PLANE STRAIN COMPRESSION BEHAVIOUR OF GEOGRID-REINFORCED SAND AND ITS NUMERICAL ANALYSIS

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ABSTRACT

Plane strain compression tests were performed on large specimens that were either unreinforced or reinforced with 6 or 11 layers of geogrid, both 57.0 cm in height and 24.4 cm × 21.4 cm in cross-section. It is shown that the effects of covering ratio for each grid layer is much more important than the total tensile stiffness of grid within the limits of the test conditions in this study. Numerical analysis of the test results by a plane strain non-linear elasto-plastic FEM was performed considering strain localisation as well as anisotropic stress-strain behaviour of sand and interface properties. The geogrid was modelled as a planar reinforcement. Not only the pre-peak stress-strain behaviour of the unreinforced and reinforced specimens, but also the peak strength, post-peak behaviour and dilatancy characteristics from the FEM analysis all compared well with those from the physical tests. The effects of reinforcement rigidity and covering ratio were also well simulated. The relationship between the reinforcement covering ratio in the physical tests and the equivalent interface friction angle for the FEM analysis that provides the same reinforcing effects is presented. The mechanism of tensile-reinforcing is analysed based on local stress paths within the reinforced sand obtained from the FEM analysis.

Key words: covering ratio, finite element method, geogrid, grid rigidity, interface friction angle, plane strain compression test, reinforced sand (JGC: D6/E13)

INTRODUCTION

Tensile-reinforcing effects on the performance of geosynthetic-reinforced soil depend on the arrangements and rigidities of reinforcement. It is misleading, therefore, to classify tensile reinforcement members into inextensible ones (like metal strips) and extensible one (like most geotextile reinforcements) based on only the material stiffness (such as the Young's modulus) of reinforcement while ignoring the effects of their three-dimensional geometry and arrangements on the behaviour of a reinforced soil mass. This classification is very common in current engineering practice, giving the impression that ordinary geotextile reinforcements are too extensible to be used for important permanent reinforced soil structures.

Yamauchi (1985) performed a series of plane strain compression (PSC) tests on small specimens of dense Toyoura sand that were either unreinforced or tensile-reinforced with various types of planar reinforcements placed horizontal, i.e. normal to the σ1 direction (Tatsuoka and Yamauchi, 1986). The dimensions of the specimens were 7.5 cm × 4.0 cm × 8.0 cm in the σ1, σ2 and σ3 directions. In order to understand the observed tensile-reinforcing effects, the PSC test results were simulated by non-linear elasto-plastic FEM analysis considering strain localisation (Kotake et al., 1999). Not only the pre-peak behaviour, but also the peak strength and post-peak behaviour, were simulated very well. Local stress and strain fields were also examined in detail for better understanding of the reinforcing mechanism. It was also shown that the friction angle μ at the interface between the reinforcement and the sand, as well as the reinforcement stiffness, could have a great effect on the reinforcing effects.

One of the popular types of tensile-reinforcement, polymer grid, was not used in the above-mentioned investigation. A geogrid has anisotropic strength and deformation characteristics between the longitudinal and transversal directions. Therefore, PSC tests are not only representative of usual field operational conditions, but also suitable for a better understanding of tensile-reinforcing mechanisms for geogrid-reinforced soil. Another series of PSC tests on unreinforced and reinforced dense To-
youra sand was performed ten years later using specimens that were much larger (57 cm × 24.4 cm × 21.4 cm in the \(\sigma_1\), \(\sigma_2\) and \(\sigma_3\) directions; Fig. 1) so that multiple layers of grid, as used in prototype reinforced backfill, could be arranged (Hirakawa et al., 1998). In these tests, effects of the total tensile rigidity per reinforcement layer and the covering ratio of each reinforcement layer were investigated by using: a) a set of different grids having different total tensile rigidities with the same covering ratio; and b) another set of different grids having different covering ratios with the same total tensile rigidity.

The first objective of this study is, therefore, to simulate the results obtained from the second series of PSC tests by plane strain non-linear elasto-plastic FEM considering strain localisation, anisotropic stress-strain properties of sand and interface properties, as employed by Kotake et al. (1997, 1999). Although grid reinforcement is globally planar, it has a three-dimensional (3D) structure in the scale of soil particle size. To rigorously simulate the local behaviour of such a 3D structure, a 3D FEM analysis with a huge number of elements is required. Considering that such an analysis is neither realistic nor feasible at the present time, the grid reinforcement was modelled as a planar reinforcement having the same total tensile rigidity per layer as the original one while having an equivalent friction angle at the interface with sand, which is equal to or smaller than the physical interface angle. By comparing the results from the FEM analysis with those from the physical PSC tests, evaluation was attempted of the effects of reinforcement rigidity and reinforcement covering ratio. It was found that the equivalent interface angle used in the FEM analysis is a function of the covering ratio of reinforcement in the physical tests. By using a respective appropriate equivalent interface angle between the reinforcement and sand, realistic global stress-strain relationships with realistic global dilatancy characteristics of geogrid-reinforced sand specimens were obtained from the FEM analysis. Local stress paths within the reinforced specimens were also examined by the FEM analysis.

**PSC TESTS**

**Specimens**

For preparing the specimens, air-dried Toyoura sand \((D_{90}=0.180 \text{ mm}; \ U_r=D_60/D_{90}=1.40; \ e_{\text{max}}=0.977 \text{ and } e_{\text{min}}=0.605)\) was pluviated through air to have a relative density \(D_r\) of about 90%. The batch of Toyoura sand used in the present study was different from that used by Tatsuoka et al. (1986). A large triaxial cell with an automated loading and measuring system, described by Hoque et al. (1996), Hoque and Tatsuoka (1998) and Tatsuoka et al. (1999), was used. The specimens were either unreinforced or reinforced with 6 or 11 layers of geogrid placed horizontally at an equal vertical spacing. Each geogrid layer was placed after the top surface of each pluviated sand layer was levelled. A 0.8 mm-thick latex rubber membrane was used. The \(\sigma_1\) surfaces of the specimen were lubricated by means of a 0.8 mm-thick latex rubber membrane sheet smeared with a 30 \(\mu\text{m}\)-thick Dow high vacuum silicone grease (Tatsuoka et al., 1984). The \(\sigma_2\) surfaces were also lubricated in a similar way. Isotropic confining pressure \(\sigma_c\) equal to 9.8 kPa was first applied to each air-dried specimen by a partial vacuum to make it self-set for setting up the measurement devices and other testing system. Then, by increasing the vertical pressure \(\sigma_1\), the specimen was loaded to an anisotropic stress state at 30 \(\sigma_3=\sigma_1/\sigma_2=0.4\) with \(\sigma_3=9.8 \text{ kPa}\), followed by anisotropic compression at 30 \(\sigma_3=0.4\) and \(\sigma_2=19.6 \text{ kPa}\). The specimen was then loaded in compression at \(\sigma_2=19.6 \text{ kPa}\) and at a constant axial strain rate \(\dot{\varepsilon}_a\) of 0.125%/min. Axial strains \(\varepsilon_a\) were measured locally by using a pair of vertical local deformation transducers (LDTs) (Goto et al., 1991) with a gauge length of 50 cm as well as an ordinary external displacement transducer set to the loading piston. Lateral strains \(\varepsilon_a\) were obtained from changes in the specimen width, which were directly measured by using three pairs of proximity transducers set at three elevations (0.5 cm, 29.5 cm and 48.5 cm from the specimen bottom). After the start of strain localisation, both unreinforced and reinforced specimens do not deform homogeneously. Therefore, the local axial strains measured with LDTs are still "global axial strains". In addition, already at loading stages before the start of strain localisation, reinforced specimens do not deform homogeneously due to the effects of tensile-reinforcing. As the proximity transducer sensed the lateral expansion around the mid-heights of sand layers sandwiched by two grid layers, the average lateral strains \(\varepsilon_a\) (positive in compression) obtained from their readings overestimated the negative true average values, in particular at larger strain levels.

**Geogrid-Reinforcement**

Seven different types of geogrid reinforcement, having different rigidities and structures, were prepared (see Fig. 2 and Table 1). Reinforcement type a is the original type, which is being used for many prototype geogrid-reinforced soil structures. The material of the geogrid is polyvinyl alcohol, called Vynilon. For the original type grid,
the longitudinal members (strands) are 0.6 cm wide and 0.08 cm thick. The transversal members, 0.06 cm wide and 0.08 cm thick, are knitted and heat-bonded to the longitudinal members. The rigidity per layer is larger by a factor of 190 in the longitudinal direction (for strands) than in the transversal direction. The aperture is 2 cm by 2 cm. Figure 3 shows the result from a tension loading test in air on a 20 cm-wide specimen of the original grid type a, performed at an axial strain rate of 1.0%/min. The measured deformation properties of the geogrid are not perfectly linear. Therefore, the geogrid was modelled both as a non-linear material, as measured, and as a linear planar material having an average stiffness of 953 kN/m, based on the results presented in Fig. 3. As the number of strands per meter width is 50 and the cross-sectional area of each strand is 0.8 mm times 6.0 mm, this stiffness value is equivalent to a material Young's modulus \( E \) of 953 kN/(50·0.8 mm·6.0 mm)=3.97 GPa. The average interface properties between the original type grid and air-dried Toyoura sand were evaluated by a series of pull-out tests using a 35 cm-long and 30 cm-wide grid specimen under normal stress of 29–98 kPa, which was applied by using an air-pressure bag. The data points presented in Fig. 4 denote the maximum pull-out strength in each test. The two theoretical linear relationships will be explained later.

The geogrid reinforcements other than the original one (type a), having different structures and rigidities, were produced by re-assembling longitudinal members that were obtained by disassembling the original grid. They were glued at nodes to transversal members using elastic epoxy resin. The same number per layer of transversal members as the original one was used for all the types of geogrid. The structure and rigidity of these reinforcement members will be represented in terms of the following normalised parameters;

\[
CR = \text{(covering ratio per each geogrid reinforcement layer)} \times 100\%.
\]

The \( CR \) is defined as the planar area covered by reinforcement in each layer, which is equal to 100% for a sheet. For defining the values of \( CR \), the areas covered by the transversal members were ignored by considering

<table>
<thead>
<tr>
<th>Reinforcement type</th>
<th>(a)*</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
<th>(g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of strand × Number of attached layers</td>
<td>9×1</td>
<td>5×1</td>
<td>5×7</td>
<td>9×4</td>
<td>18×1</td>
<td>18×2</td>
<td>36×1</td>
</tr>
<tr>
<td>Shape</td>
<td>grid</td>
<td>grid</td>
<td>grid</td>
<td>grid</td>
<td>grid</td>
<td>grid</td>
<td>planar</td>
</tr>
<tr>
<td>Total strand thickness (mm)</td>
<td>0.8</td>
<td>0.8</td>
<td>5.6</td>
<td>3.2</td>
<td>0.8</td>
<td>1.6</td>
<td>0.8</td>
</tr>
<tr>
<td>Strand width (mm)</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Strand central distance (mm)</td>
<td>20</td>
<td>46</td>
<td>46</td>
<td>20</td>
<td>7</td>
<td>7</td>
<td>—</td>
</tr>
<tr>
<td>Covering ratio, ( CR ) (%)</td>
<td>25</td>
<td>14</td>
<td>14</td>
<td>25</td>
<td>50</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>Total rigidity ratio, ( TRR )</td>
<td>1</td>
<td>0.556</td>
<td>3.89</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

*The original geogrid
that their contribution to the tensile-reinforcing effects is minor;

\[ TRR = \frac{\text{(total rigidity ratio of geogrid reinforcement)}}{100\%}. \]

(2)

The \( TRR \) is defined as the ratio of the total tensile rigidity of a given reinforcement layer to that of the original grid reinforcement (type \( a \)).

The tensile reinforcing effects on the peak strength of reinforced sand will be expressed in terms of "SR (strength ratio)" which is defined as the global compressive strength in terms of the maximum principal stress \( (\sigma_1/\sigma_3)_{\text{max}} \) of a given reinforced specimen to that of the corresponding unreinforced specimen.

Test Results

PSC tests were conducted on; a) one unreinforced specimen; b) eleven specimens reinforced with six layers of geogrid having different structures and rigidities (types \( a\sim g \) shown in Fig. 2); and c) one specimen reinforced with eleven layers of geogrid (type \( a \)) (see Table 2). For four test conditions, two tests were performed under the same conditions to confirm the repeatability of test results.

Figure 5(a) shows the global relationships between the average stress ratio \( R^* \) \( (\sigma_1/\sigma_3) = 19.6 \text{ kPa} \) and the average axial strain \( (\varepsilon_a) \) for the unreinforced specimen and two specimens reinforced with six or eleven layers of geogrid type \( a \) \( (9 \times 1) \). Here, \( (9 \times 1) \) means "the number of longitudinal members" times "the ratio of the thickness of reinforcement to that of the original reinforce-

![Graph showing reinforced and unreinforced specimens with stress ratio and axial strain.]

Table 2. Summary of PSC tests at \( \sigma_3 = 19.6 \text{ kPa} \)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Unreinforced or reinforced</th>
<th>No. of grid layers</th>
<th>Reinforcement type</th>
<th>Covering ratio ( (CR) ) (%)</th>
<th>Total rigidity ratio ( (TRR) )</th>
<th>Initial void ratio ( (\sigma_1/\sigma_3)_{\text{max}} )</th>
<th>Strength ratio ( (SR) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>Unreinforced</td>
<td>None</td>
<td>None</td>
<td>None</td>
<td>0.660</td>
<td>6.17</td>
<td>1.00</td>
</tr>
<tr>
<td>R6-a</td>
<td>Reinforced</td>
<td>Six</td>
<td>(a) ( (9 \times 1)* )* 25</td>
<td>1.000</td>
<td>0.638</td>
<td>31.56</td>
<td>5.12</td>
</tr>
<tr>
<td>R6-b</td>
<td></td>
<td></td>
<td>(b) ( (5 \times 1)** )* 14</td>
<td>0.556</td>
<td>0.632</td>
<td>15.94</td>
<td>2.58</td>
</tr>
<tr>
<td>R6-c</td>
<td></td>
<td></td>
<td>(c) ( (5 \times 7)** )* 14</td>
<td>3.890</td>
<td>0.648</td>
<td>20.70</td>
<td>25.10</td>
</tr>
<tr>
<td>R6-d</td>
<td></td>
<td></td>
<td>(d) ( (9 \times 4)** )* 25</td>
<td>4.000</td>
<td>0.656</td>
<td>50.80</td>
<td>47.65</td>
</tr>
<tr>
<td>R6-e</td>
<td></td>
<td></td>
<td>(e) ( (18 \times 1)** )* 50</td>
<td>2.000</td>
<td>0.642</td>
<td>50.76</td>
<td>8.23</td>
</tr>
<tr>
<td>R6-f</td>
<td></td>
<td></td>
<td>(f) ( (18 \times 2)** )* 50</td>
<td>4.000</td>
<td>0.663</td>
<td>57.44</td>
<td>9.31</td>
</tr>
<tr>
<td>R6-g</td>
<td></td>
<td></td>
<td>(g) ( (36 \times 1)** )* 100</td>
<td>4.000</td>
<td>0.646</td>
<td>63.30</td>
<td>10.26</td>
</tr>
<tr>
<td>R11-a</td>
<td></td>
<td></td>
<td>(a) ( (9 \times 1)* )* 25</td>
<td>1.000</td>
<td>0.640</td>
<td>84.88</td>
<td>13.76</td>
</tr>
</tbody>
</table>

\*The original reinforcement type.

\**The number of longitudinal members" times "the ratio of the thickness of reinforcement to that of the original reinforcement, type \( a \)" (see Fig. 2 and Table 1).

\*Measured before loading.

\**Average value, plotted in Fig. 15.

Fig. 5. Results from physical PSC tests on unreinforced specimen and those reinforced with 6 or 11 layers of geogrid: (a) relationships between average stress ratio and average axial strain, (b) relationships between average volumetric strain and average axial strain.
ment type a or the number of attached original strand layers (see Fig. 2 and Table 1)". The average axial stress $\sigma_1$ was obtained by dividing the total axial load as measured with the local cell attached between the specimen cap and the loading piston by the initial cross-sectional area of the specimen. The axial stress $\sigma_1$ from the FEM analysis was obtained in the same way. Therefore, the values of $\sigma_1$ presented in this paper are nominal values. This stress calculation method was chosen because the cross-sectional area becomes non-uniform in the axial direction in reinforced specimens, and it is therefore difficult to select the representative value. In addition, in the physical tests, it is very difficult to evaluate average cross-sectional areas of air-dried reinforced specimens, as discussed earlier. Despite the above, the use of nominal $\sigma_1$ values has negligible effects on the comparison of peak stress and stress-strain behaviour between the physical tests and the FEM analysis for the same test condition and between different test conditions in both physical tests and FEM analysis.

Figure 5(a) shows that the strength of the reinforced specimens is considerably larger than that of the unreinforced specimen and that a larger strength has been obtained for eleven layers of reinforcement. Figure 5(b) shows the global relationships between the average volumetric strain $\varepsilon_v$ and the average axial strain $\varepsilon_a$. With the specimens reinforced with eleven layers of grid, lateral strains at $\varepsilon_a$ larger than about 2.0% could be failed to measure. It can be seen that for the entire range of PSC loading, the reinforced specimens are more contrant that the unreinforced one, and to a larger extent with a larger number of reinforcement layers. This is likely due to the following two mechanisms, which are relevant to the behaviour before the start of strain localisation: first, a sand mass that is more effectively tensile-reinforced becomes less dilative due to the restraint by tensile reinforcement to the development of lateral tensile strains in sand. This behaviour is typically observed in direct shear tests on reinforced sand under constant normal pressure (Qui et al., 2000). Second, a better reinforced sand mass becomes more contractant in PSC at a constant confining pressure due to more compression of sand by larger local axial stresses themselves due to larger local confining pressures resulting from larger tensile-reinforcing effects.

Figures 6(a) and (b) show the results from PSC tests on the unreinforced specimen and those reinforced with six layers of geon grid having different structures and different $TRRs$ and $CRs$, obtained by Hirakawa et al. (1998). The result from one test was selected when two tests were performed under the same test condition (see Table 2). Considerable effects of different grid types can be seen from these figures. It can also be seen that the specimen becomes more contractant as the strength becomes larger, which is also due to the two mechanisms discussed above.

Photograph 1 shows the $\sigma_2$ surfaces of the unreinforced and reinforced specimens, standing under a partial vacuum of 19.6 kPa, with the confining platens having been removed after each PSC test. The grid seen on the $\sigma_2$ surfaces is deformable, as it is made of latex rubber. The spacing is nominally 1 cm. It is seen that a single shear band has developed in the unreinforced specimen (U). The thickness of shear band seen in the picture may be larger than the true value, because of the effect of using a rather thick membrane (0.8 mm thick). In the reinforced specimens, multiple-shear bands have developed, which are more diffused in more effectively reinforced specimens. It may also be seen from these pictures that with the reinforced specimens, the ends of multiple shear bands tend to appear at the levels of reinforcement on the $\sigma_2$ surfaces. This is because shear bands are very difficult to pass through each grid layer. This feature is well simulated by the FEM analysis, as shown later in this paper.

**FEM MODEL FOR PSC TESTS**

**Modeling of PSC Specimen**

The reinforced specimens were discretised into approximately 1 cm x 1 cm quadrilateral plane strain elements, divided at an equal interval in the vertical and horizontal directions (Figs. 7(a) and (b)). The specimen reinforced with six layers of grid was divided into 60 and 20 in the vertical and horizontal directions with an element size of 0.95 cm x 1.07 cm. The numbers of elements and nodal points were 1,200 and 1,281. The specimen reinforced with eleven layers of grid was divided into 66 and 20 in
the vertical and horizontal directions with an element size of 0.87 cm × 1.07 cm. The numbers of elements and nodal points were 1,320 and 1,407. In Fig. 7, the locations of truss elements for reinforcement layers are denoted by thick lateral lines. For the unreinforced specimen, the mesh for the specimen reinforced with six grid layers was used.

The full models shown in Figs. 7(a) and (b) were used when analysing the detailed behaviour of the three specimens presented in Fig. 5, such as local strain fields and local stress path within the specimen. However, the computation time for one simulation using the full models was too long; for example, a simulation of the PSC test on sand reinforced with eleven layers of grid type a using an engineering work station (HP Visualise-EG Model C180) took about 120 hours (5 days). Therefore, solely for the purpose of obtaining the global stress and strain relationships of specimens reinforced with six layers of grid (except for the one depicted in Fig. 5), the one-span model shown in Fig. 7(c), having a single layer of sand with a
Fig. 7. FEM models for multi-layer geogrid-reinforced sand specimens: (a) full model with 6 layers, (b) full model with 11 layers, (c) one span model with 6 layers, (d) comparison of global stress-strain relationships from FEM analyses using full and one-span models and that from the corresponding physical PSC test

reinforcement layer at the mid-height, was used. Figure 7(d) compares results from a pair of FEM analyses using the full and one-span models for a specimen reinforced with six layers of grid under the same test conditions, and they are compared with that from the corresponding physical test (the angle $\mu=20^\circ$ used in the analysis will be explained later in this paper). It may be seen that the global stress and strain relationships from the two FEM analyses are nearly the same, except for those at large strains in the post-peak regime (Kotake, 1998). The difference in the post-peak behaviour would due to the different degrees of kinematic restraint to strain localisation at the top and bottom boundaries between the two types of models.

Constitutive Model for Sand

As the details of the sand model have been reported in several previous papers by the authors (Siddiquee et al., 1995, 1999; Kotake et al., 1999), only the essence will be described below.

A generalised elasto-plastic model with a non-associated flow rule and isotropic strain-hardening-softening yield properties was used. The yield function $\Phi$ (Eq. (3)) and the plastic potential function $\Psi$ (Eq. (6)) were of, respectively, Mohr-Coulomb and Drucker-Prager types;

$$\Phi = -\eta I_1 + \frac{1}{g(\theta)} \sqrt{J_2 - k_1} = 0$$  \hspace{1cm} (3)$$

where $I_1$ is the first invariant of stress (i.e., hydrostatic stress component, positive in compression), $J_2$ is the second stress invariant (i.e., deviatoric stress) and $g(\theta)$ is the Lode angle function, which is defined as:

$$g(\theta) = \frac{3 - \sin \phi_{mob}}{2\sqrt{3} \cos \theta - 2 \sin \theta \sin \phi_{mob}}$$  \hspace{1cm} (4)$$

$\eta$ is the deviatoric stress at $\theta=30^\circ$ (on the $\pi$-plane), which is related to the mobilised angle of internal friction $\phi_{mob}$ as;

$$\eta = \frac{2 \sin \phi_{mob}}{\sqrt{3 (3 - \phi_{mob})}}$$  \hspace{1cm} (5)$$

$k_1$ in Eq. (3) and $k_2$ in Eq. (6) is the cohesion intersect, which is zero in the present analysis. The plastic potential function has a similar form to the yield function with $g(\theta)=1.0$ in Eq. (3), as;

$$\Psi = -\alpha' I_1 + \sqrt{J_2 - k_2} = 0.$$  \hspace{1cm} (6)$$

Equation (6) has the advantage of having differentiability in all the stress states. The factor $\alpha'$ is defined depending on the type of analysis. For plane strain conditions, $\alpha'$ is defined as;

$$\alpha' = \frac{\tan \psi}{\sqrt{9 + 12 \tan^2 \psi}}$$  \hspace{1cm} (7)$$

where $\psi$ is the mobilised angle of dilatancy, which is given as;

$$\psi = \arcsin \left[ \frac{d \varepsilon^p \varepsilon^p + d \varepsilon^g \varepsilon^g}{d \varepsilon^p \varepsilon^p - d \varepsilon^g \varepsilon^g} \right]$$  \hspace{1cm} (8)$$

where $d \varepsilon^p$ and $d \varepsilon^g$ are the major and minor plastic principle strain increments (positive in compression). In the present study, the value of $\psi$ was determined from the following equation obtained by modifying Rowe's original equation (Rowe, 1962) to simulate as closely as possible the results from the physical PSC tests on unreinforced Toyoura sand;

$$R - 1 = (R_{es} - 1) \cdot D$$  \hspace{1cm} (9a)$$

where $R$ is the principal stress ratio $\sigma_1/\sigma_3$, $R_{es}$ is the value of $R$ at the residual state, and $D$ is the principal plastic strain ratio, equal to $-d \varepsilon^p/d \varepsilon^p$. The value of $R_{es}$ is linked to the residual angle of friction $\phi_{res}$, which has been found to be a function of the confining pressure $\sigma_3$ for Toyoura sand (Siddiquee et al., 1999), as;

$$\phi_{res} \text{ (in degree)} = 35.67 - 3.00 \cdot \log_2 (\sigma_3/p_c)$$  \hspace{1cm} (9b)$$

where $p_c$ is equal to 98 kPa. Similar results have been obtained for other types of sands (Siddiquee et al., 1999). According to Eq. (9b), the values of $R_{es}$ and $\phi_{res}$ decrease with the increase in the confining pressure.

The pre-peak stress-strain relationships of the test material in PSC (i.e., Toyoura sand) reported by Tatsuoka et al. (1986) were modelled into a generalised
hyperbolic equation (Tatsuoka et al., 1993), which was used as a growth function of the yield surface defined in Eq. (3). The generalised hyperbolic equation is summarised also in Appendix A of Kotake et al. (1999). The growth functions in some typical cases are depicted in Figs. 2(d) of Siddiquee et al. (1999) and Fig. A1 of Kotake et al. (1999). Yassin et al. (1999) have shown that even with nearly the same physical properties of sand particles, the deformation and strength characteristics of Toyoura sand (and another clean sand) could be largely different among different batches. This was the case with the relationship between the batch used by Tatsuoka et al. (1986) and that used in the present PSC tests. The stress-strain properties of the former batch were modelled by Tatsuoka et al. (1993) for numerical analysis and used in the FEM analysis by Siddiquee et al. (1999) and Kotake et al. (1999). It was found that the peak friction angle \( \phi \) of the batch used in the present study was smaller by about 4% than that of the batch used by Tatsuoka et al. (1986), while general features of stress-strain properties were very similar. Based on the above, in comparison to the Toyoura sand model developed by Tatsuoka et al. (1993) and described also in Siddiquee et al. (1999) and Kotake et al. (1999), only the \( \phi \) values were reduced by 4% without any changes in the other parameters to be used in the present FEM analysis. So, in the present study, the values of \( \phi \) when the direction of \( \sigma_1 \) is normal to the bedding plane were given as follows, considering the pressure-dependency of \( \phi \):

\[
\phi \text{ (in degree)} = 0.96 \cdot \{59.47(1.5 - \varepsilon) \\
= \sigma_1 \leq (\sigma_3) \}
\]

when \( (\sigma_3) = 4(1 - \varepsilon) \cdot p_s \) (kPa), which is the value of \( \sigma_3 \) below which \( \phi \) is independent of \( \sigma_3 \). Note that as the angle \( \delta \) of direction of \( \sigma_1 \) relative to the direction of the bedding planes deviates from 90°, the \( \phi \) value decreases due to the effects of inherent anisotropy, as described in detail in Tatsuoka et al. (1993). That is, as summarised in Appendix B of Kotake et al. (1999), the \( \phi \) value at an angle \( \delta \) is given as \( \phi(\delta) = \phi(90°) \cdot g(\delta) \). Here, \( g(\delta) \) is the anisotropy function, which decreases gradually from 1.0 as \( \delta \) decreases from 90° is equal to about 0.9 at \( \delta = 40° \), has a minimum of 0.83 at \( \delta = 26° \) (which is close to 45° - \( \phi/2 \)), and increases towards 0.89 at \( \delta = 0° \) as \( \delta \) decreases. The inherent anisotropy in the pre-peak stress-strain properties was also modelled as a function of the angle \( \delta \), as summarised in Appendix A of Kotake et al. (1999). The elastic stress-strain properties were also modelled, as summarised in Appendix C of Kotake et al. (1999).

It was assumed that the deformation of a given sand element under uniform boundary stress conditions is homogeneous in the pre-peak regime, and that strain localisation into a shear band starts suddenly at the peak stress state. These assumptions are a reasonable approximation of the experimental observations in an extensive series of PSC tests (Tatsuoka et al., 1990; Yoshida et al., 1994; Yoshida and Tatsuoka, 1997). In the same way, as employed by Tatsuoka et al. (1991), Siddiquee et al. (1999) and Kotake et al. (1999), shear banding was introduced by using a strain localisation parameter \( S \) in the additive decomposition of total strain increment as follows (Tanaka and Sakai, 1993):

\[
d\varepsilon_0 = d\varepsilon_0^V + Sd\varepsilon_0^B \tag{11}
\]

where \( S = F_b/F_e \), in which \( F_b \) is the area of shear band in each finite element and \( F_e \) is the area of the finite element. Then, the rate of post-peak strain softening associated with shear banding depends on the value of strain localisation parameter \( S \), which is a function of the shear band width, among others. By ignoring the effects of the orientation of shear band in each finite element, an approximated form of \( S \) used in the present study was expressed as:

\[
S = \frac{w}{F_e} \tag{12}
\]

where \( w \) is the width of shear band (= 3 mm for Toyoura sand in the present study). To model the post-peak stress-strain properties inside shear bands, a strain softening parameter \( \varepsilon_s \) was introduced as:

\[
R = R_{vo} + (R_{peak} - R_{vo}) \exp \left[ \frac{-(\gamma - \gamma_0)^2}{\varepsilon_s} \right] \tag{13}
\]

where \( R = \sigma_l / \sigma_1 \) and \( \gamma \) is the shear strain (\( = \varepsilon_1 - \varepsilon_3 \)) inside shear bands. The value of \( \varepsilon_s \) for Toyoura sand was determined by exponential fitting of the experimental data of stress-strain behaviour inside shear bands obtained by Yoshida et al. (1994), which was equal to 0.513 for a shear band thickness equal to 3 mm. The values of \( R_{peak} \) and \( R_{vo} \) were obtained from Eqs. (9b) and (10).

This method takes into account strain localisation associated with shear banding by introducing a characteristic width of shear band and by defining specific and objective shear deformation and dilatancy characteristics inside shear bands. So, this method is similar to the one proposed by Pietruszczak and Mroz (1981). Unlike their method, however, no direction of shear banding was specified in the present study. Rather, it was implicitly assumed that the direction of shear band coincides in a broad sense with the direction of maximum shear strain.

**Modeling of Geogrid-Reinforcement**

A geogrid has a locally three-dimensional geometry, which cannot be modelled directly by a two-dimensional plane strain FEM model. Therefore, an equivalent two-dimensional model approximating the locally three-dimensional structure is required. On the other hand, in all the current practical analysis of the deformation and stability of full-scale geogrid-reinforced soil structures, the locally three-dimensional structure of a reinforcement is not directly simulated, but the geogrid is modelled as a planar reinforcing member having the
same global material characteristics, such as strength, stiffness and pull-out resistance, as the original grid. This simplification is relevant in most cases, because the aperture of geogrid is usually of the order of several centimetres, which is much smaller than the size of reinforced soil structures. In addition, 3-D numerical analysis of prototype reinforced-soil structures directly simulating a locally three-dimensional structure of grid is extremely difficult to perform at present.

In the present FEM analysis, therefore, the geogrid was modelled as planar truss elements, considering that they can simulate tensile reinforcing members having the negligible or very small bending stiffness of the grid used. The cross-sectional area per layer of the truss element was equal to the total cross-sectional area of the longitudinal members for each geogrid type. The geogrid was modelled either as a non-linear material as measured (Fig. 3) or as a linear elastic one with the average Young's modulus. In the latter case, for example, for grid type a (9 × 1), the equivalent thickness of the truss element was taken to be 0.021625 cm with an average Young's modulus of $E=3.97 \text{ GPa}$.

In the preliminary analysis (Kotake et al. 1997; Kotake, 1998), the difference between the results from numerical analysis using the non-linear and linear models of grid was evaluated. The peak strength in terms of stress ratio $R^* (=\sigma_b/\sigma_n)$ of a specimen reinforced with six layers of grid type a was larger by about 7% when the linear model was used than when the non-linear model was used. The difference was considered to be small, considering that errors associated with other uncertain factors could be larger than this difference. Therefore, a simpler model (the linear one) was used in the rest of the analysis.

Modeling of Interface between Reinforcement and Sand

When a sand mass is in contact with another material under plane strain conditions, the angle of interface friction $\mu = \arctan (\tau_s/\sigma_n)_{\text{max}}$ is upper-bounded by: a) the interface friction angle $\mu_e$ when the interface is a displacement discontinuity; and b) the interface friction angle $\mu^\ast$ when the mass of cohesionless soil in the immediate vicinity of the interface is at the limiting stress condition without producing a displacement discontinuity (Tatsuoka, 1985). The latter case b) is relevant to the present PSC tests. That is, in the physical PSC tests performed in the present study, any obvious slipping with a displacement discontinuity along the interface between the grid reinforcement and the sand (i.e., case a)) was not observed (see Photo. 1). Rather, it is very likely that in the present PSC tests, at the failure state of specimen, thin sand zones, like shear bands, developed adjacent to the interface between the reinforcement layer and the main body of sand, exhibiting plastic flow in the simple shear (or direct shear) mode without a displacement discontinuity at the interface.

The use of conventional interface elements having zero thickness is relevant to case a). However, the physical meanings of the parameters used in such conventional interface elements become unclear when applied to case b), in particular in modeling the dilatancy characteristics of the thin yielding sand zones adjacent to the interface. It was considered that relevant modeling of this factor (i.e., the effects of particle size) is essential in properly modeling the effects of the thickness of these thin layers. In the present study, therefore, conventional interface elements having zero thickness were not used, but the interface modeling method that is described in Kotake (1998) and Kotake et al. (1997, 1999) was used. In this method, ordinary sand elements are used adjacent to the interface, while the strength of the sand elements is modified as described below so as to simulate interface behaviour that is relevant to the present case.

For PSC tests on sand reinforced with actually planar rigid reinforcement having a rough surface without a displacement discontinuity (i.e., case b)), the theoretical value of the interface friction angle $\mu$ is the same as the angle of friction in simple shear and direct shear tests; $\phi_{\text{in}} = \arctan (\tau_s/\sigma_n)_{\text{max}}$, where $\tau_s$ is the shear stress and $\sigma_n$ is the normal stress acting on the shear plane, given by:

$$
\phi_{\text{in}} = \arctan \left( \frac{\sin \phi_{\text{peak}} \cdot \cos \psi}{1 - \sin \phi_{\text{peak}} \cdot \sin \psi} \right) (14)
$$

where $\phi_{\text{peak}}$ is the plane strain internal friction of sand when the angle $\delta$ of the $\sigma_n$ direction relative to the bedding plane is equal to the angle $45^\circ - \psi / 2$ in the simple shear and direct shear tests and $\psi$ is the angle of dilatancy at failure (Eq. (8)).

In the case of the present PSC tests on sand reinforced with grid layers, the restraint to the deformation of sand adjacent to the interface by the grid reinforcement is equal to or smaller than that by perfectly rigid planar reinforcement having a perfectly rough surface with the friction characteristics represented by Eq. (14) under otherwise the same conditions. This phenomenon is similar to the decrease in the interface friction angle $\mu$ from $\phi_{\text{in}}$ due to elongation along the interface resulting from tensile deformation in the interface direction of the material with which the sand mass is in contact (Tatsuoka, 1985). In the scheme of the present FEM analysis, therefore, interface friction angles $\mu$ that were equal to or smaller than $\phi_{\text{in}}$ were used to simulate the interface properties between a given grid reinforcement and the adjacent sand. To this end, the strength of the sand elements adjacent to the grid was made artificially weaker than that of the remaining part of sand as follows.

The theoretical relationship between the angles $\phi_{\text{peak}}$ and $\phi_{\text{in}}$ (Eq. (14)) is not linear, but it can be approximated by the following linear relationship:

$$
\phi_{\text{peak}} = 1.2 \cdot \phi_{\text{in}} \quad \text{or} \quad \phi_{\text{in}} = \phi_{\text{peak}} / 1.2. (15)
$$

Based on this relationship, for a given interface friction angle $\mu$, reduced angles of friction under plane strain conditions, $\phi_{\text{peak}} = 1.2 \cdot \mu$, were assigned to one layer of sand elements adjacent to the interface. By this method, the failure of the sand elements adjacent to the interface is controlled by the simple shear failure along the interface, rather than axial (vertical) compression failure, as
discussed later related to Figs. 18 and 19.

In fact, Eq. (15) is in accordance with the results from the pullout tests of the grid presented in Fig. 4 as follows. The theoretical value of \( \sigma_3 \) for the normal stress \( \sigma_n \) on the direct shear failure plane is obtained as:

\[
\sigma_3 = \frac{1 - \sin \psi}{1 - \sin \phi \sin \psi} \sigma_n. \tag{16}
\]

For the average normal stress in the pull-out tests, \( \sigma_n = 50 \text{ kPa} \), the angle of internal friction \( \phi \) in this case is about 43° (as shown below), which gives \( R = \sigma_y / \sigma_3 = 5.3 \). On the other hand, according to Eq. (9b), the residual angle of dilatancy \( \psi_{Ad} \) is about 10°, giving \( R_{Ad} = 4.0 \), for \( \sigma_3 = 40 \text{ kPa} \) (as shown below). Then, substituting \( R = 5.3 \) and \( R_{Ad} = 4.0 \) into Eq. (9a), the dilatancy ratio at failure \( D = -d \varepsilon_d^v / d \varepsilon_3^v = 1.43 \) is obtained, which gives an angle of dilatancy at failure of \( \psi = 10° \). By substituting \( \sigma_n = 50 \text{ kPa}, \phi = 43° \text{ and } \psi = 10° \) into Eq. (16), we obtain \( \sigma_3 = 40 \text{ kPa} \). By substituting \( \sigma_3 = 40 \text{ kPa} \) and the average void ratio in the present study, \( e = 0.65 \), into Eq. (10b), an angle of internal friction \( \phi \) when \( \delta = 90° \) that is equal to 47.5° is obtained. Then, the angle of internal friction under the concerned direct shear condition, which is equal to \( \phi \) at \( \delta = 45° - \psi / 2 = 40° \), is obtained as:

\[
\phi(\delta = 40°) = \phi(\delta = 90°) \cdot g(\delta = 40°) = 47.5° \cdot 0.9 = 43°. \tag{17}
\]

The value \( g(\delta = 40°) \approx 0.9 \) is obtained from Fig. B1 of Kotake et al. (1999). By substituting \( \phi = 43° \) and \( \psi = 10° \) into Eqs. (14) and (15), we obtain values of \( \phi_{Ad} = \mu \) of, respectively, 37° and 36°. The relationships \( \tau_c = \tan \phi_{Ad} \cdot \sigma_3 = \mu \cdot \sigma_3 \), for \( \phi_{Ad} = \mu = 36° \) and 37° are presented in Fig. 4. It may be seen that these relationships are well in accordance with the pull-out test results. Note that the values of \( \phi_{Ad} \) at \( \sigma_3 \) values higher than 50 kPa become smaller than 36°-37°.

One of the objectives of the present study is to examine whether the behaviour in PSC of a sand specimen reinforced with a given grid could be properly simulated by FEM analysis using a relevant value of \( \mu \), and if this is the case, to find a relationship between the value of \( \mu \) and the structure of grid (i.e., covering ratio and others). The largest drawback of the present method is that when the volume of the interface elements is no longer negligible compared with that of the rest of soil and when the value of \( \mu \) approaches zero, the effects of the soft response of the elements against axial loading become noticeable. However, for ordinary field conditions (and the test conditions in the present study), the above is not the case. Interface elements with a finite thickness exhibiting volume changes when sheared (e.g., Pande and Sharma, 1979) could have been used in the present study. However, their use was not attempted, as it is not certain whether such element can exhibits such high stiffness and strength against axial loading and low stiffness and strength with a relevant amount of dilatancy against shear loading as in the present case. Further study will be necessary in this respect.

**Non-linear Solution Techniques**

A set of non-linear equations was solved by the dynamic relaxation (DR) technique (Tanaka et al., 1988), which has a reputation in solving highly non-linear equations, especially for high friction angle materials as in the present case. The integration of the elasto-plastic equation was done by the return mapping scheme (Ortiz and Simo, 1986), which is a first order approximated Euler backward integration.

A FEM code with an optimised dynamic relaxation solver developed by Tanaka and Kawamoto (1988) was used. Four-node quadrilateral elements along with reduced integration (Zienkiewicz et al., 1971) were used to improve the bounds of solution in the pseudo-equilibrium for sand as a highly non-linear material. To prevent any probable hourglass mode, the anti-hourglass scheme proposed by Flanagan and Belytschko (1981) was adopted. Following an elastic stiffness approach of the scheme, a very small elastic stiffness (0.05%) of the actual material elastic stiffness at the start of loading was added to the non-linear system as an hour-glass resisting nodal force whenever any soil element started to form an hour-glass mode. The details are given in Siddiquie et al. (1995).

In the present analysis, the initial isotropic effective confining pressure of \( \sigma_c (=19.6 \text{ kPa}) \) was first given to all the plane strain elements. Loading by strain control in the physical PSC tests was simulated by applying prescribed uniform vertical displacements to the nodal points along both top and bottom ends of the specimen. Free lateral movements were allowed to occur at these nodal points, as the friction at the specimen ends was reduced to a very small value by means of a lubrication layer. Convergence was confirmed by checking the global residual force norm as:

\[
\frac{(\|F-P+P_{init}\|)^2}{(\|F\|)^2} \leq f_1 \tag{18}
\]

and the differential residual force norm between two successive iterations;

\[
\frac{(\|F-P+P_{init}\|)^2 - (\|F-P+P_{init}^n\|)^2}{(\|F-P+P_{init}\|)^2} \leq f_2 \tag{19}
\]

where \( F \) is the external applied force, \( P \) is the internally developed forces, \( P_{init} \) is the initially existing forces, \( n \) and \( n+1 \) mean successive iterations, \( f_1 \) and \( f_2 \) are tolerance of convergence, both of which were \( 10^{-6} \) in the present analysis. In addition, the increment of the global internal energy during each iteration (i.e., the amount of work done by the residual force on the displacement increment) was specified to be within a present tolerance \( \epsilon_E \) of the initial energy increment as:

\[
\frac{\Delta t \cdot \|\dot{u}(F-P+P_{init})\|^2}{(\Delta t \cdot \|\dot{u}\|^2 \cdot |F|^2)} \leq \epsilon_E \tag{20}
\]

where \( \dot{u} \) is \( \dot{u} \) at the time \( t=0 \). Such differential tolerance as stated above is necessary to make the solutions independent of the number and size of finite elements. This specific value was chosen as a compromise between computation time and desired accuracy. It has been checked that by one order reduction in that tolerance from the
value shown above, the computation time doubles although results did not change significantly. A displacement increment of 0.00285 × 2 cm/step, i.e., an average axial strain increment $d e_a$ of 0.01% /step, was employed, which has been found to be small enough to keep accuracy and numerical stability with an equilibrium iteration tolerance of a force norm (Eqs. (18)) and (19)) as well as an energy norm $e_k$ of $10^{-6}$.

Figure 8 compares the global stress-strain relationships for the unreinforced specimen obtained from the physical PSC test referred to in Fig. 5 and the corresponding FEM analysis. Note that the peak strength is nearly the same between the physical test and the FEM analysis, which is due to the adjustment of the $\phi$ value in the sand model, as mentioned earlier. Yet, it is very important to note that the overall pattern of stress-strain behaviour is very similar between the physical test and the analysis. Figures 9(a) and (b) show the distributions of local shear strain $\gamma = e_t - e_r$ and displacements at the nodes obtained at global axial strains of 1.0% (at the peak stress state) and 3.0% (at the residual state). It can be seen that although the details are noticeably different, strain localisation patterns in the physical test (Photo. 1(U)) and the numerical analysis (Fig. 9) is similar in a global sense. A good agreement of the global stress-strain behaviour (Fig. 8) and a reasonable simulation of strain localisation shown above are the basis for further analysis of reinforced specimens. More detailed discussions on shear banding are out beyond scope of the present paper.

EFFECTS OF GRID RIGIDITY, COVERING RATIO AND EQUIVALENT INTERFACE FRICTION ANGLE

Effects of Grid Rigidity and Covering Ratio

A parametric study by FEM analysis on the effects of the tensile rigidity of geogrid was performed for different tensile rigidities ($TTR$s), together with different equivalent interface friction angles ($\mu$). FEM analysis using different tensile rigidities of $TTR = 1/100, 1/100, 1/10, 1/5, 1/2, 1, 4, 10, 50$ and $100$ with $\mu = 25^\circ$ was first per-
formed. Figure 10 shows the global relationships between the average principal stress ratio \(R^* = \sigma_1 / \sigma_2\) and the average axial strain \(\varepsilon_a\) obtained from the FEM analysis.

Note that for the basic case of grid type a \((9 \times 1)\) with TRR = 1.0, the total tensile rigidity per layer EA is equal to the Young’s modulus of grid \(E = 3.97 \text{ GPa}\) times the total cross-sectional area per layer of the original grid \(A_t = 9.8 \text{ mm} \times 6.0 \text{ mm} = 59.5 \text{ kN}\). Figures 11(a) and (b) summarise the relationships between the strength ratio SR and the total tensile rigidity ratio TTR for \(\mu = 25^\circ\). Results for \(\mu = \phi_{\text{peak}},\ 38^\circ,\ 17^\circ\) and \(12^\circ\) obtained from similar parametric studies are also presented. The values of TTR are plotted in the arithmetic and logarithmic scales, respectively, in Figs. 11(a) and (b). Note that the analysis using \(\mu = \phi_{\text{peak}}\) gives strength values that are larger than the possible maximum values in the physical PSC tests, as this value violates the condition given by Eq. (14) or (15). Detailed discussions on this issue are given in Kotake et al. (1999).

It can be seen from Figs. 10 and 11 that the effects of increasing the tensile rigidity on increasing the strength of reinforced sand are significant when the rigidity is relatively low. However, the effects gradually decrease with the increase in the tensile rigidity, and finally disappear as the rigidity becomes higher than a certain limit. In fact, the rate of gain in the strength per unit increase in TTR, \(d(SR)/d(TTR)\), becomes very small as TTR becomes larger than unity (see Fig. 12). The curves shown in this figure were obtained from a non-linear curve fitted to the results obtained by the FEM analysis.

Figure 13 shows the measured relationships between the average stress ratio \(R^* = \sigma_1 / \sigma_2\) and the average axial strain \(\varepsilon_a\) for the specimens reinforced with six layers of geogrids having the same TTR = 4 \((EA = EA_t \times 4/\text{layer})\), but having different covering ratios \((CRs) = 14\%, 25\%, 50\%\) and 100\%. The results obtained from the FEM analysis using the same value of TTR as the physical PSC tests and having different equivalent interface friction angles \(\mu = 12^\circ, 17^\circ, 25^\circ, 33^\circ\) and \(\phi_{\text{peak}}\), are also shown. By comparing the results from the physical PSC test and the FEM analysis, it is clear that for the same value of TTR, the overall effects of CR observed in the physical PSC tests are very similar to those of the angle \(\mu\) in the FEM analysis.

The physical experimental data shown in Fig. 13 and those from the other tests are summarised and compared with those from the FEM analysis in Figs. 11(a) and (b). It may also be seen that with the physical tests, the increase in the strength with the increase in the total 8
rigidity is not significant when the covering ratio is kept constant. The relationship denoted by the letter A in Fig. 11 is for grids with the same thickness but different numbers of longitudinal members. The increase in SR from the left end to the right end of relationship A is therefore due to the increase in the number of longitudinal members. It is clearly seen that most of the increase in SR along the relationship A is due to the increase in CR, while the contribution of the increase in TTR is minor. Figure 14 shows the relationship between the ratio of the strength ratio to the total rigidity ratio (SR/TTR) and TTR obtained from the physical PSC tests and the FEM analysis in a full-log plot. The ratio SR/TTR means the contribution of reinforcement volume (or total reinforcement rigidity) to the strength of reinforced sand. It may be seen that the contribution of reinforcement volume (or total reinforcement rigidity), SR/TTR, decreases consistently with the increase in the total reinforcement rigidity TTR (or reinforcement volume). These results, presented in Figs. 11 and 14, indicate that for a fixed covering ratio, the increase in the reinforcement volume may result in insignificant or even no increase in reinforcing effects when the reinforcement volume exceeds a certain limit.

The test conditions employed in the present study cover a range of typical practical cases. Within the limit examined in the present study, the effects of increasing the covering ratio (CR) on the increase in the strength of reinforced sand are much more significant than those of increasing the total reinforcement rigidity. It can be concluded therefore that it is not relevant to over-emphasise the effects of the total tensile rigidity of reinforcement layer, and it is quite misleading to classify reinforcements solely based on the material stiffness.

Figure 15 shows the relationship between the strength ratio (SR) and the covering ratio (CR) for TTR=4 from the physical PSC tests. It may be seen that the rate of increase in the SR with the increase in CR decreases with the increase in SR; the strength for CR=50% is already nearly the same as that for CR=100%. From this point of view, it is seen that the use of sheet reinforcement with CR=100% is not the most cost-effective, but the use of a grid with CR=50% is sufficient.

Covering Ratio (CR) and Interface Friction Angle (μ)

It is important to note that the overall pre-peak stress-strain curves from the physical PSC test and the FEM analysis presented in Fig. 13 are very similar for similar peak strengths. It can also be seen from Fig. 11 that the overall relationships between SR and TTR from the physical PSC tests with different covering ratios and those from the FEM parametric study with different μ values are very similar to each other.

It can be concluded, therefore, that reinforcement having a locally three-dimensional structure with a certain covering ratio (CR), like the grids used in the present study, can be properly modelled in plane strain FEM analysis for planar reinforcement layers having an equivalent interface friction angle μ. In such FEM analysis, each μ value corresponds to a certain CR value of the original reinforcement, and the total tensile rigidity should be the same as that of the original reinforcement.

Quantitative Relationship between CR and μ

Figure 15 also shows the relationship between SR and μ for TTR=4.0 obtained from the FEM analysis. In Fig. 15 (and Fig. 16), the part of the relation for the μ values lower than 12° is denoted by a broken curve, as this part is not simulated considering that the present interface model is not relevant for such low μ values. It may be seen that the contribution of CR in the physical PSC tests and that of μ in the FEM analysis to the increase in SR are not proportional to each other. The solid curve shown in Fig. 16 is the relationship between CR in the physical PSC tests and the interface friction angle μ in the FEM analysis, giving the same strength ratio (SR), for the case of TTR=4.0. This relationship was obtained from the two relationships shown in Fig. 15, and was fitted by the equation shown in Fig. 16. The three data points presented in Fig. 16 are those obtained from other physical PSC tests in which the values of TTR were different from 4.0 and the corresponding FEM analysis. It can be seen that these additional data are in good accordance with the relationship for
Fig. 16. Relationship between reinforcement covering ratio (CR) and equivalent interface friction angle (μ)

Fig. 17. Global stress-strain relationships from physical PSC tests on unreinforced specimen and those reinforced with six or eleven layers of grid tape a (9 × 1), compared with those from FEM analysis (μ = 20° for the reinforced specimens)

TRR = 4.0. Figure 17 shows the global relationships between the average stress ratio $R^*$ and the average axial strain $ε_a$ for the unreinforced specimen and two specimens reinforced with six or eleven layers of grid type a (9 × 1) having TRR = 1.0 and CR = 25%. These results are compared with the respective result from FEM analysis. It is seen that using the same value of μ = 20°, the FEM analysis simulates the results for two different numbers of grid layers well. These results suggest that the μ and CR relationship for six grid layers having TRR = 4.0, shown in Fig. 16, is also valid for other TRR values and other numbers of grid layers.

It is likely that such a relationship between μ and CR as presented in Fig. 16 is affected by the boundary conditions and dimensions of reinforced sand mass and cannot be directly applied to general cases including prototype geogrid-reinforced sand structures. However, it can be concluded at least that the deformation and strength of prototype geogrid-reinforced sand structures can be analysed by two- or three-dimensional FEM analysis in which the grid is modelled by planar reinforcement having a relevant interface friction angle μ with the same total tensile rigidity per each layer as the original grid.

Fig. 18. Strain and displacement fields of the specimen reinforced with 6 layers type a grid (9 × 1) obtained from equivalent two-dimensional FEM analysis: (a) at peak state ($ε_a$ = 3.0%), (b) at residual state ($ε_a$ = 4.5%)

**Failure Mode**

Figures 18(a) and (b) show the distributions of local shear strain $γ = ε_1 - ε_2$ and displacements at the nodes obtained at global axial strains of 3.0% (at the peak stress state) and 4.5% (at the residual state) for the specimen reinforced with six layers of grid type a (9 × 1), obtained from the FEM analysis using a full model (Fig. 7(a)). In particular, it can be seen from Fig. 18(b) that the shear bands developed in the following way:

a) for the whole thickness in the central part of the third sand layer from the top;

b) for the whole thickness in the right part of the sec-
Fig. 19. Strain and displacement fields of specimen reinforced with 11 layers of grid (9 x 1) obtained from the equivalent two-dimensional FEM analysis: (a) at peak state ($\varepsilon_p = 7.0\%$), (b) at residual state ($\varepsilon_p = 11.5\%$)

Fig. 20. Relationships between average volumetric strain and average axial strain obtained from physical PSC tests and FEM analysis; results corresponding to: (a) Fig. 17, (b) Fig. 13

This pattern of strain localisation by the FEM analysis is similar to that from the physical PSC test shown in Photo. 1(R6-a). Figures 19(a) and (b) show similar results for the specimen reinforced with eleven layers of grid type a (9 x 1) obtained from the FEM analysis, also using a full model (Fig. 7(b)). Unfortunately, no pictures like those presented in Photo. 1 were taken in this physical test.

**Volume Change Characteristics**

Figures 20(a) and (b) compare the relationships between the average volumetric strain and the average axial strain obtained from the physical PSC tests and the FEM analysis, each corresponding to Figs. 17 and 13. Curves from some physical tests in which reasonable data of lateral strains could not be obtained are not shown here. For the FEM analysis, average volumetric strains were obtained as "the total volume change of specimen"/"the initial total volume of specimen". It may be seen that not only before the residual state, but also at the residual state where shear bands have developed, the volume change behaviour is similar between the physical PSC tests and the FEM analysis.

The specimens that were more effectively reinforced exhibited more contractant and less dilative behaviour than the unreinforced specimen in the physical PSC tests. It can be seen that the above fact is reasonably simulated by the FEM analysis. The FEM analysis of the specimen reinforced with eleven layers of grid type a considerably overestimates the volume contraction. This is likely due to the fact that the excessive compression of artificially
value equal to the confining pressure \( \sigma_c = 19.6 \text{ kPa} \). This is because this element is located around the boundary of the zone where the stress state at failure is uniquely determined by the boundary stress \( \sigma_r \), as explained by the stress characteristics method (zone A in Fig. 23). After the local peak state is attained, however, the behaviour becomes rather unstable, due likely to the phenomenon of shear banding.

On the other hand, in the elements located in inner areas, 606, 608 and 610, the value of \( (\sigma_1)_{\text{local}} \) increases consistently with global axial loading, which is to a larger extent in the elements located closer to the central axis of specimen. The increase is larger in the specimen reinforced with a larger number of grid; in the central element 610 in the specimen reinforced with six and eleven layers of grid, the peak value of \( (\sigma_1)_{\text{local}} \) is, respectively, 12 and 34 times as large as \( \sigma_c = 19.6 \text{ kPa} \) (Figs. 22(a) and (b)).

It can be seen from these figures that the stress state within a failing reinforced sand is significantly inhomogeneous and complicated. When the global peak stress ratio is attained, the value of \( (\sigma_1)_{\text{local}} \) is decreasing in elements 602, 604 and 606, while the value of \( (\sigma_1)_{\text{local}} \) is increasing or nearly at its peak in elements 608 and 610. In addition, in all the elements (except for element 602 in the specimen reinforced with eleven layers of grid), the value of \( (\sigma_1)_{\text{local}} \) starts decreasing after the value of \( (\sigma_1)_{\text{local}} \) starts increasing, which is due to the increase in \( (\sigma_3)_{\text{local}} \) developed by the effects of tensile-reinforcing. This behaviour is most clearly seen in element 610 in the specimen reinforced with eleven layers of grid (the right bottom figure in Fig. 22(b)). Due to the above, in all the elements, the respective local peak principal stress ratio \( (\sigma_1/\sigma_3)_{\text{local}} \) is attained before the state of global peak stress ratio \( (R_{\text{global}})_{\text{peak}} \) is attained. The results shown above indicate that the failure of reinforced sand is considerably progressive in the sense that the local peak strengths in terms of both \( \sigma_1 \) and \( \sigma_3 \) are never attained simultaneously. Because of the above, for numerical analysis of the large strain behaviour of reinforced sand structures, it is imperative to use a constitutive model of sand that can take into account the strain-softening properties associated with strain localisation into shear bands.

It is also important to note that the elements that locate in inner areas of a specimen support a larger part of the applied axial load, and in these elements, before and for a range of stress after the respective \( (\sigma_1/\sigma_3)_{\text{local}} \) peak is attained, the local stress ratios \( (\sigma_1/\sigma_3) \), are kept nearly constant. This is particularly the case in elements located closer to the central axis of specimen. These local stress paths are rather similar to that of anisotropic consolidation at a constant stress ratio, which is between the \( K_0 \) value \( (\geq 1 - \sin \phi_{\text{peak}}) \) in one-dimensional compression and that at the failure \( (\leq (1 - \sin \phi_{\text{peak}})/(1 + \sin \phi_{\text{peak}})) \). On the other hand, the constitutive model of Toyoura sand used in the present analysis was developed using the results from PSC tests performed at different constant confining pressures and isotropic compression tests.

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**Fig. 21. Location of elements for local stress paths shown in Fig. 22**

weakened sand elements adjacent to the reinforcement became non-negligible because of a larger ratio of the volume occupied by these elements to that of the rest of ordinary sand elements. This drawback can also be seen in the relationship between the stress ratio and the axial strain shown in Fig. 17. More detailed examination of the data shown in Fig. 20 will not be made in the present study, considering the limited accuracy of the lateral strains measurements in the physical PSC tests as mentioned before.

**LOCAL STRESS STATES**

Local stress paths in terms of the local principal stress, \( (\sigma_1)_{\text{local}} \) and \( (\sigma_3)_{\text{local}} \), were obtained for representative elements that locate between the reinforcement layers in the specimens reinforced with six and eleven layers of grid type a (9 \( \times \) 1), as shown in Figs. 21(a) and (b). As these elements locate at or around the mid-height of the respective sand layer between the vertically adjacent reinforcement layers, the local principal stresses, \( (\sigma_1)_{\text{local}} \) and \( (\sigma_3)_{\text{local}} \), are acting in the vertical and horizontal directions or nearly in those directions.

Local stress paths for the elements are shown in Figs. 22(a) and (b). The states of local peak principal stress ratio \( (R_{\text{local}})_{\text{peak}} = ((\sigma_1/\sigma_3)_{\text{local}})_{\text{peak}} \) are denoted by "Local Peak", while the states of global peak stress ratio \( (R_{\text{global}})_{\text{peak}} \) are denoted by "Global Peak". The points denoted by "PRE" and "POST" mean arbitrary pre- and post-peak states, which are indicated on the stress-strain curves by the FEM analysis shown in the top left place in the particular figure. It may be seen from Figs. 22(a) and (b) that in element 602, located close to the lateral boundary, up to the local peak stress state, the value of \( (\sigma_1)_{\text{local}} \) is kept nearly constant, close to the initial
Fig. 22. Local stress paths of representative elements shown in terms of principal stress, $\sigma_1/p_\ast$ and $\sigma_3/p_\ast$ ($p_\ast = 98$ kPa) in relation to global stress-strain behaviour: (a) reinforced with 6 layers of grid type a ($9 \times 1$) (above), (b) reinforced with 11 layers of grid type a ($9 \times 1$) (the next page)
CONCLUSIONS

From the results presented above, the following conclusions can be derived:

1) In the present experimental program, where the rupture of grid did not occur, the effects of the covering ratio of each grid layer were much more important than those of the total tensile rigidity per grid layer. The results suggest that from the viewpoint of tensile-reinforcing function, the conventional classification of reinforcements into “extensible and inextensible ones” solely based on the material stiffness is very misleading.

2) The global relationships among the average stress and the average axial and volumetric strains, both pre- and post-peak, observed in the physical PSC tests could be well simulated by the plane strain FEM described in this paper.

3) The properties of the interface between the grid layer having a given covering ratio and the sand not exhibiting slipping with a displacement discontinuity can be simulated by plastic flow in the simple shear (or direct shear) mode in the sand elements adjacent to the interface. A grid, having a locally three-dimensional structure, can be modelled by a planer reinforcement having a 100% covering ratio with a proper interface friction angle $\mu$ and the same total tensile rigidity as the original grid. For the test conditions in the present study, the value of $\mu$ is uniquely related to the covering ratio of grid.

4) The results from the FEM analysis showed that the failure of reinforced sand specimens in PSC is considerably progressive; in particular, the global peak state is attained when the local axial stress is either decreasing or increasing after the respective maximum local principal stress ratio has been attained. These results indicate that modeling of the strain-softening behaviour of sand associated with strain localisation is imperative for the numerical analysis of the large strain behaviour of reinforced sand. The results from the FEM analysis also showed that the stress paths in zones closer to the central axis of specimen are similar to those in anisotropic compression at constant stress ratios close to that at failure. This result indicates that more study into the deformation characteristics in such a case as above will be necessary.

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REFERENCES


