UNDRAINED SHEAR CHARACTERISTICS OF SATURATED SAND UNDER ANISOTROPIC CONSOLIDATION

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ABSTRACT

A series of undrained triaxial compression tests was performed on Toyoura sand in order to investigate the behavior of sand under large deformation. The present study focuses on the effects of anisotropic consolidation on the undrained behavior of sand. A wide range of initial states of sand is covered and taken into account with the behavior of sand varying from contractive to dilative. Different states of consolidation stress were shown to affect the stress-strain behavior of sand and the development of excess pore water pressure up to an axial strain of 5%. Beyond a strain in excess of 10%, the behavior of sand was shown to become independent of the stress state at consolidation. Consequently, the relation between void ratio and confining stress at steady state and quasi-steady state are independent of the extent of anisotropic consolidation. Moreover, the initial dividing curve between dilative and contractive behaviors in an e − p' diagram was shown to move down as the sand is more anisotropically consolidated.

Key words: anisotropic consolidation, sand, steady state, triaxial compression test, undrained shear (IGC: D6/D3)

INTRODUCTION

Shear failure of sandy ground is caused not only by an increased external load but by a reduced level of effective stress. It is well known that the shear strength is reduced when the pore water pressure increases while the total overburden pressure is maintained more or less constant. Such a situation is found to exist in the cyclic loading conditions during earthquakes as well as in filtration of the ground water caused by heavy rainfall and melting of snow.

It is well known that the undrained shear strength of sand is affected by several factors such as the density, the effective confining pressure, and the initial static shear stress. Different conditions make sand behave in different manners, either contractive or dilatant.

Castro (1975) proposed the concept of a “steady state” in which sand undergoes undrained shear with large strains under constant void ratio, constant shear stress and constant effective mean stress. The behavior of sand in undrained triaxial monotonic shear was classified by Castro and Poulos (1977) into three categories; complete loss of shear strength at large deformation, partial loss of strength after the peak strength and softening, and dilative behavior in which shear strength and effective stress keep increasing with shear deformation without softening. The loss of strength is associated with the contractive nature of sand in which excess pore water pressure develops substantially. The key to the three different behaviors was thought to be dominated by the initial confining pressure and the void ratio. Vaid and Chern (1985) demonstrated that complete loss of strength is followed by the development of strength (hardening) when sand is not very loose. This behavior is called limited liquefaction, while “static” liquefaction stands for unlimited deformation with zero residual strength. Vaid et al. (1990) declared that the state of the minimum shear stress after the peak but before hardening is equivalent with the phase transformation proposed by Ishihara et al. (1975).

Studies of the steady state involve establishment of the relation between void ratio and the effective mean principal stress (e − p' relation) at the steady state. Been and Jefferies (1985) proposed that the state parameter, which is the difference between the current void ratio and the void ratio at the steady state, governs the undrained behavior of sand. Vaid and Chern (1985) reported that the e − p' relation at the steady state is identical whether sand is isotropically or anisotropically consolidated.

The behavior of sand at large strains appears to be more complex than normally understood. Verdugo (1992) indicated that a steady state occurs even after dilatant behavior. The state of minimum shear stress after the peak was called the “quasi-steady state” by him and it gave a different e − p' relation from that at the
steady state. Yoshimine et al. (1998) revealed that the \( e-p' \) relation at the steady state varies with the type of shear, ranging from triaxial compression, through triaxial strain shear to triaxial extension. Meneses et al. (1998) indicated that sand is made very contractive when a cyclic loading is superimposed perpendicular to the monotonic shear.

Additionally, it is well known that the initial static shear stress is an important factor in the softening behavior during a cyclic test of very loose sand. Sladen et al. (1985) proposed the concept of the "collapse surface", and indicated that anisotropically consolidated samples often failed in undrained triaxial monotonic tests, when the stress state reached the collapse surface obtained from isotropic tests. Hyodo et al. (1994) indicated that the potential for the occurrence of flow deformation depends on the mutual situation between the initial consolidation and the phase transformation points in the \( p'-q \) stress state diagram.

A review of the past studies described above indicates that major interest has been focussed on earthquake-induced liquefaction of loose sand and, thus, that tests have been run mostly on isotropically consolidated specimens. It is, hence, important to take account of the undrained failure of denser materials caused by such non-seismic mechanisms as heavy rainfall and water submergence. Moreover, the importance of anisotropic consolidation is evident. The work in the present study has concentrated on triaxial compression tests of a specimen consolidated isotropically and a specimen consolidated anisotropically at a stress ratio of \( K_c = \sigma_1'/\sigma_3' < 1.0 \).

**METHOD OF TEST**

A series of triaxial compression tests was performed on Toyoura sand. The physical properties of this sand are: \( D_0 = 0.18 \text{ mm}, \ U_e = 1.3, \ G_s = 2.65, \ e_{max} = 0.988 \) and \( e_{min} = 0.616 \). The samples measured 5 cm in diameter and 10 cm in height. Enlarged end plates with a diameter of 62 mm were used at both the ends. The end friction at both top and bottom ends was lubricated by two layers of membrane and grease. The membrane layer was made up of eight pieces which were 0.2 mm thick, and the grease layer was of 0.15 mm in thickness.

Tested samples were prepared by the method of wet tamping in which sand of 5% water content was placed in six layers and compacted to a desired density inside a sample preparation mould. The presented method was selected because it was the most suitable method to obtain a very loose state of soil packing and to cover a wide range in density in the reconstituted sample (Ishihara, 1996). The static and cyclic deformation-strength characteristics are remarkably influenced by the method of sample preparation (Oda, 1972; Tatsuoka et al., 1982; Miura and Toki, 1982). Thus, it may be assumed that this method produces isotropic samples which possess a random fabric with respect to particle orientation.

After a hydrostatic pressure of 20 kPa was applied on a sample, carbon dioxide gas was circulated from the bottom to the top of the specimen for 30 minutes, followed by circulation of 500 cm\(^2\) of de-aired water through the specimen. To ensure water saturation with Skempton's B value exceeding 0.96, a back pressure of 196 kPa was applied.

Isotropic consolidation was attained by increasing both the axial and the lateral stresses simultaneously. In contrast, anisotropic consolidation with \( K_c = \sigma_1'/\sigma_3' > 1.0 \) other than unity was achieved by a stress history such as that illustrated in Fig. 1. Both the axial and the lateral stresses were raised simultaneously, while maintaining \( K_c \) equal to the desired value.

After consolidation was completed, triaxial undrained compression was conducted in a strain-controlled manner with an axial strain rate of 1% per minute. After the tests were over, the void ratio of a specimen was determined by measuring its dry weight and the water content. Conventionally, the void ratio of a specimen is determined by measuring the dry weight and the sample volume after deposition prior to assembling a triaxial pressure chamber. When the wet tamping of sample preparation is employed, however, significant volume change occurs while applying the cell pressure and circulating de-aired water in an unsaturated specimen (Sladen and Handford, 1987; Verdugo, 1992). Since this volume change occurs after the pressure chamber is assembled, it cannot be measured from outside, and therefore, the alternative way described above was employed. Special care was taken as described below to measure water content.

(1) After termination of the shearing, an attempt was made to drain as much pore water as possible into a burette. This goal was achieved by applying additional high-amplitude cyclic loads to the specimen, thereby causing a complete remoulding or liquefaction with high excess pore water pressure, and then opening a drainage valve. This procedure was repeated several times.

(2) The triaxial chamber was disassembled and the water on the outer surface of a rubber membrane was wiped off; the cap and the pedestal of the sample were then removed, and the weight of the wet sand measured.

![Fig. 1. Stress path for anisotropic consolidation](image-url)
together with a membrane.

(3) At the same time, the membrane was made dry and its weight was measured.

(4) The sand was then dried in an oven and its dry weight measured.

(5) The total amount of pore water was determined by using the volume of drained water in (1) and the amount of water that existed in wet sand (2) minus the weight of the membrane (3).

UNDRAINED MONOTONIC LOADING TESTS AFTER ISOTROPIC CONSOLIDATION

Triaxial compression tests were carried out in a monotonic loading condition on samples which were isotropically consolidated under $p' = 98$ kPa or 294 kPa. The stress-strain and stress-path diagrams as presented in Figs. 2 and 3 were obtained. The effective mean stress is defined by $p' = (\sigma_1 + 2\sigma_3)/3$, while the deviator stress by $q = (\sigma_1 - \sigma_3)$. It is seen in these figures that the void ratio of the sand is instrumental in changing the behavior of sand from contractive to dilative (see Fig. 4). Similar effects were reported previously by Castro et al. (1977) and Vaid et al. (1985).

Generally, the stress path of a contractive sand is bent to the left in the figures, while that of a dilative specimen is bent to the right. It is interesting that the specimen of $e = 0.917$ with $p'_c = 98$ kPa exhibited a contractive behavior in the course of a dilative deformation. It is thus difficult to state that the density of a sand alone governs the dilative or contractive behavior. When the minimum value of $p'$ in this test is denoted by $p'_s$, the isotropic consolidation stress by $p'_c$, the ratio of $p'_c/p'_s$ is 1.1. Since Sladen et al. (1985) reported that no softening of stress was possible when $p'_c/p'_s < 2.0$, it appears that the tentative decrease of $p'$ for $e = 0.917$ with $p'_c/p'_s = 1.1$ was an exceptional one, possibly related to characteristics due to wet tamping.

Figure 2(b) shows that the stress paths of all the specimens were more or less identical until their first bending, except for the one with $e = 0.950$ which led to a total loss of strength. In contrast, Fig. 3(b) with a higher $p'_c$ exhibits different shapes of path, varying in accordance with void ratio.

UNDRAINED MONOTONIC LOADING TESTS AFTER ANISOTROPIC CONSOLIDATION

Tests on anisotropically consolidated specimens were conducted with $\sigma_3' = 98$ and 294 kPa, employing $K_c$.
values of 0.5 and 0.33. The results of the tests with \( \sigma_{ic} = 98 \) kPa and \( K_c = 0.5 \) in Fig. 5 demonstrate that loose sand developed softening behavior with or without hardening in the later stage, while denser sand showed more dilative behavior. The stress path with \( e = 0.914 \) in Fig. 5(b) is a combination of intermediate contraction in the course of dilative behavior. These features are in agreement with those observed in isotropically consolidated samples.

Figure 6 presents the test results with \( \sigma_{ic} = 98 \) kPa and \( K_c = 0.33 \). It is to be noted that the stress state at consolidation with \( K_c = 0.33 \) was close to the failure line in Fig. 6(b). The effective stress state moved to the right in the initial stage of shearing, which is a dilative behavior, even though softening and contractive behavior occurred in the later stage. Although such a combination of dilatant and contractive deformations is the same as those in Fig. 2 (\( e = 0.917 \)) and Fig. 5 (\( e = 0.914 \)), the presented sample attained the peak stress at a much smaller strain.

Figures 7 and 8 illustrate the test results with \( K_c = 0.5 \) and 0.33 under an elevated consolidation pressure of \( \sigma_{ic} = 294 \) kPa. The observed behavior is similar to that under \( \sigma_{ic} = 98 \) kPa. In Fig. 8(b), for example, samples with intermediate void ratios of 0.869 and 0.888 exhibited a dilative behavior in the initial stage of shear, then became contractive and attained a peak strength, ending up with a dilative behavior again.

Figure 9 examines the effects of anisotropic consolidation by using test data with \( K_c = 0.33, 0.5, \) and 1.0. The lateral effective stress during consolidation, \( \sigma_{ic} \), was equal to 59 kPa in all the tests. It is seen in Fig. 9(a) that the different stress states during consolidation made the shapes of stress-strain curves different in the strain range less than 5%. The sample with \( K_c = 1.0 \) showed a monotonic increase in stress, while the sample with \( K_c = 0.33 \) exhibited softening. Similarly, the development of excess
pore water pressure (Fig. 9(b)) and the shape of stress path (Fig. 9(c)) were different for samples with different $K_c$ values. Immediately after the shearing was started, the excess pore water pressure decreased with $K_c=0.33$, while it increased for $K_c=1.0$. When the strain exceeded a value of 10%, however, both the level of deviator stress and excess pore water pressure became similar, irrespective of $K_c$ values, with all the effective stress states in Fig. 9(c) located along the failure line. It may also be mentioned that the value of the residual strength is independent of the extent of the anisotropic consolidation or the initial static shear stress.

INITIATION OF STRAIN SOFTENING DURING MONOTONIC SHEAR

The softening of sand after the peak strength is important from the stability viewpoint because it could lead to a large deformation involving flow failure. Vaid and Cherif (1985) studied the behavior of contractive sand and showed that the effective stress state at the outset of softening has a constant stress ratio, independent of the consolidation pressure. The present study elucidates in Fig. 10 the critical stress ratio (CSR) at the onset of softening as well. With a void ratio after consolidation of around 0.93, the line of CSR obtained for 294 kPa consolidation does not hit the peak stress state, in contrast to cases with other values of consolidation stress. The higher stress in consolidation made the sand behave more contractively, as evidenced by the test with $\sigma_{ic}=490$ kPa. When the void ratio decreased to 0.887 for $\sigma_{ic}=490$ kPa, a less contractive behavior occurred.

Figure 11 illustrates the same stress paths normalized by the effective mean stress, $p'$, during consolidation. When the void ratio was around 0.93, the lower consolidation pressure made the sand less contractive. Thus, the idea of constant CSR is not valid in the present study. When $p'$ was lower or when void ratio was smaller, softening was initiated at a stress state closer to the failure line. The failure line indicates the condition of $q/p'$ where large deformations take place involving failure of the sample. It has been reported by Zlatovic et al. (1997), that stress paths normalized by the initial confining pressure practically coincide up to the peak strength for loose sandy and non-plastic silty soils, irrespective of the initial state of the soil sample. Therefore, the onset of softening behavior is considered to be dominated by the fabric of the sample with few fines content in the present study.

The stress ratio at the onset of softening, $M_F = q/p'$ at the peak value of $q$, was plotted against void ratio, $e$, for $K_c=1.0$ in Fig. 12. There are good relationships between $M_F$ and $e$ for consolidation stress greater than 98 kPa; the looser sand has a smaller $M_F$. For the case of 59 kPa
consolidation, however, \( M_p \) always takes a value around 1.3, except for the loosest sample of \( e=0.952 \). This is probably because the onset of softening under this lowest pressure is governed by shear failure (see Fig. 11) in place of volume contraction. The solid lines in Fig. 12 were obtained by means of least-square approximation of the experimental data:

\[
M_p = A - B \times e
\]  

(1)

Figure 13 indicates the variation of "A" and "B" parameters with the consolidation pressure. Initial confining stress is evaluated as \( \sigma_c = (\sigma_{ic} + \sigma_{qc})/2 \).

The stress ratio, \( M = q/p' \), was calculated at states of consolidation, peak deviator stress, quasi-steady state, and steady state for anisotropically consolidated samples and is plotted in Figs. 14 to 17 where the lateral effective stress is 294 kPa during consolidation. The quasi-steady state is the state of the minimum shear stress after softening from the peak, while the steady state is the ultimate
Fig. 13. Numerical coefficients of $M_p$-$e$ relationships versus initial confining stress: $\sigma'_c = (\sigma_{ic} + \sigma_{ic}')/2$

Fig. 14. Ratio of deviator stress and effective mean stress at various states versus void ratio for specimen consolidated anisotropically at stress ratio of $K_c=0.67$

Fig. 15. Ratio of deviator stress and effective mean stress at various states versus void ratio for specimen consolidated anisotropically at stress ratio of $K_c=0.5$

Fig. 16. Ratio of deviator stress and effective mean stress at various states versus void ratio for specimen consolidated anisotropically at stress ratio of $K_c=0.4$

Fig. 17. Ratio of deviator stress and effective mean stress at various states versus void ratio for specimen consolidated anisotropically at stress ratio of $K_c=0.33$

state of constant deviator stress (Ishihara, 1996). When this state was not attained until 30% axial strain, the stress state at this large strain was employed. At around 30% axial strain, though shear band was not observed in samples, there seem to be non-homogeneities in the strain distribution due mainly to friction at both ends of the sample.

Figure 14 is concerned with tests for $K_c=0.67$. A solid line in the figure indicates the stress ratio at the peak strength of isotropically consolidated samples. Note that this curve and the data from $K_c=0.67$ (shown by ○) are consistent. A similar point can be made in Fig. 15 with $K_c=0.5$ as well. In Figs. 16 and 17 with $K_c=0.4$ and 0.33, in contrast, there is not such a consistency any more and the softening commenced from a higher stress ratio, $M_p$. This is probably because the stress ratio during anisotropic consolidation, $M_c$, was already close to or greater than the $M_p$ of isotropically consolidated samples. Accordingly, $M_p$ was affected more profoundly by the failure mechanism of sand than by dilatancy, taking more or less
Fig. 18. $M_p=q_p/p_p'$ versus $M_c=q_c/p_c'$ and $K_c=\sigma_{ic}/\sigma_{ic}'$ for contractive samples

constant values independent of void ratio; see in particular Fig. 17.

The effects of stress level during consolidation are studied in Fig. 18. The $M_p-M_c$ relationships for all tests were plotted in this figure. Because $M_p$ is influenced by the void ratio, the confining pressure and $M_c$ (as shown in Figs. 14-17), the relation between $M_p$ and $\sigma_{ic}$ is not clear in Fig. 18. When $M_c$ is smaller than 0.7, the value of $M_p$ is independent of $M_c$. When $M_c$ is greater than 0.7, however, there is a lower bound for $M_p$ which increases with $M_c$. As can be seen, these data points in the lower bound fall in a straight line. When $M_c$ was close to or greater than $M_p$ of isotonically consolidated samples in Figs. 16 and 17, the point of $M_p$ corresponding to its $M_c$ is located on this lower bound. The upper bound value of $M_p$ is almost equal to the average value of the stress ratio at quasi-steady state or steady state, $M_s=M_{ss}=1.27$, as described later.

STEADY STATE AND QUASI-STeadY STATE

Discussion is made of the quasi-steady state and the steady state in this section. Figure 19 illustrates data for tests of isotropic consolidation ($K_c=1.0$). The curves in the figure stand for the average relationships. The same curves are drawn in Fig. 20 where data for anisotropic consolidation are plotted. It is to be noticed that the $p'-e$ relations drawn here are independent of the extent of anisotropic consolidation.

The ratio of deviator stress and effective mean stress $M_s$ versus void ratio $e$ at quasi-steady state, are shown in Fig. 21, and $M_{ss}$ versus $e$ at steady state are illustrated in Fig. 22. Both $M_s$ and $M_{ss}$ show a slight increase with void ratio, varying more appreciably for a void ratio larger than 0.92. This characteristic is probably because of measurement error at lower pressure levels. The effective mean stress $p'$ and the deviator stress $q$ at the quasi-steady state are presented in Fig. 23. The stress ratio $M_s$ in this diagram appears to be constant and equal to 1.27, over the whole range of consolidation pressure and $K_c$ values. Moreover, the stress state at the steady state in Fig. 24 has the same stress ratio, $M_{ss}=1.27$, irrespective of consolidation stress and $K_c$. The mobilized friction angle is $32^\circ$ at $M=1.27$. 

Fig. 21. Ratio of deviator stress and effective mean stress at quasi-steady state versus void ratio
INITIAL DIVIDING CURVE

The test data presented so far have revealed that the sand behaves either in a contractive or in a dilative manner, depending on the void ratio, stress level during consolidation, and extent of anisotropic consolidation. Figure 25 shows the states of isotropic consolidation which were employed for tests. Two solid curves stand for the steady state and the quasi-steady state, respectively, as derived in the previous section. This figure discriminates the behavior of samples by using solid rectangles for contractive and open rectangles for dilative behavior. The contractive behavior is defined as shear deformation with softening, while the dilative behavior is defined as shear deformation without softening. There
Fig. 28. Comparison of initial dividing line with the stress ratio of consolidation, $K_C$

seems to be a boundary in Fig. 25 between two types of behavior, and this boundary is here called the initial dividing line, which is illustrated by a dashed curve.

Initial dividing lines were similarly drawn in Figs. 26 and 27 for $K_C=0.5$ and 0.33, respectively. It is seen in these figures that the steady state line is located on the dilative side of the initial dividing line for less anisotropic consolidation, and moves into the contractive domain for smaller values of $K_C$.

Figure 28 summarizes the initial dividing lines for all the states of consolidation. The translation of the dividing lines as mentioned above is evident here. For $K_C=0.33$, the dividing line is in close agreement with the quasi-steady state curve when $p'$ is lower than 200 kPa. The sand becomes, thus, more contractive as the extent of anisotropic consolidation increases.

CONCLUDING REMARKS

A series of undrained triaxial compression tests was performed on Toyoura sand in order to investigate the behavior of sand under large deformation. A special emphasis was placed on the study of softening in anisotropically consolidated sand specimens in the course of shearing. The conclusions drawn from this study are summarized as below.

1) Different states of consolidation stress produce different stress-strain behavior during shearing and hence, different excess pore water pressure up to the axial strain of 5%. Beyond 10% strain, on the contrary, the behavior of sand becomes independent of the stress state at consolidation.

2) For isotropically consolidated samples, the stress ratio $q/p'$ at the peak deviator stress is governed by both the void ratio and the consolidation stress, when the consolidation stress is higher than 98 kPa. When it is lower than 98 kPa, the stress ratio $q/p'$ at the peak stress is close to that at shear failure.

3) The stress ratio at the peak deviator stress $M_P$ varies in a more complex manner for anisotropically consolidated samples. The effect of anisotropic consolidation on the stress ratio at peak shear stress $M_P$ is not important when the extent of anisotropic consolidation is small. The value of $M_P$ is profoundly affected by the extent of anisotropic consolidation.

4) The $e \sim p'$ relations at steady state and quasi-steady state are independent of the extent of anisotropic consolidation.

5) The initial dividing curve differentiating between dilative and contractive behaviors demonstrated in an $e \sim p'$ diagram moves down as sand is more anisotropically consolidated.

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