ULTIMATE BEARING CAPACITY AND SETTLEMENT OF FOOTINGS ON REINFORCED SANDY GROUND

CHING-CHUAN HUANG and LI-LING HONG

ABSTRACT

The applicability of a method for predicting 'bearing capacity increase' in reinforced sandy ground was examined using tests performed under various test conditions. It was found that the present method predicted, with reasonable accuracy, the bearing capacity increase in sandy ground, reinforced with stiff reinforcement. This method may not be applicable for sandy ground reinforced with extensible reinforcement due to the unsuccessful formation of a semi-rigid zone under the footing. An investigation into the settlement of a footing on reinforced sandy ground, at ultimate footing load condition, suggested that the settlement of footing for reaching peak footing load may be correlated to the 'deep-footing' and the 'wide-slab' mechanisms. That is, the ultimate settlement ratio between reinforced and unreinforced model sandy ground, \( SR_r \), may be linearly correlated to \( BCR_d \) and \( BCR_s \), which represent 'deep-footing' and 'wide-slab' effects, respectively, on the ultimate bearing capacity increase in reinforced sandy ground.

Key words: bearing capacity, foundation, reinforcement, sand, settlement (IGC: E/3, E/2, E/0)

INTRODUCTION

A pioneer study on the effect of placing reinforcing layers under a footing to increase the bearing capacity of sandy ground underlain by soft clay was conducted by Yamanouchi (1970). He reported that placing low-stiffness polymer net under the footing is effective for increasing the bearing capacity of the soft ground. This concept, which is widely accepted in the studies of bearing capacity characteristics of reinforced ground, i.e., the so-called "earth slab," was first proposed by Binquet and Lee (1975a, b). In their study, a method based on stress distribution in the semi-finite elastic foundation with assumed failure surfaces, was proposed for estimating the ultimate bearing capacity of sandy ground reinforced with horizontal metal strips. Unfortunately, this model was not widely adopted because of some inherent disadvantages, e.g., failure to explain the mechanism of bearing capacity increase in ground reinforced with short reinforcement similar in length to the width of footing, as found by Huang and Tatsuoka (1988, 1990), and Huang and Menq (1997). Research using model tests, large-scale tests, and centrifuge tests, and focused on raising the bearing capacity of reinforced uniform sandy ground, has been conducted by Fragaszy and Lawton (1984); Guido et al. (1986); Akinmusuru and Akinbolade (1981); Huang and Tatsuoka (1988, 1990); Khing et al. (1992); Takemura et al. (1992) and Ju et al. (1996). Tests have also been performed to investigate bearing capacity characteristics of reinforced ground subjected to inclined or eccentric loading (Abdel-Baki et al., 1993; Manjunath and Dewaikar, 1996). This research has indicated that placing reinforcement in sandy ground under the footing is an effective way of increasing the ultimate bearing capacity. Schlosser et al. (1983) first suggested a failure mechanism, in horizontally reinforced ground loaded by a footing, based on the concept of 'deep-footing' and 'wide-slab', as schematically shown in Fig. 1. Huang and Tatsuoka (1988, 1990) performed a series of tests using a reduced-scale model and incorporating a photogrammetric technique. They concluded that both 'deep-footing' and 'wide-slab' mechanisms dominated the ultimate bearing capacity of reinforced sandy ground. They showed that by placing reinforcing layers with high tensile rigidity horizontally beneath the footing, the reinforced zone directly beneath the footing (A-zone, Fig. 1) behaved as a semi-rigid block due to the restraining effect on lateral deformation of sand. When the compressive strength of the semi-rigid block was larger than the ultimate bearing capacity of the unreinforced ground beneath the semi-rigid block (B-zone, Fig. 1), the ultimate bearing capacity of the reinforced ground was controlled by the failure of the B-zone. This is the 'deep-footing' mechanism. The 'wide-slab' mechanism is deemed to be an extension of 'deep-footing' when the semi-rigid reinforced block extends beyond the width of the surface footing. 'Deep-footing' may be applicable to footings reinforced with short reinforcement, i.e., the footing width (B) is equal
to the reinforcement length (L), or to that reinforced with long reinforcement (L > B). The 'wide-slab' mechanism may only be applicable for a footing reinforced with long reinforcement, (L > B). These two mechanisms have been investigated experimentally and analytically by Huang and Tatsuoka (1988, 1990). The mechanism of 'deep-footing' may be easily identified by comparing the results obtained in surface footing tests on reinforced ground and in deep footing tests on unreinforced ground (Huang and Tatsuoka, 1988, 1990), under the condition of \( D_a \) (the depth of reinforced zone)=Df (the depth of rigid footing). This mechanism has also been observed in centrifuge tests using a radiograph technique (Takemura et al., 1992). Verification of the 'wide-slab' mechanism, using loading tests similar to those used for 'deep-footing', is difficult in practical terms because it is costly and time consuming. The existence of a 'wide-slab' directly under the footing, which spreads the footing load into a deeper and wider area in the unreinforced zone, has been observed in model tests incorporated with a photogrammetric method (e.g., Yamanouchi, 1970; Huang and Tatsuoka, 1990). One of the possible ways to evaluate the 'wide-slab' effect, based on the limited test results obtained so far, is therefore, to back-calculate the wide-slab, angle \( \alpha \) (Fig. 1), based on the experimentally verified 'deep-footing' effect. This approach has been used by Huang and Menq (1997), and also forms the basis of the present study.

Huang and Menq (1997) analyzed a total of 95 tests performed by the aforementioned researchers. They found that deep-footing and wide-slab effects only exist for some of the tests (Test No. 1-16, in Table 2) in which adequate spacing and proper interface roughness for the reinforcement were employed. They also found that the tests performed by Binquet and Lee (1975a) and by Fragaszy and Lawton (1984), in which reinforcing strips with smooth soil-reinforcement interface were used, all failed to activate the deep-footing and wide-slab effects. This was despite the small spacing \( (d_i/B=0.3) \) of reinforcement employed. The criteria for judging the 'deep-footing' and 'wide-slab' mechanisms will be discussed later. Huang and Menq (1997) proposed the following equation for estimating the wide-slab angle of \( \alpha \) (Fig. 1) based on a linear regression of the test results in which deep-footing and wide-slab were successfully activated (Test No. 1-16, in Table 2):

\[
\tan \alpha = 0.68 - 2.071 \cdot \frac{d_i}{B} + 0.743 \cdot CR + 0.03 \cdot \frac{L}{B}
\]

(1)

in which

\( \alpha \): angle of wide-slab (or load-spreading angle under the footing)
\( d_i \): uniform spacing of reinforcing layers in vertical direction under the footing
\( CR \): covering ratio of reinforcement strips (Fig. 2)
\( L \): the length of reinforcement
\( B \): width of strip or rectangular footing

Eq. (1) is valid for the following conditions:

\[
0 \leq \tan \alpha \leq 1
\]

\[
0.25 \leq \frac{d_i}{B} \leq 0.5
\]

\[
0.02 \leq CR \leq 1.0
\]

\[
1 \leq \frac{L}{B} < 10
\]

\[
1 \leq n \leq 5
\]

\[
0.3 \leq \frac{D_B}{B} < 2.5
\]

in which,

\( n \): total number of reinforcing layers
$D_R$: the depth of reinforced zone.

The following five points must be mentioned in dealing with Eq. (1):

1. The physical significance of $\alpha$ in the present study is different from the load spreading angle conventionally used in estimating stress increment in elastic foundation. In the present study, $\alpha$ signifies the extent of the restrained area under the footing at the ultimate condition. While the load spreading angle, as conventionally used in foundation engineering, reflects the load-spreading behavior of homogenous ground under elastic states before the ultimate condition is attained.

2. Equation (1) is based on tests in medium to dense sand with the internal friction angle ($\phi$), as measured in triaxial tests, ranging between 37° and 45°. For other soil conditions, Eq. (1) may require further justification.

3. Equation (1) is based on tests using metal strips or geogrids as reinforcement. Huang and Tatsuoka (1990) showed that the tensile strain of metal strips, at the ultimate footing load, ranged between $0.04 \times 10^{-2}$ to $0.8 \times 10^{-2}$ under a planar equivalent tensile stiffness which ranged between $5 \times 10^2$ to $5 \times 10^3$ kN/m. Assuming that a similar stress condition occurred for the planar geogrid sheets with tensile stiffness, ranging between 300–600 kN/m as shown by McGown et al. (1985) and Takemura et al. (1992), then tensile strains ranging from $0.3 \times 10^{-2}$ to $1.5 \times 10^{-2}$ should be anticipated for the geogrids under otherwise similar test conditions. For relatively extensible reinforcement, e.g., non-woven geotextiles or non-axially-stretched plastic nets, which for yielding and breaking point strains may be as large as 40% to 1500%, respectively (Yamanouchi, 1970), the applicability of the model proposed by Huang and Menq (1997) has yet to be validated.

4. Schlosser et al. (1983) analyzed a number of tests and claimed that the depth of the uppermost layer of reinforcement $u$ has a significant influence on the wide-slab angle $\alpha$ which represents the extent of the earth-slab below the footing. In the case of single-layer reinforcement, then $u = d$. In the case of multiple-layer reinforcement, the effect of $u$ is equal to that of $d$, if the reinforcement layers are uniformly spaced in the vertical direction. To the author's best understanding, in all the multiple-layer reinforced tests found in the literature, the effect of $u$ has always been confused with the concurrent effect of 'deep-footing' because experiments have rarely been performed by only changing $u$ while $D_R$ remains a constant. E.g., Binquet and Lee, 1975a; Akinmusuru and Akinbolade, 1981; Guido et al., 1986; Yetimoglu et al., 1994). Huang and Menq (1997) investigated a series of tests, in which the effect of $u$ was the focus, and pointed out that only one very limited test had a positive value of $\alpha$. This also suggested the temporary use of the uniform vertical spacing $d$, instead of $u$.

5. It is noted in Eq. (1) that the parameter $d/B$ has a major effect on the value of $\alpha$. This supports quantitatively, the conclusions on the effect of $u/B$ reached by various researchers (e.g., Binquet and Lee, 1975a; Akinmusuru and Akinbolade, 1981; Schlosser et al., 1983; and Guido et al., 1986). This effect can be detected from the largest value of the constants for the parameter $d/B$, compared with those for other parameters, such as: $CR$ and $L$.

In this report the applicability and accuracy of the method proposed by Huang and Menq was examined by using two series of tests performed by Yetimoglu et al. (1994), and Ju et al. (1996). In addition, another important aspect of the reinforced sandy ground, i.e., the settlement of footing under ultimate loading, was also investigated. It is noted that the problem of settlement of footing under working load may be encountered occasionally in practice. The author believes, however, that the ductility of the reinforced soil structures may be required in many geotechnical applications. A design methodology which takes into account the ductility of soil structures, is founded on the knowledge of the mechanism controlling the failure (or the ultimate state) of the reinforced soil structures (e.g., Tatsuoka et al., 1998). Discussion of the settlement and bearing capacity of footings for the pre-failure condition is beyond the scope of the present study.

**ULTIMATE BEARING CAPACITY OF REINFORCED SANDY GROUND**

The ultimate bearing capacity of a reinforced sandy ground, $q_{ult}(reinforced)$, suggested by Huang and Menq (1997) is expressed as (Fig. 1):

$$q_{ult}(reinforced) = \eta \left( B + \Delta B \right) \cdot \gamma \cdot N_p + \gamma \cdot D_R \cdot N_s$$

$$= q_{ult}(D_R > 0) + q_{ult}(slab) \quad (2)$$

in which,

$$q_{ult}(D_R > 0) = q_{ult}(D_R = 0) + \gamma \cdot D_R \cdot N_s \quad (3)$$

$$q_{ult}(D_R = 0) = \eta \cdot B \cdot \gamma \cdot N_p \quad (4)$$

$$q_{ult}(slab) = \eta \cdot \Delta B \cdot \gamma \cdot N_s \quad (5)$$

$q_{ult}(D_R > 0)$, $q_{ult}(slab)$: the bearing capacity components generated by deep footing and wide-slab mechanisms, respectively.

$q_{ult}(D_R = 0)$: ultimate bearing capacity of surface footing on unreinforced ground

$N_p$, $N_s$: the bearing capacity factors for the self-weight and the overburden stress at the level of footing base, respectively.

$D_R$, $B$, and $\Delta B$: the depth of reinforcement, the width of footing, and the increase of the width of footing due to the wide-slab effect, respectively (Fig. 1).

$\eta$: shape factor ($\eta = 0.5$ for strip footings; $\eta = 0.5 - 1.0 \cdot (B/L)$ for rectangu-
lar footings as suggested by Terzaghi, 1943, L: the length of footing)

\[ \gamma: \text{ unit weight of soil} \]

The bearing capacity ratio for reinforced ground, \( BCR_{R} \), is:

\[
BCR_{R} = \frac{q_{u}(\text{reinforced})}{q_{u}(D_{y}=0)} = \frac{q_{u}(D_{y}>0)}{q_{u}(D_{y}=0)} + \frac{q_{u}(\text{slab})}{q_{u}(D_{y}=0)} = BCR_{D} + BCR_{S}\]

(6)

in which,

\[ BCR_{D}: \text{ a component of bearing capacity ratio generated by the deep-footing effect} \]

\[ BCR_{S}: \text{ a component of bearing capacity ratio generated by the wide-slab effect} \]

It is well known that the reacting pressure on the footing base is non-uniform. However, from the point of views of limit equilibrium analysis (e.g., Terzaghi, 1943) and limit analysis (e.g., Chen, 1975), and also from the point of view of practical engineering, the bearing capacity of footings can be estimated based on the assumption of uniform pressure distribution on the footing base, as expressed by Eq. (2). It is also noted that the ‘Bearing Capacity Ratio (BCR)’ and ‘Settlement Ratio (SR)’ used throughout the present study, help to minimize the influence on the bearing capacity and settlement that are possibly induced by different test conditions, such as the size of footing, the boundary conditions, and the strength of sands, etc.

From Eqs. (3), (4) and (6):

\[
BCR_{D} = 1 + \frac{1}{\eta} \frac{D_{e}}{B} \frac{N_{e}}{N_{r}}\]

(7)

From Eqs. (4), (5) and (6):

\[
BCR_{S} = \frac{\Delta B}{B} = 2 \cdot \frac{D_{e}}{B} \cdot \tan \alpha = 2 \cdot \frac{d_{1}}{B} \cdot \tan \alpha\]

(8)

Huang and Menq (1997) showed that the use of the following two bearing capacity factors, as suggested by Vescic (1973), yielded good comparisons between the theoretical values of \( BCR_{D} \) and the experimental ones obtained by Huang and Tatsuoka (1990):

\[
N_{e} = e^{-\tan \phi} \cdot \tan \left( \frac{\pi}{4} + \frac{\phi}{2} \right)\]

(9)

\[
N_{r} = 2(N_{e} + 1) \cdot \tan \phi\]

(10)

In the study by Huang and Menq (1997), the values of \( BCR_{D} \), for a total of 95 tests, were first calculated using Eq. (7). The angle \( \alpha \), was then calculated based on Eqs. (6) and (8) by substituting \( BCR_{R} \) with the measured value of bearing capacity ratio, \( BCR_{m} \), in Eq. (6). For the tests with \( BCR_{m} > BCR_{D} \), calculated values of \( \alpha \) were larger than zero. Therefore, \( \alpha > 0 \) was deemed as a criterion for judging successful development of ‘deep-footing’ and ‘wide-slab’ under the footing. In the present study, all tests listed in Table 2 failed by the ‘deep-footing’ mechanism, except some unreinforced cases, and some other cases to be discussed later. All tests in Table 2 failed by the ‘wide-slab’ mechanism, except the tests with \( L = B \), unreinforced tests, and the tests to be discussed. The calculated values of \( \alpha \) are listed in column (13) in Table 2. It should be noted that the term ‘deep-footing’, used in this paper, is for the surface footing with a depth of reinforced zone (\( D_{e} \)), or for a deep footing with a depth of embedment (\( D_{f} \)), not larger than 2.5 times the footing width (\( B \)). This is different from the conventional definition of a ‘deep foundation’.

To investigate the applicability of the method discussed above, two series of tests performed by Yetimoglu et al. (1994), and Ju et al. (1996) were analyzed. The test conditions for these studies are summarized in Table 1. In the following comparisons, the calculated values of \( BCR_{R} \) are based on Eqs. (1), (6), (7) and (8). They are also summarized in column (14) in Table 2.

**VERIFICATION OF THE ANALYTICAL MODEL**

Yetimoglu et al. (1994) performed a series of constant rate of loading tests by using a smooth, rectangular footing, 101.5 mm-wide, 127 mm-long and 12.5 mm thick, loaded on a 0.7 m-wide, 0.7 m-long, and 1.0 m-deep ground model. A dry, uniform quartz river sand was used as the test medium, and a uniaxial polypropylene geogrid, with aperture size 80 × 140 mm, was used as reinforcement. The test conditions are summarized in Table 1. Two groups of tests, Group-1 and Group-3, in Yetimoglu’s report, were analyzed using the analytical model presented in this paper. The tests in “Group 2” in Yetimoglu’s report were excluded from this comparison because the vertical spacings of reinforcement were varied in that group of tests. Figure 3 shows comparisons between \( BCR_{R} \) and \( BCR_{m} \) for the two groups of tests. The

| Table 1. Test conditions for tests investigated in present study |
|-----------------|---------|----------------|----------------|--------|-------------------|----------------|----------------|
| Series of tests | Notation | Type of footing | Type of reinforcement | \( \phi_{bi} \) (°) | \( \gamma \) (Relative density) | Ultimate bearing capacity for unreinforced surface footing | Total number of tests used |
| Yetimoglu et al. (1994) | “YT” | Rectangular | Uniaxially-drawn PP geogrid | 40 | 17.2 kN/m² (70%-73%) | 316 kPa | 8 |
| | | B=101.5 mm | L=127 mm | | | | |
| Ju et al. (1996) | “JU” | Strip | Not-drawn PE plastic net | 45 | 15.5 kN/m² (75%) | 161.7 kPa | 10 |
| | | B=100 mm | | | | | | 1000 |
Table 2. Test results used in present study

<table>
<thead>
<tr>
<th>Number (1)</th>
<th>Series of test (2)</th>
<th>(d_f/B) (3)</th>
<th>CR (5)</th>
<th>(L/B) (6)</th>
<th>(n) (7)</th>
<th>(\phi) (8)</th>
<th>(D_f/B) (9)</th>
<th>(S_f/B) (10)</th>
<th>(SR) measured (11)</th>
<th>BCRm (12)</th>
<th>Calculated tan (\alpha) (13)</th>
<th>Calculated BCRr (14)</th>
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<td>0.201</td>
<td>2.01</td>
<td>4.00</td>
<td>0.213</td>
<td>4.33</td>
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<td>22</td>
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<td>0.3750</td>
<td>0.100</td>
<td>10.75</td>
<td>6</td>
<td>40.0</td>
<td>2.25</td>
<td>0.192</td>
<td>1.92</td>
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<td>4.99</td>
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<tr>
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<td>4.00</td>
<td>6</td>
<td>40.0</td>
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<td>—</td>
<td>—</td>
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<td>—</td>
<td>—</td>
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<td>—</td>
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<td>4.62</td>
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<tr>
<td>26</td>
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<td>0.100</td>
<td>10.75</td>
<td>6</td>
<td>40.0</td>
<td>2.25</td>
<td>—</td>
<td>—</td>
<td>4.03</td>
<td>0.087</td>
<td>4.99</td>
</tr>
</tbody>
</table>

★ Test Nos. 1-16 were used by Huang & Menq (1997) for data regression.
★ Test Nos. 17-26 were used by Huang & Menq (1997) for verifying the proposed method. These tests were performed by Kihng et al. (1992) and were denoted as "K".
★ The tests denoted by "YT" were the tests performed by Yetimoglu et al. (1994); "JU" is for the tests by Ju et al. (1996); "HC" is for the tests by Huang and Tatsuoka (1990).
★ "—" in columns (10) and (11) means "not available"; in columns (13) and (14) means "the present method not applicable".

Accuracy of the method described herein is evident. Ju et al. (1996) performed a series of tests on the bearing capacity of reinforced sandy ground using low strength, non-axially-stretched polyethylene (PE) net, with 8 x 6 mm apertures, as reinforcement. A 100 mm-wide strip footing was loaded under constant displacement rate on 1200 mm-long, 300 mm-wide, and 450 mm-deep sandy ground. Test conditions are also summarized in Table 1. Figure 4 shows that the predicted values of BCRR were comparable with the BCRR for the tests in which multilayer reinforcement (n>1) was used. Figure 5 shows a comparison of BCRR and BCRR for the tests performed by Ju et al. (1996) in which the effect of \(d_f/B\) for single-layer reinforcement (n=1) condition was investigated. The trend of values of BCRm and BCRR differs considerably when \(d_f/B\geq0.7\). The values of BCRR showed that the effect of reinforcing vanishes at \(d_f/B=0.65\). This coincides with the trends indicated by various research-
to the reinforcement. The stress in the reinforcement may be a function of footing pressure and total number reinforcement layers in an effectively reinforced zone. Therefore, the applicability of the present method to some of the reinforced model ground does not ensure the applicability of the present method to the same ground loaded with higher footing pressure. The effect of stress level (or the effect of strain level in the reinforcement) should be considered for each case.

SETTLEMENT AT ULTIMATE FOOTING LOAD

Figure 6(a) shows the relationships between the settlement ratio, $SR_f$, and $BCR_m$ for the tests summarized in Table 2. $SR_f$ is defined as follows:

$$SR_f = \frac{S_f(\text{reinforced})}{S_f(\text{unreinforced})}$$

(11)

where, $S_f(\text{reinforced})$, $S_f(\text{unreinforced})$, are the vertical settlements at the ultimate loading condition for reinforced and un-reinforced ground, respectively.

The relationships between $SR_f$ and $BCR_m$ obtained in five series of tests were investigated in the present study. It was found that a linear relationship between $SR_f$ and $BCR_m$ exists in each series of tests. These well-defined trends, for various groups of tests, may reflect the fundamental differences between these tests, e.g., the types of reinforcement, the relative density of sand, the properties of soil-reinforcement interface, and the arrangement of reinforcement, etc. It is interesting to note that the tests performed by Ju et al. (1996) had distinctively higher values of $SR_f$ compared with other series of tests. This may be attributable to the low-stiffness reinforcement used. The tests performed by Ju et al. (1996) provided valuable information on the settlement of a footing situated on sandy ground reinforced with relatively extensible reinforcement. It was also found that the $SR_f$ vs. $BCR_m$ relationships for the tests, conducted by Huang and Tatsuoka (1990), Khing et al. (1992), and Yetimoglu et al. (1994), were similar to those performed in the centrifuge by Takemura et al. (1992). This similarity suggested that model tests can also provide reasonable values of $SR_f$ for ground reinforced with stiff reinforcement. Comparing the four lines provided by the different series of tests shown in Fig. 6(a), the characteristics of $SR_f$ vs. $BCR_m$ relationships appear to be dominated by two factors, i.e., the tensile stiffness of reinforcing material and the soil-reinforcement interface compatibility. Other factors (the relative density of sand, the roughness of footing base, and the shape and size of footing), seemed to have a minor influence. This concept was inferred from the distinctly higher values of $SR_f$ provided by Ju et al. (1996), and the slightly higher values of $SR_f$ provided by Huang and Tatsuoka (1990). In the tests by Huang and Tatsuoka (1990), metal strips glued with sand particles were employed. This may result in a lower soil-strap interface friction angle, when compared with that for a soil-geogrid interface. It has been shown, by using direct shear
tests, that the friction angles between dense sand and metal strips glued with sand can be smaller than those between dense sand and geogrid (Jewell and Wroth, 1987, Cancelli et al. 1992).

RELATIONSHIPS BETWEEN $SR_f$ AND VARIOUS PARAMETERS

Figure 6(a) shows the $SR_f$ vs. $BCR_m$ relationships obtained in five series of tests. The tests using relatively stiff reinforcement show similar trends in $SR_f$ vs. $BCR_m$ relationships. A $SR_f$ vs. $BCR_m$ relationship for all data shown in Fig. 6(a) except for those by Ju et al. (1996), is expressed by the following equation based on the constraint that when $BCR_m$ is equal to 1, $SR_f=1$.

\[
(SR_f)_{\text{fitted}} = 1 + b \cdot (BCR_m - 1)
\]

(12)

A linear regression analysis for all the test data in Fig. 6(a) except those by Ju et al. (1996) gives $b=0.358$ and the conditional standard deviation of 0.203. A total of 29 sample points together with the regression line and the 90% confidence limits are shown in Fig. 6(b). Theoretically, the lower limit of the 90% confidence interval should be cut at $SR_f=1$.

Figure 7 re-plotted the data shown in Fig. 6(a) based on the concept of the ‘deep-footing’ mechanism. Four data points for unreinforced deep footings, performed by Huang and Tatsuoka (1990), were included in this figure for comparison purposes. The line for $\alpha=0^\circ$ (deep footing) represented a component of the vertical settlement in the reinforced ground induced by the ‘deep-footing’ effect. The increase of settlement for a deep footing in sandy ground, may be a result of the following factors: the increase of confining pressure on the soil elements, the increase in the length of potential failure line, and so on. It is surprising to note that no data for reinforced ground was below the line for the $\alpha=0^\circ$ (deep footing). This strongly supports the assumption that the ‘deep footing’ mechanism accounts for a portion of the settlement of a footing situated on reinforced ground.

As seen in Eqs. (7) and (8), four parameters, namely, $\eta$, $D_6/B$, $N_f/N_t$, and $\tan \alpha$, are combined to give the value of $BCR_m$ and then thus affect the value of $SR_f$. These parameters vs. ($SR_f$)$_{\text{measured}}$ using the above 29 tests are shown in Figs. 8(a)~(c), except for the shape factor $\eta$, due to its low variation compared with the other parameters and its expected poor correlation to $SR_f$. Three different kinds of $BCR$ vs. ($SR_f$)$_{\text{measured}}$ are also plotted in Figs. 8(d)~(f) for comparison. In view of Figs. 8(a)~(c), there is no dominant parameter in predicting $SR_f$ individually. When they are combined together, however, a linearly increasing trend of ($SR_f$)$_{\text{measured}}$ on $BCR_m$ could be observed in Fig. 8(f). The other way of inspecting the degree of linear dependence between ($SR_f$)$_{\text{measured}}$ and these factors is to examine their correlation coefficients. Since the largest value of the correlation coefficient comes from $BCR_m$, then it should be the best choice among the others in predicting $SR_f$. Hence, Eq. (12) provides an empirical formula for the $BCR_m$ vs. $SR_f$ relation, and Fig. 6(b) demonstrates the validity.

Alternatively, instead of using Eq. (12), because $BCR_O$ and BCRs may give acceptable correlation coefficients in predicting $SR_f$ then a linear multivariate regression analysis was performed on the 29 tests again, and the result is expressed as follows:
\[(SR_f)_{\text{measured}} = 1.046 + 0.191 \cdot BCR_d + 0.329 \cdot BCR_s \]  \hspace{1cm} (13)

where the conditional standard deviation is 0.176. The conditional standard deviation in Eq. (13) is a little less than that in Eq. (12), because in Eq. (13) there is no constraint on \((SR_f)_{\text{measured}}\) when \(BCR_d + BCR_s = 1\). It is noted that Eqs. (12) and (13) are valid only within the range of the available data. In Eq. (13), the coefficients ‘0.191’ and ‘0.329’ represent the ‘deep-footing’ and the ‘wideslab footing’ effects at peak loading on \(SR_f\), respectively. The former effect comes from the combination of the depth of footing, the shape of footing, and the strength of the sandy ground, while the latter effect is due to the degree of extension of the semi-rigid reinforced zone under the footing. It is noted that the model proposed in the present study is based on a limited number of reduced-scale model and centrifuge tests. Further studies into the size effect on the bearing capacity of footings as mentioned by de Beer (1965), and Tatsuoka et al. (1991) should be performed in order to apply the proposed model to prototype foundations.

CONCLUSIONS

Applicability of a method for predicting the bearing capacity ratio of reinforced sandy ground at ultimate footing load condition was examined using two series of model tests. It was shown that this method gave reliable predictions of the ultimate bearing capacity increase for the footings on model sandy ground reinforced with high tensile stiffness reinforcement. This conclusion is valid only for the range of extentibilities of reinforcement investigated in the present study. When applying this method to the model sandy ground reinforced with exten-
sible reinforcement, the ‘deep-footing’ mechanism may not function. In this case, the present method is not applicable. Settlement of reinforced ground at ultimate footing load condition was also investigated using a series of model tests and centrifuge tests. Linear relationships between $SR_f$ and $BCR_r$ were found for all the tests in which a semi-rigid reinforced zone was successfully developed under ultimate footing load condition. The values of $SR_f$ for reinforced sandy ground may also be a linear function of $BCR_D$ and $BCR_S$. This implies that the settlement of footing required for reaching the failure of the reinforced sandy ground may increase with the increase of the degrees of 'deep-footing' and 'wide-slab' effects.

The present study focused on the load and settlement for reinforced ground at its ultimate condition. Further studies on the settlement characteristics of horizontal reinforced sandy ground under work load condition will be required to substantiate the practical application of the soil reinforcement technique. The model proposed in the present study is based on a limited number of reduced-scale model and centrifuge tests. Further studies into the accuracy of the proposed model considering the size effect of footings should be performed in the future.

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REFERENCES