CORRELATION OF PORE-PRESSURE B-VALUE WITH P-WAVE VELOCITY AND POISSON'S RATIO FOR IMPERFECTLY SATURATED SAND OR GRAVEL

TAKEJI KOKUSHO

ABSTRACT

Field data indicate that the P-wave velocity in sand or gravel is sometimes much lower than that of water, even if the soil is below the water table. It is well understood that a slight decrease in saturation normally evaluated by the B-value has a significant effect on undrained shear behavior like liquefaction of saturated soil. In the first part of this research, theoretical formulations of the B-value, P-wave velocity and Poisson's ratio are made by taking into account the decrease in bulk modulus of water due to a mixture of air bubbles. Then, computations are carried out using formulas based on the soil properties of a typical sand or gravelly soils and Masa soil from the Kobe area to make charts correlating the variables. These charts indicate that a small decrement in the B-value in the interval of B = 1.0 to 0.8 will considerably decrease the P-wave velocity. Thus, the P-wave velocity which is easily measured in the field can serve as a convenient index to quantitatively evaluate the insitu soil B-value.

Key words: B-value, gravelly soils, liquefaction, P-wave, Poisson's ratio, undrained test (IGC: D7/E8/C8)

INTRODUCTION

During the 1995 Hyogoken Nambu earthquake, vertical array records were obtained in the northern corner of the Port Island in the Kobe city. Extensive liquefaction took place there which damped the horizontal maximum acceleration at the surface into a much lower value than the underground elevation because of the deterioration in soil shear stiffness (Kokusho and Matsumoto, 1998). Contrary to this, the vertical acceleration was greatly amplified in the surface layer because the P-wave velocity, which does not deteriorate due to strain-dependent soil-nonlinearity, was relatively low near the ground surface even under the water table, leading to a high impedance ratio for the vertical motion as demonstrated by Aoyagi and Kokusho (1999).

The P and S-wave velocities (V_p and V_s, respectively) at this vertical array site measured with the down-hole logging method are shown along the depth in Fig. 1. Two different velocities are shown in the chart; one measured at the moment of the initial instrumentation of the vertical array and the other measured after the 1995 earthquake (Kobe Municipal Office, 1997). Despite a potential crudeness in velocity measurements by means of the down-hole method, it may well be judged that both V_p and V_s in the upper layers changed to some extent because of the high level of shaking during the main shock. Among their changes, the V_p variation in the fill layer just under the water table is the most remarkable. Before the earthquake, it had been only 780 m/s from the water table down to GL-13m, while it increased to 1400 m/s in the same interval after the quake. A smaller increase in V_p is also seen in deeper layers down to GL-60m. Post-earthquake changes in V_s were not so dramatic, except in the upper fill layer where it considerably decreased due, probably, to seismic disturbance. The increase in V_p is not in accordance with the shift in V_s, indicating that causes other than the change in soil density may be responsible for this. The most probable explanation at least for the fill layer may be that a higher degree of satu-

![Fig. 1. V_p and V_s distribution along depth; measured by down-hole method before and after the earthquake at Port Island vertical array site](image)

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ration in the void of the fill material was attained due to seismic shock involving liquefaction.

The fill material consisted of a decomposed granite, locally called "Masa soil", a gravelly soil composed of gravel, sand and non-plastic silt (Hara and Kokusho, 1998). This soil was dumped from a barge bottom into the sea to make the man-made island. It may well be assumed that a large number of air bubbles were trapped in the soil void, resulting in a remarkable reduction in \( V_p \) compared to a fully saturated soil.

It is not so infrequent that \( V_p \) measured in the field is evidently lower than about 1500 m/s, which is \( V_p \) in pure water, despite the fact that the soil is located below a water table. As will be shown later, if the soil void is fully saturated, its \( P \)-wave velocity should be almost equal to or larger than that in pure water. Figure 2 shows the plots for the \( P \)-wave velocity of soils normalized by that in water, \( V_p/V_p^f \), versus Poisson's ratio, \( \nu \). These values are derived from published data on \( V_p \) and \( V_s \) measured with down-hole PS-logging (e.g. Hanshin Highway Corporation, 1997). Two different symbols, solid marks or open marks, signify that the locations are either above or below the water table. Masa soils and other types of soil are also differentiated in the chart. The solid and dashed curves in the figure indicate the outer limits of the computed correlations, which will be mentioned later. It is to be noted that, in contrast to largely scattered data above the water table, data below it are all located for a Poisson's ratio greater than 0.44. Despite these high Poisson's ratios, the normalized \( P \)-wave velocity takes a wide range of values between 1.2 to 0.4.

The degree of saturation in insitu soil is significant in evaluating insitu liquefaction potential for non-cohesive soils because it greatly influences the pore-pressure buildup. To reproduce the insitu undrained shear behavior in a laboratory test, a soil sample should have not only an identical density and micro-structure but also an identical degree of saturation. A method commonly employed to evaluate the degree of saturation in a lab specimen is the pore-pressure coefficient \( B \). The \( B \)-value is defined as

\[
B = \Delta u / \Delta \sigma_m \text{, where } \Delta u \text{ and } \Delta \sigma_m \text{ are increments in porepressure and total mean stress, respectively. It serves as a sensitive indicator of saturation for soils almost fully saturated but influenced by a little inclusion of air. Considering the ready availability of insitu } P \text{-wave velocity measurement in engineering practice, it will be beneficial to establish a correlation whereby a corresponding insitu } B \text{-value can be estimated from } V_p \text{ in order to reproduce an identical degree of saturation in the laboratory.}
\]

In the first part of this paper, the \( B \)-value, \( V_p \), and also Poisson's ratio are formulated based on Biot's poro-elasticity theory incorporating the bulk modulus of the porefluid, \( K_f \). Then, computations are carried out for these equations, taking account of the variation of \( K_f \) with increasing air-bubbles for a typical sand and gravelly soils as well as the Masa soil. Based on these computed values, charts are established correlating these variables for the different types of soils and compared with field data.

### THEORETICAL EXPRESSIONS FOR B-VALUE, P-WAVE VELOCITY AND POISSON'S RATIO

According to Biot's poro-elasticity theory (Zienkiewicz and Bettes, 1982), the equilibrium for saturated elastic porous media consists of the following two equations. The overall equilibrium for the mixture of soil skeleton and pore-water;

\[
\sigma_{ij} + p g_i = \mu \ddot{u}_i + \rho \ddot{w}_i
\]

and the equilibrium for the pore-water;

\[
\ddot{p} + \rho g_i = k^{-1} \ddot{w}_i + \rho \ddot{u}_i + n^{-1} \rho \ddot{w}_i
\]

In these equations, the tensor notation is adopted with \( i \) as the rectangular coordinate direction, \( \ddot{u}_i = \partial u_i / \partial t \), and \( \sigma_{ij} \) as the total stress tensor. In addition \( g_i = \) gravity acceleration vector, \( \ddot{u}_i = \) acceleration vector of soil skeleton, \( \ddot{w}_i = \) acceleration vector of pore-water relative to soil skeleton, \( \ddot{w}_i = \) velocity of water relative to soil skeleton, \( p = \) pore-water pressure, \( \rho = \) density of total soil, \( \rho_p = \) density of pore-water, \( n = \) porosity and \( k \) = permeability constant in the Darcy's law (Zienkiewicz and Bettes, 1982). Though the pore-water considered in this research is not necessarily fully saturated but may contain some air bubbles in it, pressure \( p \) is basically considered as the water pressure and the pressure in the air bubbles is assumed to be balanced with it. The total stress tensor \( \sigma_{ij} \) is related with the effective stress tensor \( \sigma_{ij}' \) by the following equation;

\[
\sigma_{ij} = \sigma_{ij}' - \delta_{ij} p
\]

where \( \delta_{ij} \) is Kronecker's delta. Stresses \( \sigma_{ij} \) and \( \sigma_{ij}' \) are negative for compression, while pore-pressure \( p \) takes a positive value for compression. Based on the mass balance of water flow, the divergence of the flow velocity vector must be equal to the rate of the pore volume decrease and the fluid expansion;

\[
\ddot{w}_{ii} = -\ddot{u}_{ii} - (1 - n) \ddot{p} / K_f + \ddot{\sigma}_{ii}' / 3 K_f - p n / K_f
\]

where \( \ddot{\sigma}_{ii}' / 3 \) is the time derivative of the effective mean
stress, \( \sigma_{\nu} = \sigma_{\nu} / 3 \). Here, the soil skeleton is assumed to behave as an isotropic elastic material. Furthermore, \( K_s \) and \( K_f \) are the bulk modulus for solid soil particle and for pore-fluid, respectively. In undrained condition, \( \dot{w} = 0 \) and Eq. (4) results in

\[
-\dot{u}_i - (1 - n) \frac{\dot{p}}{K_s} + \sigma_{\nu} / K_s - \dot{n} / K_f = 0
\]

(5)

Because the mean effective stress, \( \sigma_{\nu} \), is correlated with the volumetric strain, \( e = \dot{w} \), as

\[
\sigma_{\nu} = K' e
\]

(6)

where \( K' \) is the bulk modulus for the soil skeleton, the following equations can be obtained;

\[
\frac{\dot{p}}{\sigma_{\nu}} = -\left( \frac{1}{K'} - \frac{1}{K_s} \right) \left( 1 - \frac{n}{K_s + K_f} \right)
\]

(7)

Let \( \Delta p, \Delta e, \Delta \sigma_{\nu} \) and \( \Delta \sigma_{\mu} \) be increments of the pore-pressure, the volumetric strain and the total and effective stresses, respectively, in a small time interval, \( \Delta t \). If \( \sigma_{\nu} \) is eliminated by using Eq. (6), then

\[
\frac{\Delta p}{\Delta e} = -\left( \frac{1}{K'} - \frac{1}{K_s} \right) \left( 1 - \frac{n}{K_s + K_f} \right)
\]

(8)

Considering \( \Delta \sigma_{\mu} = \Delta \sigma_{\nu} = \Delta p \), the B-value can also be derived from Eq. (7);

\[
B = \frac{\Delta p}{\Delta \sigma_{\mu}} = \left( \frac{1}{K'} - \frac{1}{K_s} \right) \left( 1 - \frac{n}{K_s + K_f} \right)
\]

(9)

This equation indicates that the decrease in bulk modulus of pore-water \( K_f \) due to inclusion of air-bubbles will decrease the B-value.

Next, elastic wave propagation in a poro-elastic media is considered. If the undrained condition is assumed during the wave propagation because of its transient behavior, \( \dot{w} = 0 \) and Eq. (1) becomes:

\[
\sigma_{\nu,i,j} = \dot{\rho} \dot{u}_i
\]

(10)

because increments from an initial static equilibrium condition due to elastic wave propagation are exclusively considered. In terms of effective stresses, Eq. (10) is expressed as;

\[
\sigma_{\nu,i,j} - \rho \dot{p} = \dot{\rho} \dot{u}_i
\]

(11)

By using Eq. (8)

\[
\sigma_{\nu,i,j} = \frac{\partial p}{\partial e} e_i = -\left( \frac{1}{K'} - \frac{1}{K_s} \right) \left( 1 - \frac{n}{K_s + K_f} \right)
\]

(12)

as well as the following constitutive relations for an isotropic elastic material;

\[
\sigma_{\nu,i} = K' e + 2G \left( u_{\nu,i} - \frac{e}{3} \right) \quad i = j
\]

(13)

\[
\sigma_{\nu,i,j} = \sigma_{\nu,j} = G ( u_{\nu,j} + u_{\nu,i} ) \quad i \neq j
\]

(14)

Eq. (11) yields;

\[
\dot{\rho} \dot{u}_i = \left[ K' + \left( \frac{1}{K'} - \frac{1}{K_s} \right) \left( 1 - \frac{n}{K_s + K_f} \right) + \frac{1}{3} G \right] e_i
\]

\[
+ G \nabla^2 u_i
\]

(15)

where \( \nabla^2 u_i = (u_{\nu,i})_{,i} \).

In general, the equation of wave propagation in an isotropic elastic medium is written as;

\[
\dot{p} \dot{u}_i = (\lambda + G) e_i + G \nabla^2 u_i
\]

(16)

where, \( \lambda \) is one of Lame's constants and is expressed as

\[
\lambda = 2G / (1 - v)
\]

(17)

in which \( v \) is Poisson's ratio for the overall soil. Comparing Eq. (15) for the poro-elastic media with Eq. (16) gives,

\[
\lambda = K' + \left( \frac{1}{K'} - \frac{1}{K_s} \right) \left( 1 - \frac{n}{K_s + K_f} \right) + \frac{2}{3} G
\]

(18)

It is evident from Eq. (17) that Poisson's ratio is written as

\[
v = \frac{1}{2} \left[ K' + \left( \frac{1}{K'} - \frac{1}{K_s} \right) \left( 1 - \frac{n}{K_s + K_f} \right) + \frac{1}{3} G \right]
\]

(19)

Furthermore, the equation for P-wave propagation in an isotropic elastic media is

\[
\rho \ddot{u}_i = (\lambda + 2G) \nabla^2 u_i
\]

(20)

Therefore it is obvious that the equation for the poro-elastic media is

\[
\rho \ddot{u}_i = \left[ K' + \left( \frac{1}{K'} - \frac{1}{K_s} \right) \left( 1 - \frac{n}{K_s + K_f} \right) + \frac{4}{3} G \right] \nabla^2 u_i
\]

(21)

which indicates the P-wave velocity for the poro-elastic material \( V_p \) is expressed as

\[
V_p = \frac{1}{\rho} \left[ K' + \left( \frac{1}{K'} - \frac{1}{K_s} \right) \left( 1 - \frac{n}{K_s + K_f} \right) + \frac{4}{3} G \right]
\]

(22)

If \( V_p \) is normalized by the P-wave velocity in pure water, \( V_{p0} = \sqrt{K_f / \rho_f} \), where \( K_f \) stands for the bulk modulus for pure water, then;

\[
V_p / V_{p0} = \frac{K' / \rho}{K_f / \rho} + \left( \frac{1}{K'} - \frac{1}{K_s} \right) \left( 1 - \frac{n}{K_s + K_f} \right) + \frac{4}{3} \frac{G}{K_f}
\]

(23)

In the above equations the shear modulus \( G \) and the bulk modulus of the soil skeleton \( K' \) can be expressed by the shear wave velocity \( V_s \), as;

\[
G = \rho V_s^2
\]

(24)

\[
K' = (2(1 + v') / 3(1 - 2v')) G = [2(1 + v') / 3(1 - 2v')] \rho V_s^2
\]

(25)

where \( v' \) is Poisson's ratio for the soil skeleton in a small strain range corresponding to elastic wave propagation.

A direct relationship between the B-value and \( V_p \) can be derived from Eq. (9) and Eq. (22) by using \( V_s \) and \( v' \) in Eq. (24) and Eq. (25) as parameters;

\[
B = 1 - \frac{2(1 + v')}{3(1 - 2v')} \left( \frac{V_p / V_s^2} {V_s / V_p} \right)^2 - 4 / 3
\]

(26)
Another relationship between the \( B \) value and Poisson's ratio \( v \) for the overall soil can be obtained from Eq. (9) and Eq. (19);

\[
v = \frac{3v' + (1 - 2v')B}{2(3(1 - 2v')B)}
\]  

(27)

In these two relationships the effect of imperfect saturation on \( K \) due to the air bubble mixture is only implicitly involved. Finally, the relationship between Poisson's ratio \( v \) and \( V_s \) derived from Eq. (19) and Eq. (22) results in a well-known equation for isotropic elastic material;

\[
v = \frac{(V_s / V_r)^2 - 2}{2(V_s / V_r)^2 - 1}
\]  

(28)

EVALUATION OF SOIL PARAMETERS

In order to numerically obtain the generic trend of \( P \)-wave velocity \( V_p \) or Poisson's ratio \( v \) with varying \( B \)-value for typical sands and gravelly soils, soil parameters in the above theoretical expressions will be evaluated here for soils with different particle distributions including Masa soil. The first group of soils which were employed in a laboratory test was artificially made by blending Tonegawa sandy gravel, which originally comes from hard rocks such as andesite, sandy rock, chert, etc. Their grain size distributions and physical properties are shown in Fig. 3 and Table 1. The soils were placed in a large scale soil container of 2.0 m in diameter and 1.5 m in height and their S-wave velocities were measured under given overburden stresses. Details of the tests and their results are available in other literature (Kokusho and Yoshida, 1997). Based on a series of such laboratory tests, empirical formulas for S-wave velocity \( V_s \) of these soils were proposed as

\[
V_s = [120 + \{420U_c / (U_c + 1) - 120\} D_s] (\sigma' / \rho_0)^{0.125}
\]  

(29)

in which \( V_s \) is in \( m/s \) and \( U_c \) is the uniformity coefficient; \( \sigma' \) or \( \sigma'' \) are vertical or horizontal effective stress and \( \rho_0 \) is unit stress of 98 kPa. Moreover, \( D_s \) is the relative density defined by \( D_s = (\epsilon_{\text{max}} - \epsilon_{\text{min}}) / (\epsilon_{\text{max}} - \epsilon_{\text{min}}) \), where \( \epsilon_{\text{max}} \) and \( \epsilon_{\text{min}} \) are the maximum and minimum void ratio which are listed in Table 1. The corresponding shear modulus \( G \) (in \( kN/m^2 \)) can be written as

\[
G = \rho V_s^2
\]  

\[
= [(1 - n)p_0 + np_0] \\
\times [120 + \{420U_c / (U_c + 1) - 120\} D_s] (\sigma' / \rho_0)^{0.125}
\]  

(30)

where \( \rho_s \) is the density of the soil particle and \( \rho_s \) and \( \rho_f \) are in \( t/m^3 \). Assuming \( \sigma'' = K_0 \sigma' \) with the earth-pressure coefficient at rest \( K_0 \), the normalized shear modulus \( G^* \) eliminating the influence of effective confining stresses can be formulated as

\[
G^* = G / (K_0^{0.5} \sigma' / \rho_0)^{0.5}
\]  

\[
= [(1 - n)p_0 + np_f] \\
\times [120 + \{420U_c / (U_c + 1) - 120\} D_s] (\sigma' / \rho_0)^{0.25}
\]  

(31)

In order to simplify the condition in the following computation, it is assumed that \( G^* = G \) which indicates \( K_0^{0.5} \sigma' / \rho_0 = 1 \) or \( \sigma'' = \rho_f / K_0^{0.5} \).

The bulk modulus, \( K' \) for a soil skeleton in the drained condition, is calculated by Eq. (25) using Poisson's ratio for the soil skeleton \( v' \) in a small strain range corresponding to elastic wave propagation. It may not be so easy to identify a single representative value for \( v' \) because it will reflect the anisotropy of soil fabrics as well as the properties of soil grains. In Fig. 2, Poisson's ratios calculated from \( P \) and S-wave velocities measured by in-situ wave logging tests in sandy or gravelly soils including Masa soil are summarized. Those values corresponding to locations above the water table may represent Poisson's ratio for a partially or completely drained condition according to the different degree of saturation. Though there exist enormous variations in Poisson's ratio derived from the \( V_p / V_r \)-ratio, not only because of the difference in saturation and other soil conditions but also presumably because of inaccuracy in the wave logging tests, the majority of them are judged to be larger than 0.30. Hence the lower limit is assumed here as 0.30 and may be interpreted as the soil skeleton's Poisson's ratio; \( v' = 0.30 \). However, in view of the lack of a rigorous basis for \( v' = 0.30 \), the effect of a Poisson's ratio other than \( v' = 0.30 \) will also be considered later.

The bulk modulus for solid soil grains \( K_s \) is estimated from the assumption that the S-wave velocity of a sound rock \( V_{sr} \), from which these soil grains of a hard quality are originated, is about 3000 m/s. Therefore, \( K_s \) is written as

\[
K_s = (2(1 + v_s) / 3(1 - 2v_s)) \rho_s V_{sr}^2
\]  

(32)

in which Poisson's ratio for soil grains \( v_s \) is assumed to be 0.30, as is normally assumed in engineering practice, and the density of the grains is taken as 2.6 g/cm³.

Masa soil, which was utilized as a fill material to con-

Table 1. Physical properties of typical sand, gravelly soils and Masa soil

<table>
<thead>
<tr>
<th>Soil material</th>
<th>Max density ( \rho_{max} ) (t/m³)</th>
<th>Min density ( \rho_{min} ) (t/m³)</th>
<th>Unif. coeff. ( U_c )</th>
<th>Mean size ( D_{50} ) (mm)</th>
<th>Max. void ratio: ( \epsilon_{max} )</th>
<th>Min. void ratio: ( \epsilon_{min} )</th>
<th>Spec. density ( \rho_s ) (t/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TS Sand</td>
<td>1.705</td>
<td>1.374</td>
<td>1.95</td>
<td>0.34</td>
<td>0.966</td>
<td>0.584</td>
<td>2.701</td>
</tr>
<tr>
<td>G25 Gravel</td>
<td>2.004</td>
<td>1.706</td>
<td>5.65</td>
<td>1.13</td>
<td>0.567</td>
<td>0.334</td>
<td>2.674</td>
</tr>
<tr>
<td>G50 Gravel</td>
<td>2.151</td>
<td>1.867</td>
<td>11.3</td>
<td>2.28</td>
<td>0.429</td>
<td>0.240</td>
<td>2.668</td>
</tr>
<tr>
<td>G75 Gravel</td>
<td>2.240</td>
<td>1.960</td>
<td>31.1</td>
<td>7.30</td>
<td>0.354</td>
<td>0.184</td>
<td>2.653</td>
</tr>
<tr>
<td>Masa soil</td>
<td>2.038</td>
<td>1.696</td>
<td>46.4</td>
<td>1.25</td>
<td>0.538</td>
<td>0.280</td>
<td>2.608</td>
</tr>
</tbody>
</table>
struct man-made islands and reclaimed lands in the Kobe area, mostly consists of decomposed granite transported from the nearby Rokko Mountains. It contains lots of gravels as well as silty fines. Typical gradation curves are shown in Fig. 3, and its physical properties are also listed in Table 1. The grading curve is rather similar to the above-mentioned G50 material except for a large inclusion of fines. Because it is a weathered rock, the sand or gravel grains comprising Masa are unlikely to be as stiff as normal sandy gravel of a fluvial origin. Therefore it may well be expected that the undrained properties addressed in this research are somewhat different from the typical sand or gravelly soils discussed above.

A similar soil container test as previously mentioned was carried out recently for Masa soil from the Kobe area (Kokusho et al., 1999). In this test, Masa soil was placed in a container with different initial densities, and the S-wave velocity $V_s$ vertically propagating in it was measured under varying overburden stress $\sigma'$. This test yielded an empirical equation to estimate $V_s$ in m/s as a function of $D$, and $\sigma'$, as follows;

$$V_s = 112(D_s + 1.0)(\sigma'_f/p_0)^{0.25}$$ (33)

Based on Eq. (33), the shear modulus $G$ is formulated as;

$$G = \rho V_s^2 = (1 - n)\rho_s + n\rho_f)[112(D_s + 1.0)]^2(\sigma'_f/p_0)^{0.5}$$ (34)

where $G$ is in kN/m$^2$ and $\rho_s$ and $\rho_f$ are in t/m$^3$. The bulk modulus $K'$ for the soil skeleton is calculated in the same manner as before with $\nu' = 0.30$. The bulk modulus $K_f$ for Masa soil is evaluated by assuming the S-wave velocity of weathered granite is lower than that of sound rock. It is assumed here as $V_s = 1500$ m/s and also shifted by a factor of $\sqrt{2}$ or $1/\sqrt{2}$ to $V_s = 2120$ or $V_s = 1060$ m/s. Considering the degree of weathering of soil grains, so as to examine its effect on the correlations.

**CORRELATION CHARTS**

Ishihara (1971) made a pioneering study to establish the theoretical relationship between the P-wave velocity and Poisson's ratio for fully saturated soils based on Biot's poro-elasticity theory. In order to check the reliability of the equations in the present research in comparison with the results produced by Ishihara, the normalized $V_p$ in Eq. (23) and Poisson's ratio in Eq. (19) for a fully saturated condition are actually computed by substituting $K_f^*$ for $K_f$. Here, it is assumed that $G/p_0 = 50$ and $K_f/p_0 = 5 	imes 10^5$, as in the previous research. The bulk modulus of pure water, $K_f^*$ is taken as 2220 Mpa based on a science table. This corresponds to $V_p = 1500$ m/s. The results are shown in Fig. 4, in which the variations in the normalized $V_p$, $V_p/V_p$, and Poisson's ratio, $\nu$, are plotted versus soil porosity, $n$. Almost complete agreement is obviously seen in the chart between this and the previous study for perfectly saturated condition.

The $B$-value for perfectly saturated soil is then computed by substituting $K_f^*$ for $K_f$ in Eq. (9) for the four types of soil tabulated in Table 1. Open symbols plotted in Fig. 5 on the porosity versus $B$-value chart imply that, for a higher relative density, the $B$-value attained even in a perfectly saturated soil is lower than 0.95, which is the value normally accepted as a threshold for undrained soil.
tests. It can approach unity only when the relative density is very low. This trend does not differ greatly among the four types of soil. In Fig. 5, the $B$-value for perfectly saturated Masa soil is also plotted for four relative densities and three assumed $V_{pf}$. It is understood that the trend of Masa soil is rather different from the fluvial sand and gravelly soils, and, even with $D_r=100\%$, the $B$-value can clear the value of 0.95.

For imperfectly saturated conditions with some bubbles of air in the soil void, the $B$-value is lowered from that of the perfectly saturated condition. In Eq. (9), the $B$-value below the above-mentioned maximum value is assigned to evaluate a corresponding fluid bulk modulus $K_f$ reflecting the compressibility of air-bubbles. The corresponding normalized P-wave velocity, $V_p/V_{pf}$, is computed in Eq. (23) for four different relative density. Poisson's ratio $\nu$ for the overall soil is also computed for the same cases by using Eq. (19).

The correlation between $B$-value and $V_p/V_{pf}$ or $\nu$ computed for TS sand is shown in Fig. 6 for three different values of the soil skeleton's Poisson's ratio $\nu'$. The change in the P-wave velocity is more pronounced between $B=1.0$ and 0.8 than $B<0.8$. In contrast, Poisson's ratio, $\nu$, changes almost linearly with the $B$-value. It may be said that the effect of the soil skeleton's Poisson's ratio, $\nu'$, on the evaluation of the $B$-value will be rather significant if it moves from 0.30 to 0.20 or 0.10.

In Fig. 7, the same correlations are shown for the four different soils with different particle gradation for $B=1.0$ to 0.8 assuming $\nu'=0.30$. It is obvious that the P-wave velocity $V_p$ for an imperfectly saturated soil dramatically decreases with a small decrease in the $B$-value, irrespective of soil type, and the decreasing rate is more pronounced for a lower relative density. For $D_r=50\%$, $V_p$ at $B=0.80$ decreases to 30–50% of the P-wave velocity in a pure water, $V_{pf}$. $V_p$ for a perfectly saturated soil clear-

Fig. 6. $B$-value versus $V_p/V_{pf}$ and $\nu$ relationship for TS sand of $D_r=50\%$ with three different soil skeleton's Poisson's ratios, $\nu'$

Fig. 7. $B$-value versus $V_p/V_{pf}$ and $\nu$ relationship for four types of soils with four different relative densities, $D_r$.
ly exceeds $V_{ps}$. The excess rate becomes higher for higher relative densities and for soils with a higher gravel content or higher uniformity coefficient, $U_s$. In contrast, the Poisson’s ratio for the overall soil, $v$, versus B-value relationship is completely independent of the difference in density and type of soil, as can be understood from Eq. (27), and $v$ gradually decreases almost linearly with the decrease in B-value. It should be noted however that the maximum Poisson’s ratio corresponding to a perfectly saturated soil is a little lower than 0.50, particularly for denser soils. Figure 8 indicates the same correlations of Masa soil computed for four different values of relative density. Here $V_{s0}$ is assumed to be 1500 m/s. Compared with the fluvial soils, the difference in relative density in Masa soil tends to create smaller differences in the correlations.

**INTERPRETATION OF FIELD DATA**

In order to assess the applicability of the theory to the field data plotted in Fig. 2, correlations between $V_p/V_{ps}$ and $v$ are computed for the four types of soils as well as Masa soil with four steps of relative density, $D_r=25$, 50, 75 and 100%. Instead of showing so many individual curves for the different soil types and densities, two pairs of curves corresponding to the outer limits either for sand and gravelly soils or for the Masa soil are drawn with the solid or dashed curve on the chart in Fig. 2 and compared with the plots based on in situ wave logging measurements. The field data seems to be mostly located between the pair of solid curves. Although the coincidence with corresponding field data for the Masa soil seems poorer, the dashed curves are well within the entire plots and approximate the global trend of the Masa soil.

In engineering practice, if measured $V_p$ and $V_s$ are available in the field, a specific B-value will be directly calculated by virtue of Eq. (26) if $v'$ is properly estimated. For example, in the fill layer of Masa in the Port Island vertical array site mentioned previously, $V_p$ and $V_s$ between GL-5m to -13m before the earthquake were 780 m/s and 210 m/s, respectively. This gives an in situ B-value of 0.83 if $v'=0.3$ is assumed as indicated by the thick arrow mark in Fig. 9. In the same manner, B values are calculated from Eq. (26) with the assumption of $v'=0.3$ also for all the data shown in Fig. 2 apart from the those two with Poisson’s ratio $v<0.30$. In Fig. 9 the results are plotted versus the normalized P-wave velocity. The same symbols are employed here as in Fig. 2 for the water table conditions and soil types. It is clear from this figure that, if $V_p/V_{ps}>0.9$ then $B$ is mostly equal to or larger than the normally used threshold value of 0.95 or at least larger than 0.90. The three encircled plots well below that value correspond to very stiff soils with $V_s \geq 400$ m/s and may be ignored from the viewpoint of soil liquefaction.

There exist quite a few plots with B-values smaller than 0.90 for $V_p/V_{ps}<0.8$, including that of the previously mentioned Port Island data pointed by the arrow mark. It should be noted that even soils located apparently above the water table give B-values as high as 0.95 and no clear boundary can be drawn between plots above and below the water table between $B=0.7$ and 0.95, as indicated in the figure. One explanation for this may be the difficulty in identifying an accurate level of water table in PS-logging data. However, this may also indicate the possibility that some soils above the water table still maintain a rather high degree of saturation corresponding to such high B-values.

**CONCLUSIONS**

Theoretical formulations were first made in order to correlate the B-value with P-wave velocity, $V_p$, and Poisson’s ratio, $v$, for imperfectly saturated soils based on Biot’s theory, considering the decrease in bulk modulus due to the inclusion of air bubbles. Computations have been made by the formulas employing typical soil parameters for the sand and gravelly soils including Masa soil to obtain generic correlations. Major findings thus obtained are as follows.

1) Field data derived from PS-logging tests in sandy
and gravelly soils indicate that $V_p$ in soils below the water table frequently exhibits a much lower value than that for pure water. A $V_p$ versus Poisson’s ratio relationship theoretically derived seems to be consistent with the field data.

2) Charts correlating the $B$-value with $V_p$ or $v$ have been made for typical sand and gravelly soils as well as Masa soil from Kobe Port Island.

3) The charts indicate $V_p$ is very dependent on a small change in $B$-value between $B = 1.0$ and 0.8. The difference in relative density is more influential in these correlations than that in soil types with different particle gradation.

4) In the field data, if $V_p$ is larger than 90% of the $P$-wave velocity of water, a $B$-value of about 0.95 or larger can usually be expected.

5) The soil skeleton’s Poisson’s ratio, $v’$, has a significant effect in the evaluation, although $v’ = 0.3$ seems to be justified based on current field data.

Thus, it may be stated conclusively that the $P$-wave velocity can serve as a convenient index to estimate the degree of imperfect saturation in situ soils through the $B$-value. In undrained soil tests in the laboratory, the $B$-value is likely to reflect not only the compressibility of the pore-fluid but also the system compliance of the test device, including the rubber membrane penetration effect. However, no matter how different the in situ and laboratory mechanisms in $B$-value reduction are, it is quite clear that the only requirement is to realize the same $B$-value in the laboratory as in the field.

REFERENCES


