DELAYED COMPRESSION/CONSOLIDATION OF NATURAL CLAY
DUE TO DEGRADATION OF SOIL STRUCTURE

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ABSTRACT

The superloading yield surface concept applied to the original Cam-clay model is examined for the application to the one-dimensional consolidation problems of highly structured soils. When the proposed elasto-plastic constitutive model is taken to be the case for a structured soil, the one-dimensional consolidation computation clarifies the following: (1) Delayed consolidation should occur even under one-dimensional compression condition with a considerably low stress ratio when softening of the soil occurs with volume compression. The delayed consolidation is the consolidation process that has sometimes been referred to as “secondary compression”. (2) The structure is always degrading as plastic deformation proceeds even within the period of “secondary compression”. (3) The decay of overconsolidation to normal consolidation also proceeds with plastic deformation. In this sense, the degradation of structure can not be independent of the decay of overconsolidation. However, since the decay of overconsolidation is much faster than the degradation of structure in clay, then softening becomes possible with volume compression even under a considerably low stress ratio.

Recovery of structure with time due to chemical bonding effects etc. in soil particles is beyond the scope of this study.

Key words: constitutive model, delayed compression, elasto-plasticity, finite deformation, finite element method, secondary consolidation (IGC: D6)

INTRODUCTION

As Mesri (1975), Tavenas and Leroueil (1990), and many other soil mechanics researchers have concluded, naturally deposited clays are mostly found in structured states. In addition these clays are mostly at overconsolidated states. The original Cam-clay that came to existence some three decades ago has provided a rational basis for the integrated elasto-plastic analysis of clay behavior. However, structured and overconsolidated soils are unfortunately beyond the scope of the original Cam-clay alone. Therefore, the behaviors of those natural soils are, due to the lack of a unified framework, still usually discussed in the literature in a very descriptive manner. There is also a common tendency that, whenever structure is discussed, the discussion is almost always associated with the effect of secondary compression. To take one highly recommended example, Leroueil and Vaughan (1990) maintain that increases in “preconsolidation pressure” can be expected partly from the volume change due to secondary compression but mainly due to the existence of “structure” in natural soils. Despite the fact that the effect of secondary compression is similar to the effect of structure, these two are not easy to clearly separate. Although a definition of structure in terms of elasto-plasticity has not yet been given, the key questions that arise here should be the following:

i) Why is it possible for secondary compression to occur in natural/structured soils?

ii) Does secondary compression develop structure additionally, or does it degrade the existing structure?

Between structure and overconsolidation, there is a similar confusion. Leroueil and Vaughan (1990), again state that through their observation the shape of the “yield curve” due to structure is similar to that due to overconsolidation, although the “yield” of structured soils occurs “progressively”, which may be different from “yield” due to overconsolidation. In this context, the “loss/removal of structure” can be expected to be qualitatively similar to the decay of overconsolidation, but the former seems to proceed at a much slower pace than the latter in natural clays. In the light of the facts mentioned above, many questions arise at this point: What can define the grade/degree of structure, quantita-
tively? What causes the degradation of structure? The same is also true for overconsolidation: What can define the degree of overconsolidation? Is the classical definition of OCR in terms of “stress history” still applicable? What causes the decay of overconsolidated state? How can the soil return to a normally consolidated soil? However, the key question that should be raised here may be the following:

Is the decay of overconsolidation independent of the degradation of structure, or not?

Before proceeding further, some comments relating to this question are necessary: Overconsolidated soil has been expected, for years, to behave in a purely elastic manner even at the reloading stage, which is a fundamental concept in the classical theory of elasto-plasticity. If this is the case, the soil should not generate any irreversible strain and/or plastic strain on the way back to a normally consolidated state. Therefore, the soil may hardly be subjected to any change in the degree or state of structure until the soil arrives at a normally consolidated state (Asaoka, Nakano and Noda, 1999b). In actual fact, this can be very far from the common experience and observation of soils, particularly when “sampling disturbance” is considered. An important point here is that these experiences and observations identify the demand of, not the classical but the “unconventional theory of plasticity” (Drucker, 1988), for describing the structure and overconsolidation. Concerning the unconventional plasticity, see details in Hashiguchi (1978, 1989).

As can be seen above, three different conceptions, i.e., structure, secondary compression, and overconsolidation are closely related to each other in natural soils. Asaoka, Nakano and Noda (1998, 1999a, 1999b) extended the original Cam-clay model by introducing a “superloading yield surface concept” together with Hashiguchi’s subloading yield surface concept in order to describe the elasto-plastic behavior of structured and overconsolidated soils. In their constitutive modeling, a fully remolded/destructured, and normally consolidated soil is assumed to follow the original Cam-clay, for simplicity. The constitutive model describes the evolution of both structure and overconsolidation with ongoing plastic deformation. However, “secondary compression” can hardly be imagined to occur within the framework of their invicid elasto-plastic constitutive model, alone.

The main objective of this study is to establish a rational basis for interpreting why secondary compression/consolidation occurs in structured soils. This will be shown through solving the one dimensional consolidation behavior of such structured overconsolidated soils. The so-called secondary compression and/or consolidation will be observed not in the constitutive model but in the “solution” of consolidation problems with time. In the present study, every time/ rate effect, such as secondary compression and “isotache” properties, is not considered as a material property. The time/rate effects observed particularly in engineering time scale may be no more than an integrated nature that appears only when these soils are solved with “Darcy’s law” in a certain soil-water coupled system under appropriate initial boundary conditions.

SUPER-SUBLOADING YIELD SURFACES FOR A STRUCTURED, OVERCONSOLIDATED SOIL

In order to describe the elasto-plastic behavior of structured and overconsolidated soils, super-subloading surfaces are newly introduced to the original Cam-clay. Essential concepts and key formulations for the constitutive modeling are summarized in this section (see details in Asaoka, Nakano and Noda (1999b)).

Three Yield Surfaces

When a perfectly destructured soil is in a “normally consolidated state,” the soil is assumed to follow the elasto-plastic behavior of the original Cam-clay.

It is commonly recognized that structured soils can exist at void ratios greater than those possible for fully remolded/destructured soils (Asaoka, Nakano and Noda, 1999b). Starting from this point a superloading yield surface is newly assumed to exist in the impossible states for the original Cam-clay, i.e., outside the “Roscoe surface.” The superloading yield surface is similar in shape to the original Cam-clay yield surface with the origin of the “q-p’ space” as the similarity center, where q and p’ are the deviator stress and mean effective stress, respectively. The similarity ratio of the original Cam-clay surface to the superloading surface in terms of stresses, denoted by $R^*$, lies between zero and one. For describing the plastic response of structured soils, the flow rule is applied to the superloading yield surface.

When a current stress state is on the superloading yield surface, the soil is said to be in a normally consolidated state.

Structured soil, initially on the superloading yield surface, becomes overconsolidated soil when unloading occurs. Soil in such an overconsolidated state, when reloading happens, exhibits elasto-plastic behavior. The plastic response is again assumed to satisfy the normality rule associated with the subloading yield surface (Hashiguchi, 1978, 1989; Asaoka, Nakano and Noda, 1997). Hence, a current stress state of the overconsolidated soil is always on the subloading yield surface.

The subloading yield surface is assumed to be geometrically similar to the superloading yield surface. The similarity ratio of the subloading yield surface to the superloading yield surface in terms of stresses, denoted by $R$, also lies between zero and one. The reciprocal of $R$ corresponds to overconsolidation ratio.

Three yield surfaces and the definitions of $R^*$ and $R$ are given in Fig. 1.

It is realistic to consider that natural soil is generally in a structured overconsolidated state. Since this is equivalent to considering the case that $0 < R \leq 1$ and $0 < R^* \leq 1$, then the current stress state is assumed, without loss of generality, to be on a subloading yield surface. As indicated in Fig. 1, the current stress parameters are denoted by $q$ and $p'$, while $\bar{q}$, $\bar{p}'$, and $q^*$, $p'^*$ are the corre-
Fig. 1. Cam-clay model with super-subloading yield surfaces (after Asaoa, Nakano and Noda, 1999b)

Corresponding projected stress parameters on the superloading and Cam-clay yield surfaces, respectively. Since the similarity center of the three yield surfaces is the origin of the q-p space, then

\[ R^* = \frac{q^*}{\tilde{p}} = \frac{p^*}{\tilde{q}}, \quad \text{and} \]

\[ R = \frac{p}{\tilde{q}} \]

give the similarity ratios, \( R^* \) and \( R \), from which the three yield surfaces can be written as follows:

\[ f(p^*, q^*) + \int_0^{\tau} J \text{tr} D^p d\tau \]

\[ = MD \ln \frac{p^*}{p_0^*} + D \frac{q^*}{p^*} + \int_0^{\tau} J \text{tr} D^p d\tau = 0 \] \hspace{1cm} (3)

\[ f(\tilde{p}, \tilde{q}) + \int_0^{\tau} J \text{tr} D^p d\tau + MD \ln R^* \]

\[ = MD \ln \frac{\tilde{p}}{p_0} + D \frac{\tilde{q}}{\tilde{p}} + MD \ln R^* \]

\[ + \int_0^{\tau} J \text{tr} D^p d\tau = 0 \]

\[ f(p', q) + \int_0^{\tau} J \text{tr} D^p d\tau + MD \ln R^* - MD \ln R \]

\[ = MD \ln \frac{p'}{p_0} + D \frac{q}{p'} + MD \ln R^* \]

\[ - MD \ln R + \int_0^{\tau} J \text{tr} D^p d\tau = 0 \] \hspace{1cm} (5)

where \( M \) and \( D \) are the critical state parameter and the dilatancy parameter, respectively, while \( D^p \) denotes plastic stretching. In these yield functions, \( J \) is the Jacobian determinant of deformation gradient tensor \( F \), and is expressed in terms of specific volume as follows:

\[ J = \det F = \frac{1 + e}{1 + e_0} \] \hspace{1cm} (6)

where \( 1 + e \) and \( 1 + e_0 \) are the specific volume at current (time \( t \)) and reference (time \( t=0 \)) state, respectively. The \( p_0^* \) in Eqs. (3)-(5) is the mean effective stress on the Cam-clay yield surface that corresponds to the initial mean effective stress \( p_0^* \) of the soil in the reference state.

**Associated Flow Rule and Consistency Condition**

The associated flow rule,

\[ D^p = \lambda \frac{\partial f}{\partial T'} (\lambda > 0) \] \hspace{1cm} (7)

governs the plastic response of soils in which \( \lambda \) denotes the plastic multiplier. Here \( T' \) denotes the effective Cauchy stress tensor. Prager's consistency condition

\[ \frac{\partial f}{\partial T'} + J \text{tr} D^p + MD \frac{\tilde{R}^*}{R^*} \frac{\tilde{R}}{R} = 0 \] \hspace{1cm} (8)

should determine the size of the subsequent yield surface. Equation (8) identifies the demand for the "evolution law" for both \( R \) and \( R^* \) that gives the material time derivatives of \( R \) and \( R^* \).

**Evolution Laws for \( R \) and \( R^* \)**

Both \( R \) and \( R^* \) increase to one gradually, and thus \( \tilde{R} \) and \( \tilde{R}^* \) should be positive when plastic deformation proceeds. Since the shape of the yield surface is of the original Cam-clay type, plastic stretching is always accompanied by its shear component. Thus the norm of the shear component of plastic stretching \( ||D^p|| \) is employed as the measure of ongoing plastic deformation, in which

\[ D^p = D^p - \frac{1}{3} \text{tr} D^p J = \lambda \frac{3S}{2q} \frac{\partial f}{\partial T'} \]

\[ S = T' + p'I \] and \( I \) is an unit tensor. It is then found that the evolution law of \( R \) and \( R^* \) can be expressed generally in terms of \( ||D^p|| \) as follows:

\[ \dot{R} = JU \frac{2}{3} ||D^p|| \]

\[ \dot{R}^* = JU^* \frac{2}{3} ||D^p|| \]

in which \( U \) and \( U^* \) are both positive scalar functions of \( R \) and \( R^* \), respectively and should satisfy the following:

\[ U(R=1) = 0 \]

\[ U^*(R^* = 1) = 0 \]

**Plastic Multiplier \( \lambda \) in Terms of Stresses**

Substituting evolution laws to the consistency condition, and after some manipulation, one gets the plastic multiplier \( \lambda \) in terms of stresses as follows:

\[ \lambda = \frac{\frac{\partial f}{\partial T'} \cdot \tilde{T'}}{J \frac{D}{p^2} (M_s p' - q)} \]

in which

\[ M_s = M \left( 1 - \frac{D U^*}{R^*} + \frac{DU}{R} \right) \]
is a new critical state "parameter," which increases/
decreases its value with ongoing plastic deformation; de-
tails are given later. Note here that, for a positive λ (i.e.,
loading), one gets three cases
\[
\frac{\partial f}{\partial T'} = 0 \quad \text{(i.e., hardening) when } q < M_p' \\
\frac{\partial f}{\partial T'} = 0 \quad \text{(i.e., perfectly plastic behavior)} \\
\text{when } q = M_p' \\
\frac{\partial f}{\partial T'} = 0 \quad \text{(i.e., softening) when } q > M_p'.
\]

Simple Expressions for \( U \) and \( U^* \)
Following Hashiguchi (1989), the simplest form is as-
sumed for \( U \):
\[
U = -\frac{m}{D} \ln R.
\]
Since \( M \) in Eq. (15) should be positive, \( U^* \) should satisfy
another constraint condition other than Eq. (13) (Asaoka, Nakano and Noda, 1999b), from which \( U^* \) is
assumed in the present study to be expressed as follows:
\[
U^* = \frac{1}{D} R^*(1 - R^*m^*).
\]
In Eqs. (19) and (20), \( m \) and \( m^* \) are degradation
parameters of overconsolidated state and structured
state, respectively. The greater these parameters, the
faster the decay of these states.

Constitutive Equation and Loading Criterion
With the basic background set as before, the constitutive
equation for an overconsolidated structured soil can be
obtained as follows:
\[
\dot{T}' = E\dot{D} - A\dot{E} \frac{\partial f}{\partial T'}
\]
where \( E \) denotes the elastic modulus tensor, and
\[
\dot{T}' = T' + T'\Omega - \Omega T', \quad \Omega = \dot{R}R^T
\]
is the objective co-rotational rate tensor (Dienes, 1979) in
which \( R \) is the rotation and the upper "T" denotes trans-
pose operation. The plastic multiplier \( A \) in Eq. (21) is
expressed in terms of stretching, as follows:
\[
A = \lambda = \frac{\frac{\partial f}{\partial T'}: E\dot{D}}{\frac{\partial f}{\partial T'}: E\dot{D} + \frac{D}{p^2} (M_p' - q)}.
\]
Since the denominator of \( A \) in Eq. (21) is shown positive
under some constraint conditions on elasto-plastic
parameters, the loading criterion for the constitutive equa-
tion is found to be
\[
\frac{\partial f}{\partial T'}: E\dot{D} > 0.
\]
A detailed form of the constitutive equation is given in
Appendix A.

A POSSIBLE MECHANISM OF DELAYED COMPRESSION/CONSOLIDATION

Softening with Plastic Volume Compression
Equations (16)–(18) show that the stress state \( q = M_p' \)
is now the watershed between hardening and softening
when loading; here \( M \) is called a critical state parameter.
Substituting Eqs. (19) and (20) into Eq. (15), one gets
\[
M = M \left( R^*m^* - m \ln \frac{R}{R^*} \right)
\]
which clearly indicates that the parameter \( M \) is a vari-
able. As plastic deformation proceeds, both \( R \) and \( R^* \)
should start to increase their values, approaching to one.
The increase of \( R \) (i.e., the decay of the overconsolidated
state) makes \( M \) decrease, while the increase of \( R^* \) (the
degradation of structure) makes \( M \) increase. This is illus-
trated in Fig. 2.

Since the shape of any yield surface is of the original
Cam-clay type, then the stress state \( q = M_p' \) still remains
the watershed between plastic volume compression and
plastic volume expansion (Fig. 3).

Even when the soil is initially heavily overconsoli-
dated, it can be assumed that the decay of overconsoli-
dation will be much faster than the degradation of struc-
ture, which is actually the case of a typical natural clay
(see Leroueil and Vaughan (1990)). In such a case, it is
easily supposed that the stress state of the soil arrives at
\[
M_p' < q < M_p'
\]
during the loading stage, which gives the condition that
softening occurs with plastic volume compression (see
Fig. 4). During this stage, it can also be supposed that \( p' \)
is increasing, which should give a real volume compres-
sion.

It may be interesting to consider the case where the
degradation of soil structure is much faster than the
decay of overconsolidation. In this case, even for a small
value of initial \( M \), the \( M \) should increase very rapidly
due to structure degradation and, consequently, may
become larger than \( M \) due to the remaining overconsoli-

\[\text{Fig. 2. Decrease of } M \text{ due to decay of overconsolidation and increase of } M \text{ due to degradation of structure}\]
Fig. 3. Watershed between plastic volume compression and plastic volume expansion

Fig. 4. Softening with plastic volume compression

Fig. 5. Excess pore water pressure rise due to softening

Fig. 6. Hardening with plastic volume compression

dation. This will be the case of a loose sand, in which delayed consolidation hardly occurs, as examined below.

Delayed Compression/Consolidation

When spatial distribution of excess pore pressure is convex, softening of a saturated soil generally yields not the dissipation but the generation of excess pore pressure, which causes the delay of consolidation that is sometimes referred to as “secondary compression”.

If we take a one dimensional consolidation problem for simplicity, the stress condition (26) suggests that even under a considerably low stress ratio such as the ratio in a one-dimensional compression, the structured soil can experience softening with volume compression for a certain time period. During this, if the initial spatial distribution of excess pore pressure is convex at some region in the soil, one can observe excess pore pressure rise which, as a result, delays the complete consolidation that otherwise would have ended early. The simplest explanation for this, considering the classical Terzaghi’s one-dimensional consolidation equation as a piecewise linear model of the real consolidation, is schematically illustrated in Fig. 5.

After sufficient collapse of soil structure, $M_0$ becomes closer to $M$, and the stress state should now give only the hardening (see Fig. 6). Therefore, the localized generation of excess pore pressure does not continue so long. As a result, after the delayed compression/consolidation that should occur with softening, the soil should experience the usual “primary consolidation” with plastic hardening.

Soil elements adjacent to a softening region undergo unloading due to excess pore pressure generation under constant load application. The soil elements therefore, experience complex stress histories during consolidation: loading, unloading, reloading, exhibiting hardening softening and elastic response, the tendency of which naturally varies from the top to the bottom of the soil specimen. Due to the nature of stress path dependency, an elasto-plastic structured soil, then, naturally yields inhomogeneity with depth after consolidation.

NUMERICAL SIMULATION OF OEDOMETER TESTS OF STRUCTURED SOILS

The behavior of structured clay during one-dimen-
sional consolidation was numerically examined by simulating one-dimensional consolidation tests in the laboratory. Governing equations and necessary boundary conditions together with some comments on finite element discretization are given in Appendix B.

As shown in Fig. 7, the initial thickness of soil sample was 2 cm, the top of which was set permeable while the bottom, impermeable. The elasto-plastic parameters together with necessary initial conditions of the soil sample are given in Table 1.

**Vertical Load Application with Constant Rate**

Shown in Fig. 8 is the overall compression behavior of the soil sample in "e−log σv" diagram, which was obtained under vertical load application with constant rate. The e denotes the overall void ratio of the soil sample and σv, the vertical load per unit area. As shown in the figure, the soil takes its state in the impossible region for destructured soil, i.e., above the normal consolidation line of destructured soil. Two different loading rates were considered, and as has been commonly suggested, the higher the rate, the greater the "preconsolidation pressure", where the term "preconsolidation pressure" only means the stress at which apparent "yield" is observed on the e−log σv curve. It may be worth pointing out that both the terms "yield" and "preconsolidation pressure" are no more than vague expressions in the present context and are beyond strict technical definition in elasto-plasticity.

**Settlement Behavior under Constant Load Application**

After applying a vertical load rapidly under the constant rate of \(9.8 \times 10^{-4}\) kPa/sec, the vertical load was kept constant so that the soil sample underwent one-dimensional consolidation under constant load application. Three loading magnitudes were considered here for simplicity; these were 392 kPa, below the preconsolidation pressure, 833 kPa, above the preconsolidation pressure, and 1179 kPa a sufficiently large magnitude. These three loading conditions are indicated in Fig. 8 by points A, B and C, respectively.

Three settlement behaviors from the beginning of loading are given with time in Fig. 9. Bold lines in the figure indicate the time period during which softening occurs at a certain depth in the soil sample. When small load application is the case, since no softening occurs, the soil sam-

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**Table 1. Material constants and initial conditions (after Asaoka, Nakano and Noda, 1999b)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression index ( \lambda )</td>
<td>0.131</td>
</tr>
<tr>
<td>Swelling index ( \delta )</td>
<td>0.0754</td>
</tr>
<tr>
<td>Critical state constant ( M )</td>
<td>1.53</td>
</tr>
<tr>
<td>Poisson's ratio ( \nu )</td>
<td>0.3</td>
</tr>
<tr>
<td>Degradation parameter of structure ( m^* )</td>
<td>0.5</td>
</tr>
<tr>
<td>Degradation parameter of overconsolidated state ( m^c )</td>
<td>10.0</td>
</tr>
<tr>
<td>Initial specific volume ( \rho_0 )</td>
<td>2.19</td>
</tr>
<tr>
<td>Initial vertical stress ( \sigma_{v0} ) (kPa)</td>
<td>9.8</td>
</tr>
<tr>
<td>Initial value of ( R^* (R^c) )</td>
<td>0.05</td>
</tr>
<tr>
<td>Initial overconsolidation ratio ( 1/R_0 )</td>
<td>100.0</td>
</tr>
</tbody>
</table>

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Fig. 7. Finite element array for one-dimensional consolidation computation with drainage boundaries.

Fig. 8. Vertical load application with constant rate of loading.

Fig. 9. Delayed settlement due to softening of soils with volume compression (During bold segments softening is found to occur in the sample.)
ple undergoes normal consolidation with plastic hardening. However, when loading magnitude exceeds a certain level, the soil sample undergoes softening. This is clearly observed in excess pore water pressure distribution in the soil. Figure 10 shows the pore pressure isochrones under the applied pressure of 833 kPa. In this figure, arrows indicate the points where softening occurs with pore pressure generation. Delayed consolidation is thus found to occur due to the pore pressure rise, although the rise itself is a small amount of pressure.

When vertical load is large enough, see e.g. line C, no significant delay of consolidation is observed. This phenomenon has been indicated by Mesri and Godlewski (1977) and others. Under a large vertical load the softening occurs in the very early stages of consolidation.

Inhomogeneity of the soil sample with depth develops with time during consolidation. This is of course due to a complex stress history caused by softening, which has already been explained in the former section. Figures 11(a) to (c) show the inhomogeneity obtained at the end of consolidation. They are illustrated in the reference configuration with depth, i.e., in the initial soil sample of 2 cm height. When the vertical load is small enough, no softening can occur in the soil, and thus the soil sample does not develop any significant inhomogeneity, as shown in Fig. 11(a).

A sudden increase in the rate of settlement under constant load application has also been reported for many years. Figure 12 is an example given by Leroueil, Kabbaj, Tavenas and Bouchard (1985). Figure 13 is a numerical example for this, obtained when the vertical load was kept constant at 785 kPa. The pore pressure isochrone corresponding to this settlement behavior is given in Fig. 14, in which the unloading zone adjacent to the softening point is also indicated. The effective stress path in the soil element located in the center depth of the soil sample is
given in Fig. 15. As can be seen in this figure, when $M_0$ is sufficiently small when compared with the current stress ratio, the softening of soil occurs with volume compression. However, as structure degradation proceeds with plastic deformation, $M_0$ becomes large so that no more softening may occur at this stress level. The end of consolidation is always not a "secondary consolidation" but a normal "primary consolidation" with hardening of soils. Figure 16 shows the inhomogeneity of the soil sample at the end of consolidation, which should be compared with Figs. 11. Different loading magnitudes yield somewhat different inhomogeneity.

**Element-wise Compression Behavior**

In order to give an intuitive understanding for both the occurrence of softening and the delayed consolidation,
one-dimensional compression behavior of the soil element is illustrated in “$e - \log \sigma_v$” diagram in Fig. 17. For computing the element behavior, the constitutive equation is directly integrated along the path of one-dimensional compression deformation (see details in Asaoka, Nakano and Noda (1999b)). In Fig. 15, the apparent compression behavior of the soil sample under vertical loading with constant rate (i.e., the curves of Fig. 8) is also overlapped from which, the “preconsolidation pressure” is now found to correspond to the stress level at which softening occurs with volume compression.

When the loading rate is very slow, even the consolidation test can provide almost element-wise compression behavior, which is given in Fig. 18. During the process from point x to point y, consolidation computation was made under the constant rate of vertical displacement. There is almost no discrepancy between Fig. 17 and Fig. 18 as far as the first six decimals are concerned. However, it should be strongly noted that the fully drained condition requires an enormously long time period. If the loading rate is slow enough, since the soil sample exhibits almost element-wise characteristics, then no inhomogeneity should be found. However natural soil deposits always show some inhomogeneity with depth, which suggests that the rate of depositional environment is much faster than the loading rate required for a fully drained condition. Usual consolidation tests in the laboratory with a constant rate of displacement/loading hardly provide element-wise behavior.

Apparent “Isotache” Characteristics

When the computation is made under the condition of constant rate of vertical displacement throughout, one has an apparent “isotache” behavior. This is given in Fig. 19. If one displacement rate is changed to another one, the displacement curve gradually shifts to another curve of another rate. However, the behavior is very far from element-wise behavior as examined above. Actually, the constitutive model is perfectly invidic and any rate effect comes from the resistance of the pore water squeez-
ing out of the soil element, which is of course governed by Darcy’s law. If the “isotache” behavior implies the property that is inherent in a soil element, Fig. 19 does not give any evidence for the “isotache theory”.

CONCLUSIONS

When the proposed elasto-plastic constitutive model for structured soil is taken to be the case, the one-dimensional consolidation computation clarifies the following:

1) Delayed compression/consolidation should occur when the softening of soil occurs with volume compression.

2) The structure is constantly degrading as plastic deformation proceeds, even in the period of “secondary compression”.

3) The decay of overconsolidation also proceeds with plastic deformation. In this sense, the degradation of structure can not be independent of the decay of overconsolidation. However, since the decay of overconsolidation is much faster than the degradation of structure in clay, then softening becomes possible with volume compression even under a considerably low stress ratio.

These three conclusions correspond to the three questions raised in the INTRODUCTION, above.

Recovery of structure with time due to chemical/thermal bonding effects in soil particles is, of course, beyond the scope of this elasto-plastic study.

REFERENCES


APPENDIX A

The rate type constitutive equation (Eq. (21)) for the original Cam-clay model with super-subloading yield surfaces can be written as follows:

\[
\dot{\varepsilon} = \left( \frac{\bar{K}}{\bar{\sigma}} \right) - \frac{1}{3} \left( \frac{\bar{G}}{\bar{\sigma}} \right) (\text{tr} D) I + 2 \bar{G} D \\
- \left( \frac{\bar{G}}{\bar{\sigma}} \right) S \cdot D - \bar{R} \bar{\beta} (\text{tr} D) \left( \frac{\bar{G}}{\bar{\sigma}} \right) S - \bar{R} \bar{B} I \\
\bar{G} + \bar{R} \bar{B}^2 + h
\]

(A-1)

where

\[
\bar{K} = \frac{1 + e}{\bar{K}} p' = \frac{J (1 + e_0)}{\bar{K}} - p', \quad \bar{G} = \frac{3 (1 - 2v)}{2 (1 + v)},
\]
\[
\beta = \frac{\beta}{\sqrt{3}} = \frac{1}{\sqrt{3}} \left( \frac{M - q}{\rho'} \right), \quad \tilde{\tau} = \frac{q}{\sqrt{3}},
\]
\[
h = \frac{Jp'}{\sqrt{3}D} \left( \tilde{\beta} + \frac{MD}{U} \frac{U^*}{R} - \frac{R^*}{R^*} \right).
\] (A-2)

**APPENDIX B**

*Equation of Equilibrium of Forces*

Since the incremental constitutive model is considered, the equation of equilibrium of forces assumes a rate type, where the body force due to gravity is neglected:

\[
\text{div} \hat{S}_t = 0, \quad \hat{S}_t = \hat{T} + (\text{tr } D)T - TL^T
\] (B-1)
in which \( L, T \) and \( \hat{S}_t \) denote the velocity gradient, the Cauchy (total) stress and the nominal stress rate (Yatomi et al., 1989) of soil skeleton respectively, while the material time derivative is expressed by an upper dot.

**Effective Stress and Pore Pressure**

\[
T = T' - uL,
\] (B-2)
in which \( u \) denotes the pore pressure. In this equation, the tensile stress components of \( T \) are defined as positive and \( u \) is positive when compressive.

**Linear Rate Type Constitutive Equation of Soil Skeleton**

\[
\hat{T}' = \mathcal{E}[D].
\] (B-3)

**Compatibility Condition**

\[
L = \text{grad} \left( \frac{\partial v}{\partial x} \right),
\] (B-4)
in which \( v \) and \( x \) are the velocity vector and current position vector of the material point \( X \) of the soil skeleton respectively.

**Continuity Condition of the Soil-Water System**

The rate of volume change of the soil skeleton is expressed as,

\[
\left( \int_v \text{div} v \right)' = \int_v \text{tr } Ddv = -\int_v v' \cdot nda,
\] (B-5)
in which \( v' \) is the discharge velocity of pore water, while \( n \), the unit outward normal vector at the boundary surface of the soil skeleton.

**Darcy's Law**

\[
v' = -k \frac{\partial h}{\partial x},
\] (B-6)
in which \( k \) is the coefficient of permeability and assumes here a scalar valued constant, while \( h \) is the sum of elevation head and pressure head.

**Boundary Conditions**

The following two pairs of boundary conditions were introduced in this study: one is the pair of velocity boundary \( \Gamma_v \) and cell pressure boundary \( \Gamma_n \), where \( \Gamma_v + \Gamma_n = \partial v \).

Since the cell pressure changes with time,

\[
t = cn \quad \text{on} \quad \Gamma_n, \quad c = \text{const.},
\] (B-7)
then the nominal traction rate \( \hat{t} \) is expressed simply as,

\[
\hat{t} da = (\text{tda})' = (cnda)'
\] (B-8)
\[
= c([\text{tr } D] I - L^T) n da \quad \text{on} \quad \Gamma_v.
\]

Concerning the seepage, the pair of discharge velocity boundary \( \Gamma_v \) and pore pressure boundary \( \Gamma_n \) is considered, where \( \Gamma_v + \Gamma_n = \partial v \), that is,

\[
v' = -k \frac{\partial h}{\partial x} = 0 \quad \text{on} \quad \Gamma_v
\] (B-9)
for an undrained boundary and

\[
u = \bar{u} \quad \text{on} \quad \Gamma_u
\] (B-10)
for a drained boundary, in which \( \bar{u} \) denotes the prescribed pore pressure on \( \Gamma_v \).

**Application of the Finite Element Method**

The weak form of Eq. (B-1) is expressed as

\[
\int_V \{ \hat{T}' \cdot \delta D + (\text{tr } D)T \cdot \delta L - TL^T \cdot \delta L \} dv
\]
\[
\quad - \int_{\Gamma_v} \hat{u} (\text{tr } \delta D) d\Gamma
\]
\[
\quad = \int_{\Gamma_n} \hat{s}_t \cdot \delta n da - \int_{\Gamma_v} (\Omega T' - T' \Omega) \cdot \delta D dv,
\] (B-11)
in which \( \delta v \) is the virtual velocity satisfying necessary boundary conditions, while \( \delta L \) and \( \delta D \) are the consequent virtual velocity gradient and virtual stretching, respectively. The first integrand part on the left hand side of Eq. (B-11) yields the global stiffness matrix \( K \) of the soil skeleton in finite element discretization. The coupling equations, Eqs. (B-5) and (B-6) are discretized based on the physical model proposed by Christian and Boehmer (1970) and Akai and Tamura (1978); for details of the finite deformation version, see Asaoka, Nakano and Noda (1994).