ESTIMATION OF TOE LOAD-SETTLEMENT RELATION OF NON-DISPLACEMENT PILES BASED ON BACK-ANALYSIS OF IN-SITU TESTS

HIROAKI NAGAOKA, MASASHIRO YAMAZAKI, YING ZHANG and TAKESHI OKAMURA

ABSTRACT

In this paper, a new estimation method is presented for the toe bearing behavior of non-displacement piles embedded into bearing strata of sand. The method is developed by numerical experiments. First, a new in-situ test is proposed whose results can identify five parameters of a bearing stratum uniquely. These are parameters of the stress-strain relations of sand and coefficient of earth pressure at rest. In the in-situ test, a cylinder of, for example, 10 cm in diameter is embedded into the bearing stratum. The measured results are a load-settlement relation at the end of the cylinder and a friction-settlement relation on the surface of the cylinder. These relations are back-analyzed, using elastic-plastic finite element method and neural network, and the parameters are identified. Then using the identified parameters, the toe bearing behavior of a pile is estimated by the elastic-plastic finite element method. The estimation method can be applied to piles embedded into over-consolidated sand as well as normally consolidated sand.

Key words: (bearing capacity of pile toe), (coefficient of earth pressure at rest), (ground parameter identification), in-situ test, (non-displacement pile), over-consolidation, pile, sandy soil, stress-strain curve, vertical load (IGC: E4/E13)

INTRODUCTION

In the design of pile foundations, the load-settlement relations at the pile toes of non-displacement piles, e.g., cast-in-place concrete piles and bored precast piles, are usually estimated by two methods. One is a pile loading test and the other is a method using N-values of the standard penetration test (e.g., Nagaoka and Yamazaki, 1998). Estimation by the former is highly reliable but expensive, while the latter is convenient to use but less reliable. Development of intermediate methods is both desirable and necessary, i.e., methods which are not as expensive as pile loading tests but which can estimate load-settlement relations with higher reliability than the method using N-values. In this paper, “load-settlement relation of a pile” means the relation at the pile toe.

Nagaoka and Yamazaki (1999) proposed one intermediate method. The load-settlement relation of a large diameter pile is estimated as follows. First, a loading test on a small diameter pile is carried out and the load-settlement relation is measured. Then the relation of the small diameter pile is back-analyzed and the parameters of the constitutive equations of soil and the coefficient of earth pressure at rest are calculated. Parameters of constitutive equations of soil and coefficient of earth pressure at rest are called “ground parameters” in this paper. Using the ground parameters, the load-settlement relation of the large diameter pile is calculated by the elastic-plastic finite element method, denoted by FEM. Because the load-settlement relation of the small diameter pile is insufficient input information, the ground parameters can not be identified uniquely and consequently the load-settlement relation of the large diameter pile can not be determined uniquely. Thus a probable range of the relation is determined.

We propose an intermediate method by which the load-settlement relation of a non-displacement pile can be determined uniquely. Input information consists of the load-displacement relations of a newly proposed in-situ test. The load-displacement relations of the in-situ test are back-analysed to identify ground parameters uniquely. Then the load-settlement relation of a pile is predicted by FEM, using the identified parameters. The purpose of this paper is to construct a framework for the method. In order to construct this framework, the following questions must be answered. What analytical tools are effective for identification of the parameters? What input information can identify the parameters uniquely? By what in-situ test can the input information be measured? When the parameters are identified, is it possible to...
predict pile toe behavior with accuracy? The present paper will solve these problems by numerical experiments and present the framework of the method. Translation of the method into practice in the field is not within the scope of the present paper.

The following studies can be found on back-analyses; inputs are the load-displacement relations of conventional in-situ tests and outputs are soil parameters.

Arai (1993) proposed a method by which three parameters are identified, i.e., initial Young's modulus, cohesion and angle of internal friction $\phi$. In an application example, using the load-settlement relation of a plate loading test as input, the parameters are identified. An optimization technique is applied in order to minimize error between the relations given as input and calculated using the parameters so that the parameters are identified uniquely.

Fahey and Soliman (1994) proposed a method by which four parameters of soil are identified by the pressure-displacement relation of a pressuremeter test. The parameters are determined not by optimization techniques but by trial and error so that the measured and calculated relations may agree. In their calculation example, three different sets of the parameters were shown to calculate an identical pressure-displacement relation, i.e., the four parameters can not be identified uniquely by the method.

Consoli et al. (1997a, 1997b) proposed a method to identify two parameters of deformation and strength. Input information is the pressure-displacement relation of a pressuremeter test. Though not explicitly written in their paper, the parameters seemed to be identified by trial and error, as in the paper by Fahey and Soliman. A plate loading test and a pulling test of a footing were carried out at the same site. Using the identified parameters, they calculated the load-displacement relations of the plate loading test and the pulling test and compared them with the test results. Only partial agreement could be seen.

Compared with the studies reviewed above, features of this study from the viewpoint of back analyses are as follows:

1) Ground parameters to be identified uniquely are five in number, compared with three in Arai's study or two in the Consoli et al. study.
2) Coefficient of earth pressure at rest is identified, whereas no other studies can be found which try to identify the coefficient uniquely.
3) In order to get input information which can identify the five parameters uniquely, the idea of a new in-situ test is introduced.
4) Stress-strain relations with identified parameters can be applied to over-consolidated sand as well as normally consolidated sand.

**CONSTITUTIVE EQUATIONS**

Since studies by de Beer (1963), it has been pointed that the influence of grain crushing by shear under high pressure on the bearing behavior of pile toes is significant. Many studies on the mechanical behavior of sand under high pressure can be found, e.g., Vesic et al. (1968), but a few of them present constitutive equations. In studies by Duncan et al. (1980), Bardet (1985), Yasufuku et al. (1991) and Crouch et al. (1994), constitutive equations were presented. The equations hold from low to high mean normal stress.

The equations of Duncan et al. are chosen in the present paper for the following reason. In order to examine applicability of the equations to the analysis of pile toe behavior, Yamazaki et al. (1995) analyzed model tests of non-displacement piles in dense sand. The tests were carried out and reported by Kishida et al. (1977). Calculated load-settlement relations and contact pressure distribution at model pile toes were compared with those of the model tests. Good coincidence between them was seen and they concluded that the constitutive equations are applicable to the analysis of pile toe behavior in dense sand.

Constitutive equations in this study are expressed as follows.

$$E_1 = \frac{1 - R_1 (1 - \sin \phi) \left(\sigma_1 - \sigma_3\right)^2}{2c \cdot \cos \phi + 2\sigma_3 \sin \phi} K \cdot P_a \left(\frac{\sigma_3}{P_a}\right)^n$$

for $\sigma_1 - \sigma_3 < (\sigma_1 - \sigma_3)$

$$E_1 = 0.0 \quad \text{for} \quad (\sigma_1 - \sigma_3) = (\sigma_1 - \sigma_3)$$

$$E_1 = \frac{2c \cdot \cos \phi + 2\sigma_3 \sin \phi}{1 - \sin \phi}$$

(1)

$$E_w = K_w \cdot P_a \left(\frac{\sigma_3}{P_a}\right)^n$$

(2)

$$B = K_b \cdot P_a \left(\frac{\sigma_3}{P_a}\right)^m$$

(3)

Figure 1 shows the stress-strain relation of triaxial compression test by solid line schematically. Here $\sigma_1$ and $\sigma_3$ are major and minor principal stresses. First, soil is consolidated isotropically under $\sigma_1 = \sigma_3$. Secondly, $\sigma_3$ is increased while $\sigma_1$ is kept constant, which is a loading process expressed by the path O to A. $\varepsilon_1$ is an increment of major principal strain from O. $\sigma_1$ is decreased and in-

![Fig. 1. Stress-strain relation of triaxial compression test](image)
creased, which is an unloading-reloading process expressed by the paths A to C and C to A. $\sigma_i$ is increased further, which is again a loading process expressed by the path A to D. $E_i$ and $E_{ui}$ denote tangent Young's moduli for loading and unloading-reloading processes, respectively. $B$ denotes tangent bulk modulus for loading and unloading-reloading processes. $\phi$ and $c$ are angle of internal friction and cohesion respectively. Since the soil in the bearing strata is sand in this paper, $c=0$ and $\phi$ is defined as the secant angle of internal friction. $R_i$ is failure ratio and $P_i$ is atmospheric pressure. $K_i$, $K_w$, and $K_0$ are initial tangent modulus and tangent moduli non-dimensionlized with respect to $P_i$, when $\sigma_j$ is $P_i$; $n$ and $m$ are modulus exponents.

It was found in triaxial tests that $\phi$ depends on mean normal stress (e.g., Vesci et al., 1968) or $\sigma_3$ (Duncan et al., 1980). $\phi$ is expressed in this paper by the following equations.

$$\phi = \phi^* \quad \text{for} \quad \sigma_j < \sigma^*_j$$
$$\phi = \phi^* - \frac{\phi^{**} - \phi^*}{\log (\sigma^*_j / \sigma^{**}_j)} \log \left( \frac{\sigma^*_j}{\sigma_j} \right) \quad \text{for} \quad \sigma^*_j \leq \sigma_j \leq \sigma^{**}_j$$
$$\phi = \phi^{**} \quad \text{for} \quad \sigma^{**}_j < \sigma_j$$

(4)

Duncan et al. (1980) proposed the second equation which holds for all ranges of $\sigma_j$, which is modified as above for the following reasons. When mean normal stress is high enough, $\phi$ is equal to the angle of internal friction at critical state $\phi^{**}$ (Bolton, 1986), which introduces the third equation. When $\sigma_j$ is low enough, $\phi$ is independent of $\sigma_j$ (Tatsuoka, 1987), which introduces the first equation.

In FEM calculation, when the stress state is known, $\sigma_1$ and $\sigma_3$ are calculated. Then, using $\sigma_1$ and $\sigma_3$, $E_i$, $E_{ui}$, $B$ and $\phi$ are calculated by Eqs. (1)-(4). Using them, incremental analyses are carried out. When principal stresses are changed from $\sigma_1$ and $\sigma_3$ to $\sigma_1 + d\sigma_1$ and $\sigma_3 + d\sigma_3$, the criterion on loading and unloading-reloading is as follows (Duncan et al., 1980). If $(\sigma_1 + d\sigma_1) - (\sigma_1 + d\sigma_3)$ is greater than the maximum principal stress difference previously experienced, then it is a loading process and $E_i$ is used, and if less, it is an unloading-reloading process and $E_{ui}$ is used.

TWO STAGES IN STUDY

There are two stages in this study. In the first stage, we will determine load-displacement relations of a new in-situ test which can identify ground parameters uniquely. In the second stage, using the load-displacement relations determined in the first stage, the load-settlement relation of a pile will be estimated.

The First Stage

The following assumptions are made to reduce the number of parameters to be identified.

1) $n=0.5$, $m=0.3$
2) $\phi^{**} = 30^\circ$, $\sigma^{**}_j = 10$ MPa, $\sigma^*_j = 100$ kPa
3) Sand of a bearing stratum is normally consolidated.
4) $K_w = K$

The values of $m$ and $n$ are representative values of dense sands, which are quoted from the study by Duncan et al. (1980). Figure 2 shows relations between secant angle of internal friction $\phi$, and mean principal stress $p$ (Yasufuku et al., 1991). $\phi$, is denoted by $\phi$ in this paper. $\phi^{**}$ of the critical state depends largely on the mineral components of sand (Bolton, 1986). Toyoura sand is rich in quartz and Dogs Bay sand contains predominantly molluscan carbonate. In the case of a sand rich in feldspar, an example of $\phi^{**}$ is 40$^\circ$ (Bolton, 1986). It is assumed in this paper that sand of a bearing stratum is rich in quartz and $\phi^{**}$, $\sigma^*_{ij}$, $\sigma^{**}_{ij}$ are determined by characteristics of Toyoura sand. For Toyoura sand, $\phi$ is almost constant and nearly equal to 30$^\circ$ when $p$ is greater than 20 MPa, i.e., $\phi^{**}$ is set to 30$^\circ$ at $p=20$ MPa. In triaxial compression tests, $p=20$ MPa corresponds to $\sigma_j = (3/5)$ $p=12$ MPa and $\sigma^{**}_j$ is set to be 10 MPa. Bolton (1987) proposed that $\phi$ is constant for $p<150$ kPa which corresponds to $\sigma_j < 90$ kPa and $\sigma^*_j$ is set to be 100 kPa. If mineral components are largely different, different values must be specified.

When numerical values for $K_i$, $\phi^*$, $R_i$, and $K_0$ are given, the behavior of a pile toe can be calculated numerically, if the initial stress state is prescribed. The initial stress state is expressed by coefficient of earth pressure at rest $K_0$, $\alpha$ is $K/K_0$. Ground parameters to be identified from load-displacement relations of an in-situ test are $K_i$, $\phi^*$, $R_i$, $\alpha$, and $K_0$.

When ground parameters have expression relating to identification, the parameters are $K$, $\phi^*$, $R_i$, $\alpha$ and $K_0$, e.g., ground parameters to be identified, identified ground parameters or we are going to identify ground parameters, etc. $n$, $m$, $\phi^{**}$ and $K_w/K$ whose numerical values are prescribed in the four assumptions are called "the prescribed parameters".

Fig. 2. Mean normal stress and secant angle of internal friction (Yasufuku et al., 1991)
Three in-situ tests are shown in Fig. 3, which are named Test 1, Test 2 and Test 3. Cylinders are embedded into a bearing stratum. In Test 1, vertical load is applied to the cylinder and the relation between load and settlement at the end, denoted by $P-\delta_p$ relation, is measured. In Test 2, vertical load is applied and friction acts on the surface of the cylinder between A and B in the Fig. (b). Average friction per unit area of the surface is called friction in this paper. The relation between the friction and the settlement at the lower point, denoted by $F-\delta_f$ relation, is measured. In Test 3, the cylindrical surface is expanded laterally and uniformly as a pressuremeter and the relation between pressure for the expansion and radial displacement, denoted by the $q-\delta_q$ relation, is measured. The radial displacement being uniform over the expanding part, the pressure for expansion may be non-uniform and $q$ is defined as average pressure.

Using identification method which will be developed at this stage, a test or a set of tests is examined by numerical experiments as to whether its load-displacement relations can identify ground parameters uniquely. In the experiments, five parameters $K$, $\phi^*$, $R_1$, $\alpha$ and $K_0$ of a bearing stratum at an imaginary site are specified. The five specified parameters and the prescribed parameters, i.e., parameters whose values are prescribed in the four assumptions as defined before, are called "ground parameters of the site", though imaginary. Using the ground parameters of the site, the $P-\delta_p$, $F-\phi_f$ and $q-\delta_q$ relations are calculated by FEM. They are called the "load-displacement relations of in-situ tests at the site", which are also imaginary. The examination is carried out sequentially as shown in Fig. 4. First, Test 1, Test 2 and Test 3 are examined individually. If these load-displacement relations cannot determine ground parameters uniquely, which are equal to the ground parameters of the site, then a set of Tests 1 and 2, a set of Tests 1 and 3 and a set of Tests 2 and 3 are examined. A set of Tests 1 and 2, for example, means that $P-\delta_p$ and $F-\phi_f$ relations are simultaneously used to identify the parameters. If the load-displacement relations cannot, then a set of Tests 1, 2 and 3 will be examined.

Details are explained in IDENTIFICATION METHOD OF GROUND PARAMETERS and SELECTION OF IN-SITU TEST.

The Second Stage

In the first stage, the load-displacement relations of an in-situ test were selected which can identify ground parameters uniquely. It is examined and confirmed in the second stage that the load-settlement relation of a pile can be predicted with good accuracy, using the load-displacement relations of the in-situ test which were selected in the first stage. Instead of the load-displacement relations of the in-situ test and the load-settlement relation of a pile loading test, both of which are carried out in practice, relations calculated by FEM are used in numerical experiment at this stage. The calculation is carried out as follows.

In a bearing stratum, ground parameters and stress history are specified. The ground parameters and the stress history do not satisfy all of the four assumptions of the first stage, e.g., sand is over-consolidated, or the numerical values of $n$ and $m$ are different from 0.5 and 0.3, etc. The specified ground parameters are called "ground parameters of the site", as in the first stage. Using the ground parameters of the site as well as the specified stress history, load-displacement relations of the in-situ test and the load-settlement relation of the pile in a loading test are calculated by FEM. The load-displacement relations of the in-situ test are called "load-displacement relations of an in-situ test at the site", as in the first stage and the load-displacement relation of the pile is called "a load-settlement relation of a pile loading test at the site".
which is also imaginary.

At first it is assumed that we know neither the numerical values of the ground parameters of the soil nor the load-settlement relations of the pile loading test at the site, but only the load-settlement relations of the in-situ test at the site. Using the load-displacement relations of the in-situ test at the site and the identification method developed in the first stage, ground parameters are identified. The identified and prescribed ground parameters are usually different from the ground parameters of the site. Parameters of constitutive equations are appropriate to a soil and independent of its stress history, present stress state and future stress path. The identified ground parameters are not appropriate to the soil but dependent on its stress history, present stress state and future stress path. Stress-strain relations with the identified and prescribed parameters can not be regarded as constitutive equations but only as approximate. From now on, constitutive equations without any assumptions are called "constitutive equations". Stress-strain relations with the four assumptions are called "approximate stress-strain relations".

Using the approximate stress-strain relations and the identified and prescribed ground parameters, the load-settlement relation of the pile is calculated by FEM. It is assumed that after the calculation we are given information of the load-settlement relations of the pile loading test at the site and the ground parameters of the site. Examination is made as to whether the calculated load-settlement relation accurately simulates the load-settlement relation of the pile loading test at the site and, as a result, it is shown that the calculated relation simulates with good accuracy and that the method to estimate pile toe behavior developed in this paper is effective.

The approximate stress-strain relations are expected to play the following role. For example, let the initial stress state of a soil element be a state on the unloading-reloading process, as point C of Fig. 1. As a cylinder of the in-situ test or the pile is loaded, stress is assumed to change from C to A, which is a reloading process, and then from A to D, which is a loading process. The approximate stress-strain relations are expected to approximate, with good accuracy, the process of C to A to D by a dotted line curve.

The details will be explained in ESTIMATION METHOD OF PILE TOE BEHAVIOR.

IDENTIFICATION METHOD OF GROUND PARAMETERS

Numerical Example

The influence of the soft strata above a bearing stratum is expressed by effective overburden pressure \( \sigma_{o} \) which is 257 kPa. Figure 5(a) shows a FEM model of Tests 1 and 2 and Fig. 5(b) shows one of Test 3. Cylinders are 10 cm in diameter. In Test 1, \( P \) and \( \delta_{f} \) are redefined as values measured at A, 20 cm above the end. Friction in Test 2 is measured between A and B which is at 50 cm above the end. \( \delta_{f} \) is measured at A. The expanding part of the cylinder in Test 3 is between C and D which are at 20 cm and 80 cm above the end. The finite element is an isoparametric four-noded element with four Gauss integral points. It has been confirmed by preliminary analyses that distant boundaries from the cylinders are far enough away not to influence load-displacement relations. Radial displacement on the distant side boundary and vertical displacement on the distant bottom boundary are zero.

A bearing stratum which satisfies the four assumptions is examined, i.e., its constitutive equations are the same as the approximate stress-strain relations. The bearing stratum is named G1 and its ground parameters are shown in the first row of Table 1. The parameters are ground parameters of the site. The bearing stratum is below the ground water table and the submerged unit weight of sand of the stratum is 8 kN/m\(^3\). Young's modulus and Poisson's ratio of the cylinders of Tests 1 and 2 are 2.1 \times 10^5 \text{kPa} and 0.167. Load is applied at the head of the cylinders. In Test 3, radial displacement is uniform and vertical displacement is zero over the expanding part. Calculated results of Tests 1, 2 and 3 are shown in Fig. 6, which are the \( P-\delta_{f}, F-\delta_{f} \) and \( q-\delta_{f} \) relations respectively. They are the load-displacement relations of in-situ tests at the site.

Neural Network

Ground parameters are identified by back-analyses, using FEM and neural network, denoted by NN. A multi-layered NN is used and shown in Fig. 7(a) (e.g., Anderson, 1995). Here circles denote neurons, the first and the last layers are input and output layers and the layers between them are hidden layers. Neuron \( i \) in the \( k \)-th layer and neuron \( j \) in the \((k+1)\)th layer are connected by a synapse with connection weight \( w_{ij} \) as shown in (b). Outputs of neurons \( i \) and \( j \) are \( x_{i} \) and \( x_{j} \) respectively. \( x_{i} \) is calculated by Eqs. (5) and (6), where \( \theta_{j} \) is a threshold value and \( n \) is the total number of neurons in the \( k \)-th layer.

\[
f(x) = \frac{1}{1 + \exp(-x)} \quad (5)
\]

\[
x_{j} = f \left( \sum_{i=1}^{n} w_{ij} x_{i} - \theta_{j} \right) \quad (6)
\]

When there are two data sets, i.e., input and output data, the learning of the NN is to determine \( w_{ij} \) and \( \theta_{j} \) of all neurons by the two data set. Learning is carried out by back-propagation, which is a fundamental technique in multi-layered NN, to minimize total error (e.g., Anderson, 1995). The total error is the sum of the squared difference between the output data and the calculated results by NN whose inputs are the input data.

Inputs are the load-displacement relations of in-situ tests and outputs are ground parameters to be identified. Load-displacement relations have to be expressed by discrete values as inputs, which are shown in Fig. 8. In Test 1, inputs are loads when settlements are 1, 2, 4, 6, 8 and 10% of the diameter of the cylinder; these are denoted by \( P_{1}, P_{2}, P_{4}, P_{6}, P_{8} \) and \( P_{10} \). In Test 2, inputs are maximum
friction and settlements when friction is equal to 20, 40, 60 and 80% of the maximum friction; these are denoted by $F_{\text{max}}$, $\delta_{F1}$, $\delta_{F4}$, $\delta_{F6}$ and $\delta_{F9}$. Since in Fig. 6(b), a curve bends abruptly just before reaching $F_{\text{max}}$ and afterward friction is constant, the inputs are the maximum friction as well as the settlements. In Test 3, inputs are pressures when radial displacements are equal to 0.5, 1, 2, 3, 4, 5, 7 and 10% of the diameter; these are denoted by $q_6$, $q_1$, $q_2$, $q_3$, $q_4$, $q_5$, $q_7$, and $q_9$. Numerical values of the load-displacement relations of the in-situ tests at the site are shown in Table 2, which are read from Fig. 6.
Identification Method

The following explains how to examine whether the load-displacement relations of in-situ tests at the site can identify ground parameters uniquely or not. As an example, a set of Tests 1 and 2 is examined.

It is well known that a multi-layered NN can express any continuous functions with arbitrary accuracy if the structure of the NN is appropriate. This needs a tremen-
dous amount of trial and error to determine its proper structure, i.e., the proper number of hidden layers and proper numbers of neurons in the layers. In order to avoid a huge amount of operations, the number of hidden layers is set at one in the present study. Figure 9 shows the structure of the NN. Inputs are load-displacement relations of in-situ tests $P_1, P_2, P_3, P_4, P_5, P_6, P_{10}, F_{\text{max}}, \delta_{P_1}, \delta_{P_2}, \delta_{P_4}, \delta_{P_6}$ and $\delta_{P_8}$. Outputs are $K, \phi^*, R_f, \alpha$ and $K_0$ to be identified. Only in this section, "ground parameters" means ground parameters which relate to identification, i.e., $K, \phi^*, R_f, \alpha$ and $K_0$. Since one hidden layer may be an improper structure and introduce learning error, a new iteration method is presented to overcome this difficulty; its flow is shown in Fig. 10 and will be explained in the following.

Step 1: Setting of initial ranges of ground parameters

Initial ranges of ground parameters are set wide enough, e.g., the ranges are set in this paper such that their upper and lower limits are the greatest and the smallest values of data for dense sands which were collected and presented in the paper by Duncan et al. (1980). The ranges are called Ranges 1 and are shown in the second row of Table 1.

Step 2: Generation of learning data

Numerical values of ground parameters for learning are upper and lower limits of Ranges 1 and their average values, e.g., numerical values of $K$ are 500, 2500 and 1500. From 243 (=3^3) combinations of the five parameters, 80 combinations are selected randomly to reduce the number of FEM analyses. Carrying out FEM analyses, 80 sets of learning data are generated, which are ground parameters $K, \phi, R_f, \alpha$ and $K_0$, and load-displacement relations $P_1, P_2, P_3, P_4, P_5, P_6, P_{10}, F_{\text{max}}, \delta_{P_1}, \delta_{P_2}, \delta_{P_4}, \delta_{P_6}$ and $\delta_{P_8}$.

Step 3: Learning of NN

Using the load-displacement relations and ground parameters of the learning data as input and output respectively, the learning of the NN is carried out. Since total error decreases as the number of the hidden layer’s neurons increases to 10 and thereafter the error is almost constant, the number is set at 10.

Step 4: Estimation of ground parameters

The load-displacement relations of the in-situ tests at the site are input to the learned NN and ground parameters are calculated which are named “estimated

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**Table 2. Load-displacement relations of in-situ tests at G1 site**

<table>
<thead>
<tr>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>$P_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>37.2</td>
<td>49.3</td>
<td>65.2</td>
<td>77.5</td>
<td>86.0</td>
<td>91.7</td>
<td></td>
</tr>
</tbody>
</table>

Unit: kN

<table>
<thead>
<tr>
<th>$\delta_{P_1}$</th>
<th>$\delta_{P_4}$</th>
<th>$\delta_{P_6}$</th>
<th>$\delta_{P_8}$</th>
<th>$F_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>0.13</td>
<td>0.193</td>
<td>0.273</td>
<td>207.7</td>
</tr>
</tbody>
</table>

Unit of $\delta_{P_1}, \ldots, \delta_{P_8}$ is cm and that of $F_{\text{max}}$ is kPa

<table>
<thead>
<tr>
<th>$q_{0.5}$</th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$q_3$</th>
<th>$q_4$</th>
<th>$q_5$</th>
<th>$q_7$</th>
<th>$q_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>824</td>
<td>1213</td>
<td>1755</td>
<td>2130</td>
<td>2390</td>
<td>2558</td>
<td>2725</td>
<td>2905</td>
</tr>
</tbody>
</table>

Unit: kPa

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Fig. 9. Neural network for set of Tests 1 and 2

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Fig. 10. Examination flow of identification of ground parameters

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ground parameters by the load-displacement relations of the in-situ tests at the site” and are ordinates of horizontal dotted lines in Fig. 11.

The load-displacement relations of the learning data are input once again to the NN and ground parameters are calculated. Figure 11 shows the parameters of the learning data in abscissa and the calculated parameters by the NN in ordinate. If there is no learning error, the points are on a solid line with an inclination of 45°, but the points deviate from the line. The errors are caused by setting the number of hidden layers at one and the estimated ground parameters may not be sufficiently accurate.

Step 5: Setting of relearning ranges
How to set the relearning range of $K_0$ is explained as an example. In Fig. 12, two straight broken lines CD and EF with an inclination of 45° are drawn such that all the points are between the lines. The ordinate of G is the estimated parameter $K_0$ by the load-displacement relations of the in-situ tests at the site, i.e., 1.044. A horizontal line is drawn through G. H and I are intersections of the horizontal line and the lines CD and EF. HI is a relearning range and is named Range 2. As in the case of K shown in Fig. 11, if either of H or I is outside the initial range, H or I is reset at the limit of the initial range.

Step 6: Judgement on identification
Ground parameters are considered to be identified if convergence to the ground parameters of the site can be proved, i.e., if relearning ranges become narrow enough and estimated ground parameters are close enough to the ground parameters of the site. Conditions for identification are expressed as follows. 1) For each parameter, the
width of its relearning range is less than 5% of the corresponding parameter of the site. 2) Estimated ground parameters are nearly equal to the ground parameters of the site with errors of less than 5%. If these conditions are not satisfied, the following step is carried out. The results in Fig. 11 do not satisfy the conditions.

Step 7: Repetition of Steps 2 to 5
Ranges 1 are changed to Ranges 2, or generally Ranges i are changed to Ranges i+1, and operations of Steps 2 to 5 are repeated.

Figure 13 shows convergence of the relearning ranges. Ordinates are ground parameters and abscissas are iteration numbers. Upper and lower limits of initial and relearning ranges are shown. The conditions on identification are satisfied at the 7th iteration and estimated ground parameters at the 7th iteration, which are identified parameters, are shown with the ground parameters of the site in parentheses. Relearning ranges, Ranges 8, and the identified ground parameters are shown in the last two rows of Table 1. The identified ground parameters agree well with the ground parameters of the site. It is judged that ground parameters can be identified uniquely by the load-displacement relations of Tests 1 and 2.

**SELECTION OF IN-SITU TEST**

**Selection**
As explained in the section entitled "The First Stage" of the chapter entitled "TWO STAGES IN STUDY", the sequential examination shown in Fig. 4 will be carried out in this section.

The first examination is that of Test 1. Iteration results of Test 1 are shown in Fig. 14. Relearning ranges do not converge by the 10th iteration and it is judged that ground parameters can not be identified by the load-displacement relation of only Test 1. Figure 15 shows results of Test 2. Though \( K, R, \phi^* \) and \( K_0 \) seem to converge, \( \alpha \) does not converge. Figure 16 shows results of Test 3. Relearning ranges of all the parameters may converge if iterations are continued more than ten times, but the numerical value of \( \alpha \) of the site is outside the relearning range at the 10th iteration. It is judged that ground parameters can not be identified by the load-displacement relation of only Test 2 or 3.

As explained in IDENTIFICATION METHOD OF GROUND PARAMETERS, a set of load-displacement relations of Tests 1 and 2 can identify ground parameters uniquely, i.e., a set of \( P-\delta_p \) relation and \( F-\delta_f \) relation. The set of Tests 1 and 2 is selected and no further examinations on sets of tests, such as a set of Tests 2 and 3 and so on, are carried out.

**A New In-Situ Test**
Load-displacement relations of Tests 1 and 2 can be obtained by the test shown in Fig. 17. The cylinder is 10 cm in diameter. Settlement at 20 cm above the end and axial strains at 20 cm and 50 cm above the end are measured. Since \( \delta_p \) and \( \delta_f \) are the same, both are represented by \( \delta_f \). \( P \) and \( F \) can be calculated by the axial strain at the lower position and the difference between the two strains.

It is necessary to set the cylinder in a bearing stratum without disturbing any of the states of the stratum, such as stress state, soil parameters and so on, which will be investigated in the next and near future studies. An idea of the test apparatus is given in Fig. 18. The construction process is like that of pile installation by pre boring. A hole 10 cm in diameter is drilled by an auger and mortar is poured into the hole. A steel pipe 7 cm in outer diameter and with strain gages at two points is inserted and set into the mortar. A rod is set in a hole of the pipe to
the lower end to measure $\delta_p$. After hardening of the mortar, a load is applied at the top of the steel pipe.

ESTIMATION METHOD OF PILE TOE BEHAVIOR

Numerical Examples

Figure 19 shows a large diameter pile and a cylinder for the proposed in-situ test installed in a bearing stratum. The pile is 2 m in diameter and embedded 1.5 m into the stratum. The cylinder is also embedded 1.5 m. Effective overburden pressure acting on a surface of the stratum is 257 kPa. The stratum is below the ground water table and a submerged unit weight of sand of the stratum is 8 kN/m$^3$. Cases of four bearing strata named G2, G3, G4 and G5 are investigated, whose ground parameters are shown in Table 3 as parameters of the site. In these strata $n$, $m$ and $K_s/K$ are different from the prescribed values of 0.5, 0.3 and 1.0 of the approximate stress-strain relations. $K_s$'s of G2 and G3 are 1.0 and 0.8 respectively. In the stratum G5, $\phi^{**}$ is different from the prescribed value of 30°.

The bearing stratum G4 is of over-consolidated sand. Though numerical values of $K_s$ are specified in case G2, G3 and G5 to define initial stress states, $K_s$ in case G4 is determined by the following preliminary analyses. In the finite element model in Fig. 5, the cylinder of the in-situ test is replaced with the sand of the bearing stratum, i.e., all finite elements in the model are the sand of the bearing stratum. Using the constitutive equations of Eqs. (1), (2) and (3), elastic-plastic analyses are carried out. First a nu-
Numerical value of over-consolidation ratio OCR is specified. Overburden pressure of 257 kPa multiplied by OCR is applied to the surface of the bearing stratum and then reduced to 257 kPa. Soil elements are first loaded and then unloaded. Using the calculated stress state after unloading to 257 kPa, $K_0$ is calculated. OCR is determined by trial and error such that $K_0$ is 1.5. As a result, the over-consolidation ratio is 4.67 when $K_0$ is 1.5.

Using the FEM model of Fig. 5, load-displacement relations are calculated and shown by curves in Fig. 20, which are $P$-$\delta_P$ and $F$-$\delta_F$ relations of the in-situ tests at the sites. The FFM model for the large diameter pile is shown in Fig. 21. Calculated load-settlement relations of the piles are shown by four curves in Fig. 22, which are the load-settlement relations of pile loading tests at the sites.
Table 3. Ground parameters of G2, G3, G4 and G5

<table>
<thead>
<tr>
<th></th>
<th>$K$</th>
<th>$K_0$</th>
<th>$R_f$</th>
<th>$\phi^*$</th>
<th>$K_0$</th>
<th>$n$</th>
<th>$m$</th>
<th>$\phi^{**}$</th>
<th>$K_{0s}/K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G2</td>
<td>1400</td>
<td>1080</td>
<td>0.900</td>
<td>37.0</td>
<td>1.000</td>
<td>0.74</td>
<td>0.15</td>
<td>30.0</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>1740</td>
<td>985</td>
<td>0.935</td>
<td>35.7</td>
<td>1.036</td>
<td>(0.5)</td>
<td>(0.3)</td>
<td>(30.0)</td>
<td>(1.0)</td>
</tr>
<tr>
<td>G3</td>
<td>1400</td>
<td>1080</td>
<td>0.900</td>
<td>37.0</td>
<td>0.800</td>
<td>0.74</td>
<td>0.15</td>
<td>30.0</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>1577</td>
<td>1033</td>
<td>0.942</td>
<td>36.5</td>
<td>0.828</td>
<td>(0.5)</td>
<td>(0.3)</td>
<td>(30.0)</td>
<td>(1.0)</td>
</tr>
<tr>
<td>G4</td>
<td>1780</td>
<td>1300</td>
<td>0.670</td>
<td>51.0</td>
<td>1.500</td>
<td>0.69</td>
<td>0.16</td>
<td>30.0</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>2167</td>
<td>1050</td>
<td>0.743</td>
<td>51.0</td>
<td>1.611</td>
<td>(0.5)</td>
<td>(0.3)</td>
<td>(30.0)</td>
<td>(1.0)</td>
</tr>
<tr>
<td>G5</td>
<td>1780</td>
<td>1300</td>
<td>0.670</td>
<td>51.0</td>
<td>1.000</td>
<td>0.69</td>
<td>0.16</td>
<td>35.0</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>1522</td>
<td>1083</td>
<td>0.673</td>
<td>51.5</td>
<td>1.005</td>
<td>(0.5)</td>
<td>(0.3)</td>
<td>(30.0)</td>
<td>(1.0)</td>
</tr>
</tbody>
</table>

Unit of $\phi^*$ and $\phi^{**}$ is degree and the other parameters are of zero dimension.

In the following, it is assumed that, when we are going to estimate load-settlement relations of the piles, we know only the load-displacement relations of the in-situ tests at the sites and do not know either the load-settlement relations of the pile loading tests at the sites or the ground parameters of the sites. After the estimation has been carried out, we will know the load-settlement relations of the pile loading tests at the site and the ground parameters of the sites and will be able to compare the estimated results with them.

Identification of Ground Parameters

The criterion of convergence or identification used in this section is as follows. Using estimated parameters by the load-displacement relations of the in-situ tests at the site, the load-displacement relations of the in-situ test are calculated, and are compared with the relations of the in-situ test at the site. Differences of loads when settlements are equal are calculated. For all settlements, if the differences are less than 5% of the load of the load-displacement relations of the in-situ test at the site, ground parameters can be said to converge or to be identified. The criterion of identification is different from that described in the chapter entitled IDENTIFICATION METHOD OF GROUND PARAMETERS because in that chapter it is assumed that we know the ground parameters of the site, as well as the load-displacement relations of its in-situ test, in order to examine and select or determine in-situ tests. On the other hand, in estimating the toe bearing behavior of a pile in the present chapter, it is assumed that we only know the load-displacement relations of the in-situ test at the site.

The criterion was satisfied at the second iteration in the case of G2 and at the 3rd iteration in cases of G3, G4 and G5. Identified parameters are shown in Table 3 with the prescribed parameters in parentheses. $P-\delta_{p}$ and $F-\delta_{p}$ relations calculated using the identified and prescribed parameters are shown by white marks in Fig. 20.

In G2 and G3, since parameter $n$ or $m$ of the site, which is 0.74 or 0.15, is greater or less than the prescribed value of 0.5 or 0.3 in the approximate stress-strain relations, identified parameter $K$ or $K_0$ is greater or less than the parameter $K$ or $K_0$ of the site, respectively. In G4, though the sand is over-consolidated, the load-displacement relations coincide fairly well.

As stated in the INTRODUCTION, this study is the first to try to uniquely identify the coefficient of earth pressure at rest $K_0$. $K_0$s in all cases are identified with errors less than 7% of $K_0$'s of the site and it can be said that the identification method is effective for estimating $K_0$.

As stated in TWO STAGES IN STUDY, the approximate stress-strain relations are expected to simulate with good accuracy relations by the constitutive equations. The stress-strain relations which most influence $P-\delta_{p}$ and $F-\delta_{p}$ relations are the $\sigma_r-\epsilon_r$ relation of soil elements below the end of the cylinder and the $\tau_{r,\gamma}$ relation of soil elements next to the shaft of the cylinder; $\sigma_r$ and $\epsilon_r$ are normal stress and strain in the $r$-direction and $\tau_{r,\gamma}$ are shear stress and strain with respect to the $r$- and $z$-axes. In the cases of G2 and G4, the $\sigma_r-\epsilon_r$ and $\tau_{r,\gamma}$ relations of elements indicated by numbers in Fig. 23(a) are shown in (b) and (c). Solid lines indicate the relations by the constitutive equations with the parameters of the site, and broken lines those by the approximate stress-strain relations with the identified and prescribed parameters. Fairly good agreement can be seen. Though the same relations for G3 and G5 are not shown, the agreement is also excellent.

Estimation of Pile Toe Behavior

Load-settlement relations of piles in the cases of G2, G3, G4 and G5 are calculated by FEM, using the iden-
tified and prescribed parameters. These are shown by white marks in Fig. 22. The calculated relations using the identified and prescribed parameters and those of the pile loading tests at the sites coincide fairly well and it is confirmed that the estimation method of pile toe behavior developed in this study is effective.

The stress-strain relation which most influences the load-settlement relations of the piles is the $\sigma_s-\varepsilon_s$ relation of soil elements below the pile toes. The $\sigma_s-\varepsilon_s$ relations of the elements indicated by numbers in Fig. 24(a) for G2 and G4 are shown in (b) and (c). The relations by the constitutive equations with the parameters of the site and by the approximate stress-strain relations with the identified and prescribed parameters agree fairly well. Though those for G3 and G5 are not shown, the agreement is also excellent.

In the case of G4, the sand is over-consolidated and stresses may change during the processes of first reload-
ing and then loading. When stress states are analyzed using the constitutive equations, equations for reloading and loading are used. On the other hand, the unloading-reloading process is the same as loading process in the approximate stress-strain relations. In spite of this, a good coincidence of the stress-strain relations is seen in Figs. 23 and 24. It can be said that the approximate stress-strain relations have excellent ability in simulating those
of the constitutive equations, which may be the main reason for the good coincidence of the load-settlement relations of the piles.

CONCLUSIONS

We have presented a method to estimate the bearing behavior of a pile toe in a sand bearing stratum, using a newly proposed in-situ test. The main conclusions are as follows.

1) We determine the load-displacement relations of an in-situ test to identify five ground parameters uniquely. The parameters are coefficient of earth pressure at rest and four parameters of stress-strain relations of sand. In the in-situ test, a cylinder is embedded into a bearing stratum and a vertical load is applied to the head of the cylinder. The relation between axial force and settlement near the end of the cylinder and
the relation between friction acting on the surface of the cylinder and the settlement are measured. The load-displacement relations are these two relations.

2) An effective method is presented to identify the five ground parameters by back-analysis in which FEM and neural network are used. Because of practical calculations, the number of hidden layers of the neural network is set at one, which may be an oversimplification and may introduce significant errors. A new iteration technique is proposed in order to overcome this difficulty.

3) By this method, we can identify the five parameters. The coefficient of earth pressure at rest is identified with errors of less than 7%.

4) It is shown by numerical experiments that the load-settlement relation of a pile toe can be estimated fairly accurately using the finite element method and the identified parameters. The method can be applied not only to the bearing strata of normally consolidated sand but also to strata of over-consolidated sand, such as diluvial sand.

The method is developed by numerical experiments. In order to complete the method, it is necessary to prove its effectiveness in field tests, which will be left to a future study.

REFERENCES