning resistance moment \( M \) of the tracked vehicle is shown in Fig. 13(c) relating the turning radius \( R \) for the five steering ratios \( \varepsilon \). \( M \) decreased hyperbolically with increments of the turning radius \( R \) as predicted by Ito (1995).

**Energy Balance**

Figure 14 shows the energy balances of the input energy \( E_i \) and the output energies of the compaction energy \( E_s \), the slippage energy \( E_s \), the effective tractive energy \( E_t \) and the turning moment energy \( E_o \), relating to the slip ratio \( i_o \) of the outer track for the steering ratio of \( \varepsilon = 1.50 \). The input energy \( E_i \) and the slippage energy \( E_s \) increased almost linearly with the slip ratio \( i_o \). The compaction energy \( E_s \) increased parabolically with the slip ratio \( i_o \). The effective tractive energy \( E_t \) had a maximum value of 0.210 kN/cm/s at the optimum combination of slip ratios of \( i_i = 1.75 \% \) and \( i_o = 42.4 \% \) and then it decreased gradually. The turning moment energy \( E_o \) decreased hyperbolically with the slip ratio \( i_o \).
Fig. 13(a). Relationship between turning resistance moment $M_i$ and slip ratio $i_i$ of inner track for five steering ratios $\varepsilon$

Fig. 13(b). Relationship between turning resistance moment $M_o$ and slip ratio $i_o$ of outer track for five steering ratios $\varepsilon$

Fig. 13(c). Relationship between turning resistance moment $M$ of tracked vehicle and turning radius $R$ for five steering ratios $\varepsilon$

Fig. 14. Relationship between input energy $E_i$, compaction energy $E_c$, slippage energy $E_s$, effective tractive energy $E_t$, turning moment energy $E_3$ and slip ratio $i_o$ of outer track (Steering ratio $\varepsilon = 1.50$)

Fig. 15. Relationship between turning moment energy $E_3$ and turning radius $R$ for five steering ratio $\varepsilon$

Figure 15 shows the numerical relationship between the turning moment energy $E_3$ and the turning radius $R$ for five steering ratios $\varepsilon$. It was clear that $E_3$ decreased hyperbolically with $R$ while it increased very little with the decrement of the steering ratio $\varepsilon$.

Figure 16 shows the numerical relationship between the tractive efficiency of power $E_t$ and the turning radius $R$ for five steering ratios $\varepsilon$. It was clear that $E_t$ decreased hyperbolically with $R$ while it increased greatly with the decrement of the steering ratio $\varepsilon$.

**Stress Distribution**

As an example of the stress distribution, the distributions of the normal stress and the longitudinal shear resistance under the flexible inner and outer track belt, the distribution of the lateral shear resistance acting on
Fig. 16. Relationship between tractive efficiency of power $E_t$ and turning radius $R$ for five of steering ratios $\varepsilon$

![Graph showing the relationship between tractive efficiency and turning radius.](image)

Fig. 17(a). Distribution of normal stress $p_n$ and shear resistance $\tau_{os}$ developed under inner track (Steering ratio $\varepsilon = 1.50$, $i_{opt} = 17.0\%$)

![Diagram showing stresses and shear resistance](image)

Fig. 17(b). Distribution of normal stress $p_n$ and shear resistance $\tau_{os}$ developed under outer track (Steering ratio $\varepsilon = 1.50$, $i_{opt} = 42.5\%$)

![Diagram showing stresses and shear resistance](image)

The side of the inner and the outer track belt, and the distribution of the track tension of the inner and outer track belt, for the steering ratio of $\varepsilon = 1.50$ at the optimum combination of slip ratios of $i_{opt} = 17.0\%$ and $i_{opt} = 42.5\%$ and the optimum resultant effective tractive effort $T_{4\text{opt}} = 0.218$ kN, where the effective tractive effort energy $E_t$ takes the maximum value of 0.209 kNcm/s are shown as follows: Figures 17(a) and (b) show the distributions of the normal stress $p_n$ and $p_o$ and shear resistance $\tau_{os}$ under the flexible inner and outer track belt, respectively. Both the distributions of $p_n$, $p_o$, and $\tau_{os}$, $\tau_{os}$ show some sinusoidal distributions, where amplitudes at the front of the flexible track belt are comparatively larger than those at the rear due to the increasing thrust. The amplitudes of deflection of the flexible inner and outer track belt vary from 0.93 mm to 1.57 mm, and from 0.70 mm to 1.42 mm respectively.

Figure 18 shows the distributions of the lateral shear resistance $\tau_{lati}$ and $\tau_{lato}$ acting on the sides of the inner and the outer track belt. The maximum lateral shear resistances of $(\tau_{lati})_{max} = 0.1047$ kPa and $(\tau_{lato})_{max} = 0.074$ kPa occurred on each turning point $P_1$ and $P_2$ at a distance of $X = D/2 + Y = 20.1$ cm respectively and then decreased rapidly to negative values. Usually, the lateral shear resistance of the inner track became larger than that of the outer track corresponding to each lateral amount of slippage as shown in Eq. (29).

Figure 19 shows the distributions of the track tension $H_i$ and $H_o$ of the inner and outer track belt in this case. In general, $H_i$ was smaller than $H_o$ due to the corresponding thrust and increased almost linearly with increments of the distance $X$ from the initial track tension of 147 N. $H_i$ and $H_o$ took a maximum value of 235 N and 292 N respectively at the rear end of the flexible track belt of $X = D$.

**EXPERIMENTAL VERIFICATION**

**Experimental Apparatus and Test Procedures**

Two identical flexible tracked vehicle models of 617 N
weight having the same vehicle dimension and specification as mentioned before in Table 2 were fabricated. For the steering motion test of the tracked vehicle, the same soil bin of size 270 cm × 270 cm × 30 cm and the same air dried sandy soil sample as mentioned before in the plate loading and unloading test and the plate traction and slip depression test of the track plate were used.

Photograph 1 shows a general view of the experiment, with the front or left hand side tracked vehicle as the test vehicle and the rear or right hand tracked vehicle as the towed vehicle controlling the slip ratios and the effective tractive effort of the test vehicle. Both the tracked vehicles were driven by four electric motors mounted on the top of the vehicles so that the rotation speed of the rear sprocket of each track belt could be controlled continuously using a speed regulator by an electric AC motor of 90 W. The turning motion of the front test vehicle occurred, for example, when the circumferential speed of the outer track was fixed at 3.20 cm/s while that of the inner one was changed in 3 steps of 2.56, 2.13 and 1.60 cm/s. The steering ratio $\epsilon$, which was defined in Eq. (18), becomes 1.25, 1.50 and 2.00 accordingly. In this case, the rear vehicle should be controlled to develop the effective tractive effort for another combination of smaller circumferential speeds of the inner and outer track to keep a constant steering ratio $\epsilon'$ as mentioned in Eq. (19) in which both vehicles could turn on the same turning radius of circle. In all, 7 to 11 series of combinations of circumferential speeds of the inner and outer track were carried out for each steering ratio above to develop various values of effective tractive effort. For each setting of the combination, at least three experiments were executed to average the measured data.

The measured parameters were the effective tractive effort, the amount of depression of the track belt, the turning radius, and the slip ratios of the test vehicle. The resultant effective tractive effort $T_{ak}$ was measured using a load cell with a capacity of 1.0 kN and the longitudinal effective tractive effort $T_{lak}$ could be calculated using the measured angle $\delta$. The amount of depression of the front idler and rear sprocket $s_{01}, s_{10}$ of the inner track and $s_{02}, s_{20}$ of the outer track were measured with a depth gauge as the height difference from the surface to the bottom plane of the track. The turning radius $R$ of the center of gravity of the tracked vehicle was measured from the circular travelling loci of the inner and outer track. The slip ratios of the inner and outer track $i_{a}, i_{o}$ could be calculated from the running speed $V_1 \cos \beta_1, V_2 \cos \beta_2$, which were measured as the travelling locus divided by the travelling time at steady running state, and the measured circumferential speeds of the inner and outer track, respectively.

**Resultant Effective Tractive Effort**

Figure 20 shows the relationship between the slip ratio of outer track $i_{o}$ and inner track $i_{a}$ for three steering ratios $\epsilon$ of 1.25, 1.50 and 2.00. The solid lines mean the calculated values using the above simulation analytical method and the marked points show the experimental measured values. Both the slip ratios $i_{a}$ and $i_{o}$ increase with the increment of resultant effective tractive effort $T_{ak}$ and it was observed that $i_{a}$ was always larger than $i_{o}$. It was confirmed that the simulation results coincided fairly well with the measured values. Figure 21 shows the relationship between the resultant effective tractive effort $T_{ak}$ and the slip ratio $i_{o}$ of the outer track for three steering ratios $\epsilon$ of 1.25, 1.50 and 2.00. The solid lines mean the calculated values using the above simulation analytical method and the marked points show the experimental measured values. In general, the resultant effective tractive effort $T_{ak}$ increased gradually with the increment of slip ratio of outer track $i_{o}$ and it had no peak value. It was observed that the resultant effective tractive effort decreased with the increment of steering ratio for the same slip ratio of outer track due to the occurrence of turning moment as mentioned in Figs. 13(a), (b). It was also confirmed that the simulation results agreed well with the measured values. Figure 22 shows the relationship between the resultant effective tractive effort $T_{ak}$ and
the turning radius \( R \) for three steering ratios \( \varepsilon \) of 1.25, 1.50 and 2.00. The solid lines mean the calculated values using the above simulation analytical method and the marked points show the experimental measured values. In general, the turning radius \( R \) increases parabolically with the increment of the resultant effective tractive effort \( T_{tr} \) for the given steering ratios. The resultant effective tractive effort \( T_{tr} \) decreases with the increment of steering ratio at the large range of turning radius due to the turning resistance. It was confirmed that the simulation results closely agreed with the measured values.

**Amount of Depression**

Figure 23 shows the relationship between the amount of depression \( S \), i.e. the amount of front idler \( s_{f1} \) and rear sprocket \( s_{r1} \), and the slip ratio of the inner track \( \iota_i \) for three steering ratios \( \varepsilon \) of 1.25, 1.50 and 2.00. The solid lines mean the calculated values using the above simulation analytical method and the marked points show the experimental measured values. In general, the amount of depression of the front idler of the inner track \( s_{f1} \) tends to decrease with the increment of the slip ratio \( \iota_i \), while the amount of depression of the rear sprocket \( s_{r1} \) tends to increase with \( \iota_i \) for the given steering ratios. It was confirmed that the simulation results closely agreed with the measured values. Figure 24 shows the relationship between the amount of depression \( S \) i.e. the amount of front idler \( s_{f0} \) and rear sprocket \( s_{r0} \) and the slip ratio of the outer track \( \iota_o \) for three steering ratios \( \varepsilon \) of 1.25, 1.50 and 2.00. The solid lines mean the calculated values using the above simulation analytical method and the marked points show the experimental measured values. In general, the amount of depression of the front idler of the outer track \( s_{f0} \) and the amount of depression of the rear sprocket \( s_{r0} \) tend to increase with the increment of the slip ratio \( \iota_o \), while \( s_{r0} \) is always larger than \( s_{f0} \) for the given steering ratios. It was also confirmed that the simulation results were in good agreement with the measured values.

**CONCLUSIONS**

The turnability properties of a flexible tracked vehicle under traction on a loose accumulated sandy flat surface
have been simulated and verified experimentally for the case of the inner and the outer track while being driven. New conclusions are:

1) The vehicle speed decreases with increments of the turning radius which is accompanied by increments of the resultant effective tractive effort and decrements of the steering angular velocity of the tracked vehicle for each steering ratio. The running speed of the outer track is always larger than that of the inner track. Both the running speeds decrease with increments of the turning radius, the steering ratio, and slip ratios of the inner and the outer track respectively.

2) The slip ratio of the outer track is always larger than that of the inner track, and is a function of the slip ratio of the inner track and the steering ratio as shown in Eq. (42). Both the slip ratios of the inner and the outer track increase with increments of turning radius as shown in Eqs. (43) and (44), and the slip ratio of the outer track increases greatly with increments of the steering ratio.

3) For the inner and the outer tracks, the thrust which can be calculated as the integral of the shear resistance of soil at the interface between track and surface is always larger than the longitudinal effective tractive effort due to the occurrence of the compaction resistance. The lateral effective tractive effort decreases slightly with increments of the slip ratio.

4) For the inner track, with increments of slip ratio, the thrust and the longitudinal effective tractive effort increase parabolically and the compaction resistance increases slightly at the lower end of slip ratios. For the outer track, the thrust increases greatly with increments of the slip ratio and the longitudinal effective tractive effort takes a maximum value at a given slip ratio because of parabolically increasing compaction resistance.

5) The longitudinal effective tractive effort of a tracked vehicle increases with increments of turning radius and increases with the decrement of steering ratio at larger turning radii. The additional lateral force and the lateral effective tractive effort of a tracked vehicle decreases with increments of turning radius, and the additional lateral force is always larger than the lateral effective tractive force for all steering ratios.

6) The resultant effective tractive effort of tracked vehicle under traction does not show a peak value relating to slip ratio because the longitudinal effective tractive effort of outer track was larger than that of the inner track due to the occurrence of turning moment i.e. the resultant effective tractive effort has a maximum value when the longitudinal effective tractive effort of the outer track takes a peak value in the conventional Hump type curve. The resultant effective tractive effort increases exponentially with the turning radius and it decreases with increments of the steering ratio at larger turning radii.

7) The amounts of depression of the front idler of the inner and the outer track decrease with increments of the slip ratio while those of the rear sprocket of the inner and the outer track increase. These relationships are independent of steering ratio, and the amounts of depression of the rear sprocket are always larger than those of front idler because of the increasing eccentricity of resultant normal surface reaction with increments of the slip ratio. The angles of longitudinal inclination of the inner and the outer track increase parabolically with increments of the slip ratio. The angles of lateral inclination of a tracked vehicle increase with increments of the turning radius and steering ratio.

8) The turning resistance moment of the inner track is always larger than that of the outer track corresponding to each lateral shear resistance. Both the turning resistance moments of the inner and the outer track decrease hyperbolically with increments of each slip ratio. And the total turning resistance moment of the tracked vehicle decreases hyperbolically with increments of the turning radius.

9) The input energy, slippage energy and compaction energy increase almost linearly with increments of the slip ratio, while the effective tractive effort energy takes a maximum value at an optimum combination of slip ratios of the inner and the outer track. The turning moment energy decreases hyperbolically with increments of the slip ratio and turning radius.

10) The distributions of normal stress and longitudinal shear resistance of the inner and the outer track show some sinusoidal distributions whose amplitudes at the front of the flexible track belt are comparatively larger than those at the rear due to increasing thrust. The distribution of lateral shear resistance of the inner and the outer track have a given maximum value at each turning point. The lateral shear resistance of the inner track becomes larger than that of the outer track corresponding with each lateral amount of slippage.

11) The distribution of the track tension of the inner and the outer track belt increase almost linearly with increments of distance from the bottom-dead-center of front idler. The track tension of the outer track is larger than that of the inner track corresponding to each thrust.

REFERENCES