FREQUENCY DEPENDENT PROPERTY OF WAVES IN SATURATED SOIL

KINYA MIURAⁱ⁾, NOZOMU YOSHIDAⁱⁱ⁾ and YOON-SANG KIMⁱⁱⁱ⁾

ABSTRACT

Elastic wave propagation in saturated soil is investigated in order to grasp the frequency dependent wave propagation property of actual soil. First, a closed form solution of Biot's equation is derived under a one-directional wave propagation condition. Then, the frequency dependent properties of waves in various soils such as clay, silt, sand, gravel and soft rock are computed and discussed. The behavior is shown to be classified into three states: low frequency, high frequency and transient. Properties hardly change in the low and high frequency states. The degree of saturation is shown to hardly affect the shear wave property, but it greatly affects the dilatational wave property, especially in a highly saturated condition. Permeability hardly affects the properties at low and high frequency states, but it does change the frequency range of the transient state. Next, the applicability of Biot's formulation is examined and the high frequency state derived from Biot's equation is shown to be valid when permeability is larger than that of silt. Finally, it is shown that various mechanical properties can be evaluated using the frequency dependent characteristics of the wave, some of which are difficult to measure in situ with conventional tools.

Key words: attenuation, Biot's equation, coupled analysis, elastic wave, frequency, poro-elasticity, soil (IGC: E8/D8)

INTRODUCTION

Soil consists of an assemblage of soil particles that form a skeleton, and a pore material such as a fluid or gas. It is usually classified into three states depending on its constituents: dry, partially saturated and fully saturated states. Since the interaction between the soil skeleton and pore materials can be neglected for dry soil, the behavior is described as a single-phase continuum. On the other hand, since the interaction between them plays an important role for a fully saturated soil, it is described as a binary phase material in order to investigate its behavior in detail. The interaction may or may not be important in a partially saturated soil depending on the degree of saturation.

The theory to treat saturated soils as a binary phase material was developed by Biot (1941) for the elastic consolidation problem. It has since been improved into dynamic problems and nonlinear problems by Biot himself and many other researchers (e.g., Biot, 1956a and 1956b; Zienkiewicz et al., 1980; Prevost, 1993). This theory is usually called Biot's equation or the binary phase media equation in geomechanics.

Many studies using Biot's equation have been conducted in consolidation and liquefaction problems in which computing the change in an effective stress is important in evaluating the behavior of the ground, because, as

known to be the principle of effective stress (Lambe and Whitman, 1979), effective stress controls the stiffness and strength of a soil.

The other point of engineering interest of the binary phase media is wave propagation in a saturated soil. Ishihara (1968) solved Biot's equation for one-directional compressional waves and discussed the nature of waves in two extreme situations, i.e., very low and very high frequencies. He also showed approximate formulations of compressive and shear waves (Ishihara, 1970) and showed the frequency dependent nature of soil. Prevost (1993) solved wave equations of a two-phase material under a one-dimensional condition, but characteristics of waves propagating in a saturated soil have not been well discussed. Wyllie et al. (1962) measured the wave propagation through the saturated rock specimen in the laboratory with attention to the attenuation of the elastic waves and its frequency dependent characteristics. Hamilton (1972) and Yamamoto (1990) measured in situ dilatational waves propagating in marine sediments, and showed that wave attenuation has a frequency dependent characteristic. Yamamoto et al. (1992) demonstrated the applicability of the frequency dependent properties of elastic waves in identifying the soil profile by means of cross-hole measurement. Bowen (1982) derived basic equations for one-dimensional wave propagation in the saturated media from the general mixtures' theory,

ⁱ⁾ Associate Professor, GeoMechanics Group, Graduate School of Engineering, Hokkaido University, Sapporo 060-0813.

ⁱⁱ⁾ General Manager, Engineering Research Institute, Sato Kogyo Co., Ltd., Tokyo 103-8639.

ⁱⁱⁱ⁾ Graduate Student, GeoMechanics Group, Graduate School of Engineering, Hokkaido University, Sapporo 060-0813.

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which is equivalent to Biot's equation under linealization of the field equation and constitutive equation. These studies have indicated that waves propagating in a saturated soil clearly have frequency dependent characteristics. Discussions in these studies are, however, frequently limited in the extreme cases or just of mathematical interest; frequency dependent characteristics of actual soil were not well grasped. Moreover, in some studies, approximate formulae were used in discussing the frequency dependency nature.

In this paper, we investigate the frequency dependent characteristics of actual soil starting from the complete equation for binary-phase material through case studies. Then, we show the applicability of the derived equations by evaluating the in-situ property of soil. Since some mechanical properties are difficult to obtain in situ, derived formulation will help the accurate investigation of ground behavior in many fields.

GOVERNING EQUATIONS IN TERMS OF DISPLACEMENT

Since soils consist of soil particles, pore fluids and pore gases, it must in general be treated as a multiphase material, i.e., solid phase for soil skeleton, liquid phase for pore water and gas phase for pore air. In the followings, soil is idealized as a binary phase continuum; the mixture of pore water and pore air is modeled as a uniform fluid phase that has an averaged property.

Equilibrium conditions for solid and fluid phase media are written in the tensor form as

$$\begin{cases} (1-n)\rho_s\ddot{u}_{si} - (1-n)\rho_s b_i + (1-n)\rho_f(b_i - \ddot{u}_{fi}) \\ + s_{ij}(\dot{u}_{sj} - \dot{u}_{fj}) + \sigma_{sji,j} = 0 \end{cases} \quad (1a)$$

$$\begin{cases} n\rho_f\ddot{u}_{fi} - n\rho_f b_i - (1-n)\rho_f(b_i - \ddot{u}_{fi}) \\ - s_{ij}(\dot{u}_{sj} - \dot{u}_{fj}) + p_{,j}\delta_{ji} = 0. \end{cases} \quad (1b)$$

Here, compression is taken as positive for stress according to the convention in soil mechanics, the summation convention is employed for dummy indices that take values 1, 2 and 3, and the upper dot denotes the derivation with respect to time. Notations u_{si} and σ_{sij} denote displacement and effective stress of the solid phase, respectively, u_{fi} denotes displacement of the fluid phase, p denotes pore pressure, ρ_s and ρ_f denote mass densities of solid and fluid, respectively, n denotes porosity, b_i denotes components of the acceleration of gravity, and δ_{ij} is Kronecker's delta. Darcy's law for isotropic permeability gives

$$s_{ij} = \frac{n\rho_w g}{k} \delta_{ij} \quad (2)$$

where ρ_w is the mass density of water. The relationship between ρ_f and ρ_w is given by

$$\rho_f = S_r \rho_w + (1 - S_r) \rho_a \quad (3)$$

where ρ_a is mass density of the pore air and S_r is degree of saturation of the pore.

For the elastic solid phase, stress-strain relationships

are expressed as

$$\sigma_{sij} = 2G_s \varepsilon_{sij} + \delta_{ij} \lambda_s \varepsilon_{skk} \quad (4)$$

where λ_s and G_s are Lamé's constants of the solid phase, and ε_{sij} is strain of solid phase such that:

$$\varepsilon_{sij} = -(u_{si,j} + u_{sj,i})/2 \quad (5)$$

Here, contraction is taken as positive for strain corresponding to the definition of stress.

The constitutive law of the fluid phase is expressed as

$$p = K_f \varepsilon_{fv} = K_f u_{fi,i} \quad (6)$$

where K_f is the bulk modulus of the pore fluid and $\varepsilon_{fv} = u_{fi,i}$ is the volumetric strain of the fluid phase. When the pore fluid consists of pore water and pore air, we assume that it can be treated as a single liquid phase with averaged bulk modulus, i.e.,

$$\frac{1}{K_f} = \frac{1}{K_w} S_r + \frac{1}{K_a} (1 - S_r) \quad (7)$$

where K_a and K_w are bulk moduli of the pore air and pore water, respectively. This assumption holds when soil is in the insular air saturation state, i.e., suction is less than air entry value, under which state pore air can only exist in the state surrounded by pore water (Kohgo, 1994).

The change of the pore water must be equal to the volume change of the solid phase. With the use of Eq. (6), this condition is represented as

$$(1-n)\Delta\varepsilon_{sii} + n\Delta u_{fi,i} = (1-n)\frac{\Delta p}{K_s} + n\frac{\Delta p}{K_f} = \frac{\Delta p}{B_f} \quad (8)$$

where K_s is the bulk modulus of soil particles. When the condition $K_s \gg K_f$ holds, the first term in the middle part of Eq. (8) can be neglected, in which case $B_f (= K_f/n)$ is the averaged bulk modulus of the pore fluid. This condition is known to be generally acceptable (Skempton, 1954), but need not hold in the formulation here. The porosity n also changes depending on volume change, but its effect is second order quantity so that n is treated as constant. The substitution of Eq. (5) into Eq. (8) gives the pore pressure increment Δp in terms of displacement increments

$$\Delta p = -B_f \{ (1-n)\Delta u_{si,i} + n\Delta u_{fi,i} \}. \quad (9)$$

In order to eliminate stresses and pore pressure terms from Eq. (1), we first take increments of Eq. (1). Then by substituting Eqs. (4), (5), and (9) into Eq. (1), we can get governing equations in terms of displacement. If we consider only dynamic behavior, i.e., deviation from the static equilibrium state, the influence of the gravitational force can be disregarded, and we can take off the notation Δ that expresses increment without confusion. Finally, we obtain governing equations in terms of displacement as

$$\begin{cases} (1-n)\rho_s\ddot{u}_{si} - (1-n)\rho_f\ddot{u}_{fi} + s_{ij}(\dot{u}_{sj} - \dot{u}_{fj}) \\ - G_s u_{si,jj} - (G_s + \lambda_s) u_{sj,ji} = 0 \end{cases} \quad (10a)$$

$$\begin{cases} n\rho_f\ddot{u}_{fi} + (1-n)\rho_f\ddot{u}_{fi} - s_{ij}(\dot{u}_{sj} - \dot{u}_{fj}) \\ - B_f(1-n)u_{sj,ji} - B_f n u_{fi,jj} = 0. \end{cases} \quad (10b)$$

ONE DIMENSIONAL WAVES IN SATURATED SOIL

Two types of waves, i.e., shear wave and dilatational wave, are analyzed. They are designated to propagate in the direction of the z -axis. As shown in Fig. 1, soil particles are assumed to vibrate in the x -direction under shear wave propagation, whereas they vibrate in the z -direction under dilatational wave propagation. The plane strain condition under which the displacement in the y -direction is restricted is assumed to be fulfilled.

Shear Wave

Displacements in y - and z -directions are zero under the shear wave propagation. Displacements u_{sx} and u_{fx} can be designated as

$$u_{sx} = U_{sx}(z) \exp(i\omega t) \quad (11a)$$

$$u_{fx} = U_{fx}(z) \exp(i\omega t) \quad (11b)$$

where ω is an angular frequency, and $U_{sx}(z)$ and $U_{fx}(z)$ are amplitudes of the wave. Substituting Eq. (11) into Eq. (10), we obtain

$$\begin{cases} (\Omega - i)G_s D^2 U_{sx} + [\Omega(1-n)\rho_s - i\{(1-n)\rho_s + n\rho_f\}] \\ \quad \times \omega^2 U_{sx} = 0 \\ (\Omega - i)G_s D^2 U_{fx} + [\Omega(1-n)\rho_s - i\{(1-n)\rho_s + n\rho_f\}] \\ \quad \times \omega^2 U_{fx} = 0 \end{cases} \quad (12)$$

where $\Omega = \omega/\omega_0$ is a normalized angular frequency, where

$$\eta_s = \omega \sqrt{\frac{-\{(1+\Omega^2)(1-n)\rho_s + n\rho_f\} + i\Omega n\rho_f}{(1+\Omega^2)G_s}} = i\lambda_s + \kappa_s \quad (14a)$$

$$\lambda_s = \omega \sqrt{\frac{\sqrt{\Omega^2 n^2 \rho_f^2 + \{(1+\Omega^2)(1-n)\rho_s + n\rho_f\}^2} + \{(1+\Omega^2)(1-n)\rho_s + n\rho_f\}}{2(1+\Omega^2)G_s}} \quad (14b)$$

$$\kappa_s = \omega \sqrt{\frac{\sqrt{\Omega^2 n^2 \rho_f^2 + \{(1+\Omega^2)(1-n)\rho_s + n\rho_f\}^2} - \{(1+\Omega^2)(1-n)\rho_s + n\rho_f\}}{2(1+\Omega^2)G_s}} \quad (14c)$$

Therefore, the general solution is represented as

$$\begin{cases} u_{sx}(t, z) = A_{ss} \exp\{i(\omega t - \lambda_s z) - \kappa_s z\} \\ \quad + B_{ss} \exp\{i(\omega t + \lambda_s z) + \kappa_s z\} \end{cases} \quad (15a)$$

$$\begin{cases} u_{fx}(t, z) = A_{fs} \exp\{i(\omega t - \lambda_s z) - \kappa_s z\} \\ \quad + B_{fs} \exp\{i(\omega t + \lambda_s z) + \kappa_s z\} \end{cases} \quad (15b)$$

where A_{ss} , B_{ss} , A_{fs} and B_{fs} are the undetermined coefficients. The relationships between the coefficients are obtained from Eq. (15) and the characteristic equation of Eq. (12), yielding

$$A_{fs} = \xi_s A_{ss} \quad (16a)$$

$$B_{fs} = \xi_s B_{ss} \quad (16b)$$

where $\xi_s = (1 - i\Omega)/(1 + \Omega^2)$. The relationship between u_{sx} and u_{fx} is obtained by substituting Eq. (16) into Eq. (15), resulting in

Fig. 1(a-c). Ground motion due to one-dimensional propagation of waves: (a) shear wave, (b) dilatational wave and (c) displacement

$\omega_0 = ng\rho_w/(k\rho_f)$ is a characteristic angular frequency, and D denotes the derivation operator $\partial/\partial z$.

The solution of Eq. (12) can be computed as

$$\begin{cases} D U_{sx} = \pm \eta_s U_{sx} = \pm (i\lambda_s + \kappa_s) U_{sx} \\ D U_{fx} = \pm \eta_s U_{fx} = \pm (i\lambda_s + \kappa_s) U_{fx} \end{cases} \quad (13)$$

$$\frac{u_{sx}}{u_{fx}} = \xi_s = \frac{A_{fs}}{A_{ss}} = \frac{B_{fs}}{B_{ss}} = \alpha_s \exp(i\Delta\theta_s) \quad (17)$$

where α_s is an amplitude factor and $\Delta\theta_s$ is phase lag, which are

$$\alpha_s = |\xi_s| = \frac{1}{\sqrt{1+\Omega^2}} \quad (18a)$$

$$\Delta\theta_s = \Im \ln \xi_s = \arctan(-\Omega). \quad (18b)$$

Figure 2 shows fundamental waveforms at $t=0$ obtained from Eq. (15) schematically. Here, since both solid and fluid phases in Eq. (15) are similar to each other, only one wave is good enough to grasp the waveform. Moreover, only the first term (wave traveling in the positive direction along the z -axis) is shown, and the coefficient of the first term is replaced by u_0 . As seen in the figure, the amplitude decreases in the z -direction. In other words, the wave attenuates even during the

propagation in one direction. This is one of the important characteristics of the wave in the binary phase medium. The wave velocity V_s , wave length L_s , exponential attenuation distance δ_s , logarithmic attenuation ratio ψ_s and critical damping ratio h_s are calculated as follows based on Fig. 2

$$V_s = \frac{\omega}{\lambda_s} \quad (19a)$$

$$L_s = \frac{2\pi}{\lambda_s} \quad (19b)$$

$$\delta_s = \frac{1}{\kappa_s} \quad (19c)$$

$$\psi_s = \frac{2\pi\kappa_s}{\lambda_s} \quad (19d)$$

$$h_s = \frac{\psi_s}{\sqrt{\psi_s^2 + 4\pi^2}} = \frac{\kappa_s}{\sqrt{\kappa_s^2 + \lambda_s^2}}. \quad (19e)$$

Dilatational Waves

Displacements in x- and y-directions are zero under the dilatational wave propagation. Displacements u_{sz} and u_{fz} can be designated as

$$u_{sz} = U_{sz}(z) \exp(i\omega t) \quad (20a)$$

$$u_{fz} = U_{fz}(z) \exp(i\omega t) \quad (20b)$$

where $U_{sx}(z)$ and $U_{fx}(z)$ are amplitude of the wave. Following the same process with the shear wave, Eq. (10) yields

$$\begin{cases} \Omega n E_{us} B_f D^4 U_{sz} + \omega^2 [\Omega (E_{us} + B_f) \rho_f \\ + \Omega n B_f \{(1-n)\rho_s - (2-n)\rho_f\} \\ - i \rho_f \{E_{us} + B_f\}] D^2 U_{sz} + \omega^4 [\Omega (1-n)\rho_s \rho_f \\ - i \{(1-n)\rho_s + n\rho_f\} \rho_f] U_{sz} = 0 \\ \Omega n E_{us} B_f D^4 U_{fz} + \omega^2 [\Omega (E_{us} + B_f) \rho_f \\ + \Omega n B_f \{(1-n)\rho_s - (2-n)\rho_f\} \\ - i \rho_f \{E_{us} + B_f\}] D^2 U_{fz} + \omega^4 [\Omega (1-n)\rho_s \rho_f \\ - i \{(1-n)\rho_s + n\rho_f\} \rho_f] U_{fz} = 0 \end{cases} \quad (21)$$

where $E_{us} = \lambda_s + 2G_s$ is a one-dimensional elastic modulus of the solid phase. Unlike the shear wave case expressed in Eq. (12), Eq. (21) is a fourth order equation, therefore there are four solutions. They are

$$\begin{cases} D U_{sz} = \begin{cases} \pm \eta_{d1} U_{sz} \\ \pm \eta_{d2} U_{sz} \end{cases} = \begin{cases} \pm (i\lambda_{d1} + \kappa_{d1}) U_{sz} \\ \pm (i\lambda_{d2} + \kappa_{d2}) U_{sz} \end{cases} \\ D U_{fz} = \begin{cases} \pm \eta_{d1} U_{fz} \\ \pm \eta_{d2} U_{fz} \end{cases} = \begin{cases} \pm (i\lambda_{d1} + \kappa_{d1}) U_{fz} \\ \pm (i\lambda_{d2} + \kappa_{d2}) U_{fz} \end{cases} \end{cases} \quad (22)$$

Here

$$\eta_{d1} = \omega \sqrt{\frac{b + \sqrt{c}}{a}} = i\lambda_{d1} + \kappa_{d1} \quad (23a)$$

$$\eta_{d2} = \omega \sqrt{\frac{b - \sqrt{c}}{a}} = i\lambda_{d2} + \kappa_{d2} \quad (23b)$$

where

$$a = 2\Omega n E_{us} B_f$$

$$b = -\Omega \rho_f (E_{us} + B_f) - \Omega n \{(1-n)\rho_s - (2-n)\rho_f\} \\ \times B_f + i \rho_f (E_{us} + B_f)$$

$$c = (\Omega^2 - 1) \rho_f^2 (E_{us} + B_f)^2 \\ + 2\Omega^2 n \rho_f [\{(1-n)\rho_s - (2-n)\rho_f\} \\ \times (E_{us} + B_f) - 2(1-n)\rho_s E_{us}] B_f \\ + \Omega^2 n^2 \{(1-n)\rho_s - (2-n)\rho_f\}^2 B_f^2 \\ + 2i\Omega [-\rho_f^2 (E_{us} + B_f)^2 - n \{(1-n)\rho_s - (2-n)\rho_f\} \\ \times \rho_f (E_{us} + B_f) B_f \\ + 2n \{(1-n)\rho_s + n\rho_f\} \rho_f E_{us} B_f]$$

$$\lambda_{d1} = \omega \sqrt{\frac{\sqrt{B^+} - \Re(b) + \sqrt{C^+}/2}{2a}}$$

$$\kappa_{d1} = \omega \sqrt{\frac{\sqrt{B^+} + \Re(b) + \sqrt{C^+}/2}{2a}}$$

$$\lambda_{d2} = \omega \sqrt{\frac{\sqrt{B^-} - \Re(b) - \sqrt{C^+}/2}{2a}}$$

$$\kappa_{d2} = \omega \sqrt{\frac{\sqrt{B^-} + \Re(b) - \sqrt{C^+}/2}{2a}}$$

$$B^+ = |b|^2 + |c| + \Im(b) \sqrt{2C^-} + \Re(b) \sqrt{2C^+}$$

$$B^- = |b|^2 + |c| - \Im(b) \sqrt{2C^-} - \Re(b) \sqrt{2C^+}$$

$$C^+ = |c| + \Re(c)$$

$$C^- = |c| - \Re(c). \quad (24)$$

Equation (23) implies that there are two kinds of dilatational wave, which are called dilatational waves of the first and second kind, respectively, hereafter, and are distinguished from each other by suffices 1 and 2.

The general solution of Eq. (10) is expressed as

$$\begin{cases} u_{sz}(t, z) = A_{sd1} \exp \{i(\omega t - \lambda_{d1} z) - \kappa_{d1} z\} \\ + B_{sd1} \exp \{i(\omega t + \lambda_{d1} z) + \kappa_{d1} z\} \\ + A_{sd2} \exp \{i(\omega t - \lambda_{d2} z) - \kappa_{d2} z\} \\ + B_{sd2} \exp \{i(\omega t + \lambda_{d2} z) + \kappa_{d2} z\} \end{cases} \quad (25a)$$

$$\begin{cases} u_{fz}(t, z) = A_{fd1} \exp \{i(\omega t - \lambda_{d1} z) - \kappa_{d1} z\} \\ + B_{fd1} \exp \{i(\omega t + \lambda_{d1} z) + \kappa_{d1} z\} \\ + A_{fd2} \exp \{i(\omega t - \lambda_{d2} z) - \kappa_{d2} z\} \\ + B_{fd2} \exp \{i(\omega t + \lambda_{d2} z) + \kappa_{d2} z\} \end{cases} \quad (25b)$$

where A_{sd1} , B_{sd1} , A_{sd2} , B_{sd2} , A_{fd1} , B_{fd1} , A_{fd2} and B_{fd2} are undetermined coefficients. The relationships between the coefficients are as follows:

$$A_{fd1} = \xi_{d1} A_{sd1} \quad (26a)$$

$$B_{fd1} = \xi_{d1} B_{sd1} \quad (26b)$$

$$A_{fd2} = \xi_{d2} A_{sd2} \quad (27a)$$

$$B_{fd2} = \xi_{d2} B_{sd2} \quad (27b)$$

where

$$\xi_{d1} = \frac{\{(1-n)\Omega + i\} [2n\{\Omega(1-n)\rho_s - i\rho_f\} B_f + b + \sqrt{c}]}{2n\{1 + (1-n)^2\Omega^2\} \rho_f B_f}$$

Fig. 2. Waves with attenuation in z-direction

$$\xi_{d2} = \frac{\{(1-n)\Omega + i\}[2n\{\Omega(1-n)\rho_s - i\rho_f\}B_f + b - \sqrt{c}]}{2n\{1 + (1-n)^2\Omega^2\}\rho_f B_f}.$$

Equations (25) to (27) resulted in

$$\frac{u_{fz1}}{u_{sz1}} = \xi_{d1} = \frac{A_{fd1}}{A_{sd1}} = \frac{B_{fd1}}{B_{sd1}} = \alpha_{d1} \exp(i\Delta\theta_{d1}) \quad (28a)$$

$$\frac{u_{fz2}}{u_{sz2}} = \xi_{d2} = \frac{A_{fd2}}{A_{sd2}} = \frac{B_{fd2}}{B_{sd2}} = \alpha_{d2} \exp(i\Delta\theta_{d2}) \quad (28b)$$

$$\alpha_{d1} = |\xi_{d1}|, \quad \Delta\theta_{d1} = \Im \ln \xi_{d1} \quad (29a)$$

$$\alpha_{d2} = |\xi_{d2}|, \quad \Delta\theta_{d2} = \Im \ln \xi_{d2} \quad (29b)$$

where α_{d1} and α_{d2} are amplitude ratios, and $\Delta\theta_{d1}$ and $\Delta\theta_{d2}$ are phase differences as schematically shown in Fig. 2.

In a similar manner to the case of the shear wave, the wave velocities V_{d1} and V_{d2} , the wave lengths L_{d1} and L_{d2} , the exponential attenuation distances δ_{d1} and δ_{d2} , the logarithmic attenuation ratios ψ_{d1} and ψ_{d2} and the critical damping ratios h_{d1} and h_{d2} are calculated to be

$$V_{d1} = \frac{\omega}{\lambda_{d1}} \quad (30a)$$

$$L_{d1} = \frac{2\pi}{\lambda_{d1}} \quad (30b)$$

$$\delta_{d1} = \frac{1}{\kappa_{d1}} \quad (30c)$$

$$\psi_{d1} = \frac{2\pi\kappa_{d1}}{\lambda_{d1}} \quad (30d)$$

$$h_{d1} = \frac{\psi_{d1}}{\sqrt{\psi_{d1}^2 + 4\pi^2}} = \frac{\kappa_{d1}}{\sqrt{\kappa_{d1}^2 + \lambda_{d1}^2}} \quad (30e)$$

$$V_{d2} = \frac{\omega}{\lambda_{d2}} \quad (31a)$$

$$L_{d2} = \frac{2\pi}{\lambda_{d2}} \quad (31b)$$

$$\delta_{d2} = \frac{1}{\kappa_{d2}} \quad (31c)$$

$$\psi_{d2} = \frac{2\pi\kappa_{d2}}{\lambda_{d2}} \quad (31d)$$

$$h_{d2} = \frac{\psi_{d2}}{\sqrt{\psi_{d2}^2 + 4\pi^2}} = \frac{\kappa_{d2}}{\sqrt{\kappa_{d2}^2 + \lambda_{d2}^2}}. \quad (31e)$$

CASE STUDIES AND DISCUSSION

Solutions of Biot's equation were derived in the previous section. In order to grasp the frequency dependent nature of wave propagation in actual soils, we first visualize it for typical soils. Seven kinds of soil, i.e., loose sand, dense sand, silt, normally consolidated clay (NC clay), overconsolidated clay (OC clay), gravel and soft rock, are chosen for this purpose. They are shown in Table 1 with the material properties used in the analysis. Here, values from ρ_t to k are assigned a priori as typical values in the practice of soil mechanics by JSCE (1994), and the other values are calculated from them. Computed velocity, logarithmic damping, amplitude ratio and phase difference are shown in Figs. 3 to 5 for shear wave, dilatational wave of the first kind, and dilatational wave of the second kind, respectively.

Extreme States

From Figs. 3 to 5, the frequency dependent property can be separated into three states, i.e., low frequency state, high frequency state and transient state. Both in the low frequency state and high frequency state, the property is almost constant, and therefore can be represented by two extreme states, i.e., $\omega \rightarrow 0$ ($\Omega \rightarrow 0$) and $\omega \rightarrow \infty$ ($\Omega \rightarrow \infty$). These two states are discussed first in this section and the general features of frequency dependent characteristics will be discussed in the next section. Responses of each wave in these states are obtained in the following, in which suffices (0) and (∞) correspond to the low and high frequency states, respectively:

Shear wave

Low frequency state:

$$V_{s(0)} = \sqrt{\frac{G_s}{\rho_t}} \quad (32a)$$

$$\psi_{s(0)} = 0 \quad (32b)$$

$$\alpha_{s(0)} = 1 \quad (32c)$$

$$\Delta\theta_{s(0)} = 0. \quad (32d)$$

High frequency state:

$$V_{s(\infty)} = \sqrt{\frac{G_s}{\rho_d}} \quad (33a)$$

$$\psi_{s(\infty)} = 0 \quad (33b)$$

$$\alpha_{s(\infty)} = 0 \quad (33c)$$

$$\Delta\theta_{s(\infty)} = -\frac{\pi}{2} \quad (33d)$$

where ρ_d and ρ_t are bulk densities of soil in dry and wet state,

Fig. 3(a-d). Frequency dependent properties of shear wave for typical soils: (a) wave velocity ratio, (b) amplitude ratio, (c) phase lag and (d) logarithmic damping

$$\rho_d = (1-n)\rho_s \quad (34a)$$

$$\rho_t = (1-n)\rho_s + n\rho_f. \quad (34b)$$

$$\psi_{d1(0)} = 0 \quad (35b)$$

$$\alpha_{d1(0)} = 1 \quad (35c)$$

$$\Delta\theta_{d1(0)} = 0. \quad (35d)$$

Dilatational wave of the first kind

Low frequency state:

$$V_{d1(0)} = \sqrt{\frac{E_{us} + B_f}{\rho_t}} \quad (35a)$$

High frequency state:

$$V_{d1(\infty)} = \sqrt{\frac{2nE_{us}B_f}{\rho_f(E_{us} + B_f) - n(2\rho_f - \rho_t)B_f - \sqrt{\{\rho_f(E_{us} + B_f) - n(2\rho_f - \rho_t)B_f\}^2 - 4n(\rho_t - n\rho_f)\rho_f E_{us}B_f}}} \quad (36a)$$

$$\psi_{d1(\infty)} = 0 \quad (36b)$$

$$\alpha_{d1(\infty)} = \frac{\sqrt{\{\rho_f(E_{us} + B_f) - n(2\rho_f - \rho_t)B_f\}^2 - 4n(\rho_t - n\rho_f)\rho_f E_{us}B_f} + \rho_f(E_{us} + B_f) - (n\rho_t + 2n\rho_f - 2n^2\rho_f)B_f}{2n(1-n)\rho_f B_f} \quad (36c)$$

$$\Delta\theta_{d1(\infty)} = 0. \quad (36d)$$

Dilatational wave of the second kind

Low frequency state:

$$V_{d2(0)} = 0 \quad (37a)$$

$$\psi_{d2(0)} = 2\pi \quad (37b)$$

$$\alpha_{d2(0)} = \frac{(1-n)B_f + E_{us}}{nB_f} \quad (37c)$$

$$\Delta\theta_{d2(0)} = -\pi. \quad (37d)$$

High frequency state:

Fig. 4(a-d). Frequency dependent properties of dilatational wave of first kind for typical soils: (a) wave velocity, (b) amplitude ratio, (c) phase lag and (d) logarithmic damping ratio

$$V_{d2(\infty)} = \sqrt{\frac{2nE_{us}B_f}{\rho_f(E_{us} + B_f) - n(2\rho_f - \rho_t)B_f + \sqrt{\{\rho_f(E_{us} + B_f) - n(2\rho_f - \rho_t)B_f\}^2 - 4n(\rho_t - n\rho_f)\rho_f E_{us}B_f}}} \quad (38a)$$

$$\psi_{d2(\infty)} = 0 \quad (38b)$$

$$\alpha_{d2(\infty)} = \frac{\sqrt{\{\rho_f(E_{us} + B_f) - n(2\rho_f - \rho_t)B_f\}^2 - 4n(\rho_t - n\rho_f)\rho_f E_{us}B_f} - \rho_f(E_{us} + B_f) + (n\rho_t + 2n\rho_f - 2n^2\rho_f)B_f}{2n(1-n)\rho_f B_f} \quad (38c)$$

$$\Delta\theta_{d2(\infty)} = -\pi. \quad (38d)$$

In the low frequency state, the fluid phase moves the same as the solid phase under shear wave propagation, i.e., $u_{si} = u_{fi}$. This indicates that the undrained condition holds. Solution in this state is identical to the solution by uncoupled analysis because fluid does not resist shear stress. There is no attenuation in this state. In the high frequency state, on the other hand, only the solid phase vibrates as indicated by $\alpha_{s(\infty)} = 0$, i.e., the drained condition holds. The velocity at the high frequency state is larger than that at the low frequency state because $\rho_d < \rho_t$ (see Eqs. 32(a) and 33(a)). The wave does not attenuate the same as the low frequency state.

The wave propagation characteristics of a dilatational wave of the first kind are nearly the same as a shear wave except that $\alpha_{cd1(\infty)} \neq 0$ and $\Delta\theta_{cd1(\infty)} = 0$. Ishihara (1968) pointed out that the solid phase and fluid phase

propagate independently under the undrained condition in the high frequency range. In summary, both shear wave and under the dilatational wave of the first kind propagate under the undrained condition at low frequency and drained condition at high frequency. The difference between them is that the fluid phase does not vibrate under shear wave propagation, but does so under dilatational wave of the first kind propagation.

Compared with these two waves, dilatational waves of the second kind show quite different features. The wave velocity V_{d2} asymptotically becomes zero as the frequency tends to zero, and attenuation is very large. Ishihara (1968) explained that it cannot propagate as a wave, but rather in a form similar to diffusion or heat condition, and thus classified it as a consolidation process. In the high frequency range, the wave propagates un-

Fig. 5(a-d). Frequency dependent properties of dilatational wave of second kind for typical soils: (a) wave velocity, (b) amplitude ratio, (c) phase lag and (d) logarithmic damping ratio

der the drained condition the same as with other waves.

General Feature of Frequency Dependency and Its Effect

All three different waves show the tendency of increasing wave velocity increases with angular frequency. The dilatational wave of second kind exhibits the largest attenuation in all frequency regions. The shear wave and dilatational wave of the first kind show notable attenuation only at around $\Omega=1$. Amplitude ratio is largest in the dilatational wave of the second kind in the low frequency region. It is obvious that this kind of frequency dependent characteristic comes from the interactive behavior between solid and fluid phases shown in Eq. (2).

The properties of the waves change sharply around the characteristic state where $\Omega=1$ or $\omega=\omega_0$. The leap or shift of the velocity, attenuation, amplitude ratio and phase lag are observed at different angular frequencies depending on soil type, whose tendency depends on the mechanical properties of soils. Since ω_0 is equal to $ng\rho_w/(k\rho_f)$, the angular frequency where wave properties of soils change is strongly controlled by the permeability, or practically by grain size. This explains why gravel, the highest permeable soil, changes its wave properties at the lowest angular frequency and overconsolidated clay, the lowest permeable soil, at the highest

angular frequency.

Loose sand and soft rock show a different tendency from other soils in amplitude ratio and phase lag of the dilatational waves, as shown in Figs. 4(b, c) and 5(b, c). This difference comes from the low Skempton's B' value; B' of loose sand is less than 0.5 because of the low degree of saturation, as seen in Table 1, and that of soft rock is 0.3 because of the high rigidity of the solid phase. The effect of degree of saturation will be discussed in the subsequent section.

The frequency range that is important in earthquake engineering practice is usually less than several tens hertz or a few hundreds rad./sec. of angular frequency for the shear wave, because the behavior of the ground and structures is predominantly controlled by earthquake waves in this frequency range. The effect of the attenuation and phase difference derived from the interactions between solid and fluid phases is negligible and the wave velocity is constant in this frequency range. The only exception may be gravel, because, as seen in Figs. 3, 4 and 5, frequency dependency of the shear wave begins to be clear when ω is greater than about 100 rad./sec., i.e., frequency is greater than about 16 Hz. Apart from this case, the single-phase assumption based on the undrained condition is valid in this frequency range. Here, it is noted

Table 1. Mechanical properties of typical soils selected for demonstration of frequency dependent wave properties (JSCE, 1994)

Material type	Loose sand	Dense sand	Silt	NC clay	OC clay	Gravel	Soft rock	Note
ρ_i : bulk density of wet material (kg/m ³)	1.90 × 10 ³	2.00 × 10 ³	1.80 × 10 ³	1.80 × 10 ³	2.00 × 10 ³	1.75 × 10 ³	2.10 × 10 ³	
ρ_s : density of solid phase (kg/m ³)	2.65 × 10 ³	2.65 × 10 ³	2.65 × 10 ³	2.75 × 10 ³	2.75 × 10 ³	2.36 × 10 ³	2.61 × 10 ³	
n : porosity	0.454	0.394	0.515	0.543	0.429	0.448	0.315	
G_s : shear modulus of solid phase (N/m ²)	0.4 × 10 ⁸	1.2 × 10 ⁸	0.3 × 10 ⁸	0.3 × 10 ⁸	0.8 × 10 ⁸	1.0 × 10 ⁸	9.0 × 10 ⁸	
ν_s : Poisson's ratio	0.30	0.30	0.30	0.30	0.30	0.30	0.30	
B' : Skempton's B -value in 1-D	0.40	0.70	0.80	0.80	0.80	0.60	0.30	
k : coefficient of permeability (m/s)	1.0 × 10 ⁻⁴	1.0 × 10 ⁻⁵	1.0 × 10 ⁻⁶	1.0 × 10 ⁻⁸	1.0 × 10 ⁻⁹	1.0 × 10 ⁻²	1.0 × 10 ⁻⁷	
e : void ratio	0.832	0.650	1.062	1.188	0.750	0.812	0.460	$e = n/(1-n)$
λ_i : Lamé's constants (N/m ²)	0.6 × 10 ⁸	1.8 × 10 ⁸	0.45 × 10 ⁸	0.45 × 10 ⁸	1.20 × 10 ⁸	1.50 × 10 ⁸	13.5 × 10 ⁸	$\lambda_s = 2G_s\nu_s/(1-2\nu_s)$
E_s : Young's modulus of solid phase (N/m ²)	1.04 × 10 ⁸	3.12 × 10 ⁸	0.78 × 10 ⁸	0.78 × 10 ⁸	2.08 × 10 ⁸	2.60 × 10 ⁸	23.4 × 10 ⁸	$E_s = 2(1+\nu_s)G_s$
E_{us} : stiffness in 1-D of solid phase (N/m ²)	1.40 × 10 ⁸	4.20 × 10 ⁸	1.05 × 10 ⁸	1.05 × 10 ⁸	2.80 × 10 ⁸	3.50 × 10 ⁸	31.5 × 10 ⁸	$E_{us} = 2(1-\nu_s)G_s/(1-2\nu_s)$
K_i : bulk modulus of fluid phase (N/m ²)	0.424 × 10 ⁸	3.86 × 10 ⁸	2.16 × 10 ⁸	2.28 × 10 ⁸	4.80 × 10 ⁸	2.35 × 10 ⁸	4.25 × 10 ⁸	$K_i = nE_{us}B'/(1-B')$
B_i : averaged bulk modulus (N/m ²)	0.933 × 10 ⁸	9.80 × 10 ⁸	4.20 × 10 ⁸	4.20 × 10 ⁸	11.2 × 10 ⁸	5.25 × 10 ⁸	13.5 × 10 ⁸	$B_i = K_i/n$
S_i : degree of saturation of pore (%)	99.30	99.93	99.87	99.88	99.95	99.88	99.94	$S_i = (1/K_i - 1/K_r)/(1/K_r - 1/K_w)$
ρ_i : bulk density of fluid phase (kg/m ³)	9.930 × 10 ²	9.993 × 10 ²	9.987 × 10 ²	9.988 × 10 ²	9.995 × 10 ²	9.988 × 10 ²	9.994 × 10 ²	$\rho_i = \rho_a(1+S_i) + \rho_w S_i$
ρ_d : bulk density of dry material (kg/m ³)	1.45 × 10 ³	1.61 × 10 ³	1.29 × 10 ³	1.26 × 10 ³	1.57 × 10 ³	1.30 × 10 ³	1.79 × 10 ³	$\rho_d = (1-n)\rho_s$
ω_0 : characteristic angular frequency (rad./sec.)	4.48 × 10 ⁴	3.87 × 10 ⁵	5.06 × 10 ⁶	5.33 × 10 ⁸	4.21 × 10 ⁹	4.40 × 10 ²	3.09 × 10 ⁷	$\omega_0 = ng\rho_w/k\rho_i$
Maximum B' value	0.974	0.932	0.976	0.976	0.952	0.938	0.706	$B' = K_w/(K_w + nE_{us})$

Bulk modulus of saturated water, $K_w = 2.31 \times 10^9$ (N/m²), bulk modulus of air, $K_a = 3.03 \times 10^5$ (N/m²), bulk density of water, $\rho_w = 1000.0$ (kg/m³), bulk density of air, $\rho_a = 0.0$ (kg/m³)

that only elastic behavior is dealt with in this paper; binary phase treatment is important when other factors, such as liquefaction phenomena in which dilatancy plays an important role, are taken into account. For example, if a change in excess porewater pressure occurs due to dilatancy, the undrained assumption may not hold. Although water flow may be very small during an earthquake, the bulk modulus of fluid is large, and therefore, change of excess porewater pressure that is roughly obtained as the product of bulk modulus of water and the volume of water flow can be a finite value (Yoshida and Tsujino, 1989).

The important frequency range may be a little higher, probably up to 100 Hz or more, for dilatational waves because the predominant frequency of structures in the vertical direction is higher than that in the horizontal direction and a dilatational wave usually propagates in the vertical direction near the structure. The frequency range where attenuation becomes significant is, however, higher in the dilatational waves than in the shear wave. Therefore, single-phase treatment is also valid in most soils the same as in the case of the shear wave.

Apart from earthquakes, there are fields in which the frequency range of interest is higher than those discussed in the preceding section, in which case the interactions between solid and fluid phases become more important. An example is PS logging by suspension method (Kitsunozaki, 1982). This method uses waves with several hundreds hertz, i.e., circular frequency larger than a few thousands. This circular frequency is larger than the characteristic circular frequency ω_0 of gravel. Therefore, obtained S- and P-wave velocities of gravel may not be these of single-phase analysis. Even in this frequency range, however, other soils hardly show frequency dependent characteristics. Another example is acoustic emission in which frequencies of several thousands hertz are of interest. Loose sand will enter the affected frequency range and dense sand may enter the affected range in this frequency range. Other fields such as shock wave, i.e., waves caused by blasting, by pile driving and by impulse shock, etc. also use high frequency waves, and therefore frequency dependent characteristics may become important.

As discussed, a coupled treatment is not necessary in many instances in geotechnical engineering, except for very high permeable soil such as gravel. Therefore, conventional expression of wave velocities, one shear and one dilatational wave velocity, is justified in many practical situations. This does not, however, indicate that the formulation presented here is not valuable. Of course, this formulation will be useful in fields that consider a higher frequency range as described above. Moreover, as explained in the subsequent section, the presented formulation may be useful in evaluating the mechanical properties of in situ soil, some of which is difficult to measure by the conventional method.

Effect of Degree of Saturation

We investigated various soils with typical material

Fig. 6(a-d). Change of shear wave propagation properties with B' value for $\Omega=1$: (a) wave velocity ratio, (b) amplitude ratio, (c) phase lag and (d) logarithmic damping

properties. Among the material properties in Table 1, the B' value may be the most scattered quantity in each soil. Therefore, in order to grasp the wave propagation properties, it seems important to make the effect of degrees of saturation clear. Figures 6, 7 and 8 show wave velocities, amplitude ratios, phase lags and damping ratios of each wave at characteristic frequency state ($\Omega=1$) versus B' value relationships. Figures 9, 10 and 11 show figures similar to Figs. 3 to 5, but drawn for only dense sand for the B' value as a parameter. The B' value dependent characteristics have nearly the same tendency for all Ω values although the value itself is different. Here, it is noted that the B' value does not become unity even for completely saturated soil, although it is frequently approximated to 1. Maximum possible values of B' are shown in Table 1, and figures are drawn up to this B' value. There should be a lower boundary value of B' below which the present theory is not valid because pore air forms channel shape and, in this case the behaviors of the pore material cannot be expressed as a fluid with an average property. As discussed in the preceding section, this depends on the suction and air entry values and is not a unique value. Therefore, we compute a low B' value up to $B'=0.1$.

As shown in Fig. 6, all lines are nearly horizontal for shear wave; therefore the shear wave propagation property is hardly affected by the degree of saturation. This can

be also confirmed in Fig. 9 in which all relationships lie in one line. On the other hand, the wave velocity of dilatational waves increases as the B' value increases, as shown in Figs. 7 and 8. This can be recognized because the bulk modulus of fluid B_f increases toward K_w/n as B' increases. The increase becomes very rapid to infinite for a dilatational wave of the first kind when B' approaches unity. As described above, however, B' cannot become unity even in a completely saturated state. The maximum possible wave velocities can be obtained by replacing B_f by K_w/n for complete saturation. The wave velocity of a dilatational wave of the second kind is zero regardless of the B' value as indicated in Eq. (37a) when $\Omega=0$. When $\Omega>0$, it also increases with the B' value. Unlike the dilatational wave of the first kind, however, the rate of increase becomes smaller as B' increases. The velocity at a high frequency state tends to $\sqrt{nE_{us}/[(1-2n)\rho_f+n\rho_t]}$ when B' approaches 1.

Both amplitude ratio and phase lag increase as the B' value increases for a dilatational wave of first kind, but decrease for a dilatational wave of the second kind. The important behavior in Fig. 7 may be the logarithmic damping ratio of the dilatational wave of first kind Ψ_{d1} ; it once decreases sharply when B' is about 0.4 and then increases sharply again. Figure 10 suggests that, when Ψ_{d1} takes a minimum value 0, wave velocities at low and high

Fig. 7(a-d). Change of dilatational wave of first kind propagation properties with B' value for $\Omega=1$: (a) wave velocity, (b) amplitude ratio, (c) phase lag and (d) logarithmic damping ratio

frequency states are identical, amplitude ratio is 1, and there is no phase difference. All these result suggest that both solid and fluid phases behave in a similar manner. If the degree of saturation becomes smaller than this state, the amplitude of the fluid phase becomes smaller than that of the solid phase.

Effect of Permeability

Permeability is also different for the materials listed in Table 1, because it is affected strongly by the existence of fines. Figures 12, 13 and 14 show results of the parametric study of dense sand by changing permeability. Since the purpose here is to grasp the effect of changes in permeability, unrealistic values of permeability for dense sand may also be included. It is clearly seen that properties in both low frequency and high frequency states do not change regardless of the permeability. This is easily recognized as equations expressing the properties at two extreme states (Eqs. (32), (33), (35), (37), (36) and (38)) do not include permeability and normalized circular frequency. Therefore permeability affects only the frequency range of the transient state.

As shown in the preceding, ω_0 is equal to $ng\rho_w/(k\rho_f)$. Therefore, n , ρ_w , and ρ_f affect the characteristic frequency in addition to permeability. The variance of these quantities is, however, not large compared with the vari-

ance of permeability, therefore their effect can be said to be much smaller than permeability.

Applicability of the Coupled Formulation

The soil is modeled into a continuum in Biot's equation. The validity of this assumption must be examined in the high frequency range because the wavelength becomes shorter as the frequency increases. Let us consider the loose sand as an example. The shear wave and dilatational wave of the first kind velocities are about 150 m/s and 350 m/s, respectively, in the high frequency range as seen in Figs. 3 and 4. Several soil particles may be required to exist within one wavelength in order to maintain the continuum assumption. When the diameter of the sand is to be 0.2 mm as a typical value, the wavelength must be larger than about 1 mm. Therefore the frequency must be smaller than 1.5×10^5 Hz and 3.5×10^5 Hz for the shear wave and dilatational wave of the first kind, respectively. Corresponding angular frequencies are about 10^6 rad./sec. and 2.2×10^6 rad./sec. As seen in Figs. 3, 4 and 5, a transient state from low to high frequency states (characteristic state) is included in these frequency ranges.

Next, let us consider typical silt and normally consolidated clay whose diameter are 0.01 mm and 0.0005 mm, respectively. A procedure similar to that with sand gives

Fig. 8(a-d). Change of dilatational wave of second kind propagation properties with B' value for $\Omega = 1$: (a) wave velocity, (b) amplitude ratio, (c) phase lag and (d) logarithmic damping ratio

a frequency range in which the formulation is valid of less than about 2.5×10^6 Hz and 5×10^7 Hz, respectively; corresponding angular frequencies are about 1.5×10^7 rad./sec. and 3×10^8 rad./sec. The characteristic state of the silt is included in this range, but that of the clay is not included. When the diameter of the soil particle becomes smaller, the frequency range in which the present theory is valid expands into a higher frequency. At the same time, characteristic states also move toward a high frequency region because permeability becomes smaller as soil particles become smaller. Therefore, the validity of the theory depends on which effect becomes predominant in a high frequency range.

Permeability of many soils is proportional to the square of their effective grain size (Kubota et al., 1976). As characteristic circular frequency ω_0 is proportional to the inverse of permeability, it is inversely proportional to the square of the grain size. On the other hand, the minimum wavelength above which the assumption of continuum is valid is nearly proportional to the diameter. As wave velocities are constant in a high frequency region, maximum frequency under which the assumption of continuum is valid is roughly inversely proportional to the grain size. Therefore, if the grain size becomes smaller, the characteristic circular frequency may become larger

than the circular frequency under which the assumption of continuum holds because the former is inversely proportional to the square of the grain size whereas the latter is inversely proportional to the grain size itself. The above examination indicates that this boundary exists between silt and clay. Namely, wave propagation property in the high frequency region cannot be predicted by Biot's theory or the continuum assumption for clay or soils with a permeability smaller than that of clay.

Applicability of Darcy's law may be another item to be discussed in order to discuss the applicability of Biot's equation. There are, however, no available data on the applicability of Darcy's law in the high frequency region as far as the authors know. If the frequency within which Darcy's law holds is smaller than the above-discussed frequency, the applicability of Biot's equation becomes narrower. However, even in this case, if a linear relationship holds between relative velocity and interaction force, the presented formulation is valid when k in Eq. (2) is replaced by the relevant value.

EVALUATION OF MECHANICAL PROPERTIES FROM WAVE PROPERTIES

As explained in the preceding sections, wave properties

Fig. 9(a-d). Effect of B' value on shear wave propagation properties: (a) wave velocity ratio, (b) amplitude ratio, (c) phase lag and (d) logarithmic damping

and their frequency dependent characteristics are essential to the mechanical properties of the soils through which the waves travel. This indicates that mechanical properties can be evaluated by measuring the frequency dependent properties of the wave. The flow of the evaluation is as follows:

—Mass densities of solid particles ρ_s and pore fluid ρ_f :

- ρ_s is directly measured in tests on disturbed soil samples obtained from in situ.

- ρ_f is a function of degree of saturation S_r , which can be evaluated in the latter step of the proposed procedure as indicated by Eq. (3); hence, ρ_f is tentatively estimated as a primary approximated value with $\rho_f \approx \rho_w$ for the materials under the ground water table, or by Eq. (3) with an assumed value of S_r .

—Porosity n and bulk densities of soils ρ_d and ρ_t :

- n is calculated from the shear wave velocities $V_{s(0)}$ and $V_{s(\infty)}$ which are determined from the observed V_s - ω relationship by

$$n = \frac{(V_{s(\infty)} / V_{s(0)} - 1) \rho_s}{\rho_f - (V_{s(\infty)} / V_{s(0)} - 1) \rho_s} \quad (39a)$$

or from the dilatational wave velocities at the high frequency state $V_{d1(\infty)}$ and $V_{d2(\infty)}$ by

$$n = \frac{2V_{de(\infty)} / V_{d2(\infty)} \rho_f \rho_t - 1}{2V_{d1(\infty)} V_{d2(\infty)} \rho_f^2} \quad (39b)$$

- ρ_d and ρ_t are calculated from Eq. (34).

—Wave properties at characteristic state, $\omega = \omega_0$:

In this state, properties are derived from the solutions by substituting $\Omega = 1$ resulting in

$$V_{s(1)} = \sqrt{\frac{G_s}{\sqrt{n^2 \rho_f^2 + \{2(1-n)\rho_s + n\rho_f\}^2 + 2(1-n)\rho_s + n\rho_f}}} \quad (40a)$$

$$\psi_{s(1)} = \sqrt{\frac{\sqrt{n^2 \rho_f^2 + \{2(1-n)\rho_s + n\rho_f\}^2} - \{2(1-n)\rho_s + n\rho_f\}}{\sqrt{n^2 \rho_f^2 + \{2(1-n)\rho_s + n\rho_f\}^2 + \{2(1-n)\rho_s + n\rho_f\}}}} \quad (40b)$$

$$\alpha_{s(1)} = \frac{1}{\sqrt{2}} \quad (40c)$$

$$\Delta\theta_{s(1)} = \frac{\pi}{4} \quad (40d)$$

where subscript (1) indicates that they are values at $\Omega = 1$. By substituting the known quantities, we can obtain

$$V_{s(1)} = \frac{2V_{s(0)} V_{s(\infty)}}{\sqrt{2(V_{s(0)}^4 + V_{s(\infty)}^4) + V_{s(0)}^2 + V_{s(\infty)}^2}} \quad (41)$$

Fig. 10(a-d). Effect of B' value on dilatational wave of first kind propagation properties: (a) wave velocity, (b) amplitude ratio, (c) phase lag and (d) logarithmic damping ratio

$$\psi_{s(1)} = 2\pi \sqrt{\frac{\sqrt{2(V_{s(0)}^4 + V_{s(\infty)}^4) - (V_{s(0)}^2 + V_{s(\infty)}^2)}}{\sqrt{2(V_{s(0)}^4 + V_{s(\infty)}^4) + (V_{s(0)}^2 + V_{s(\infty)}^2)}}} \quad (42)$$

—Characteristic angular velocity ω_o :

ω_o can be determined by either one of five options:

1. from the observed V_s - ω relationships as the angular frequency corresponding to $V_{s(1)}$
2. from the observed ψ_s - ω relationships as the angular frequency corresponding to $\psi_{s(1)}$
3. from the observed α_s - ω relationships as the angular frequency corresponding to $\alpha_s = \alpha_{s(1)} = 1/\sqrt{2}$ from Eq. (40c)
4. from the observed $\Delta\theta_s$ - ω relationship as the angular frequency corresponding to $\Delta\theta_{s(1)} = \pi/4$, from Eq. (40d)
5. with the angular frequency $\omega_{\psi_{s,\max}}$ which corresponds to the maximum attenuation $\psi_{s,\max}$ from the ob-

served ψ_s - ω relationships as

$$\omega_o = \omega_{\psi_{s,\max}} \sqrt{\frac{(1-n)\rho_s}{(1-n)\rho_s + n\rho_f}} = \omega_{\psi_{s,\max}} \frac{V_{s(0)}}{V_{s(\infty)}} \quad (43)$$

Among these 5 methods, the first one seems the simplest. Shear wave velocity of several hundred hertz is measured in practice in suspension PS logging, for example. Measurement at higher frequency may be possible by improving this apparatus. If waveforms at two point can be measured, methods using phase lag (4th method) becomes possible. A method using logarithmic damping ratio (2nd method) also becomes possible because the number of waves within two points can be evaluated from the wave velocity. Other methods seem difficult to use at present because it is difficult to measure them. In the future, however, this may become possible if a relevant method to measure them is developed.

—Permeability k :

$$k = \frac{ng\rho_w}{\rho_f\omega_o} \quad (44)$$

—Stiffness of solid phase: G_s , E_{us} , E_s and ν_s :

$$G_s = \rho_t \cdot V_{cs(0)}^2 \quad (45a)$$

Fig. 11(a-d). Effect of B' value on dilatational wave of second kind propagation properties: (a) wave velocity, (b) amplitude ratio, (c) phase lag and (d) logarithmic damping ratio

or

$$G_s = \rho_d \cdot V_{cs(\infty)}^2 \quad (45b)$$

$$E_{us} = \frac{(1-n)\rho_s \rho_f (V_{d1(\infty)}^2 + V_{d2(\infty)}^2) - (1-n)\{n\rho_f + (1-n)\rho_s\}\{(1-n)\rho_f + n\rho_s\} V_{d1(0)}^2}{n\{(2-n)\rho_f - (1-n)\rho_s\}} \quad (46)$$

$$E_s = \frac{3E_{us} - 4G_s}{E_{us} - G_s} G_s \quad (47)$$

$$v_s = \frac{E_s - 2G_s}{2G_s} \quad (48a)$$

or

$$v_s = \frac{E_{us} - 2G_s}{2(E_{us} - G_s)} \quad (48b)$$

—Stiffness of fluid phase: B_f , K_f and B' :

$$B_f = \frac{\rho_f \{n\rho_f + (1-n)\rho_s\} V_{d1(0)}^2 - (1-n)\rho_s \rho_f (V_{d1(\infty)}^2 + V_{d2(\infty)}^2)}{n\{(2-n)\rho_f - (1-n)\rho_s\}} \quad (49)$$

$$K_f = nB_f \quad (50)$$

$$B' = \frac{E_{us}}{E_{us} + B_f} \quad (51)$$

—Degree of saturation S_r :

$$S_r = \frac{K_f K_w - K_a K_w}{K_f K_w - K_f K_a} \quad (52)$$

Fig. 12(a-d). Effect of permeability on shear wave propagation properties: (a) wave velocity, (b) amplitude ratio, (c) phase lag and (d) logarithmic damping

In the preceding, we showed that there is a limitation of frequency above which Biot's equation is not applicable. This seems to indicate that any method that uses quantities defined at the infinite frequency cannot be used. As seen in Figs. 3 to 5, however, wave velocity, amplitude ratio and phase lag are constant in the high frequency region and are equal to the values at infinite frequency. Therefore, one can use these values instead of values at infinite frequency. Of course, if no region in which these quantities show a constant value at high frequency can be found within the applicable frequency range of Biot's equation, the proposed method cannot be used.

CONCLUDING REMARKS

Frequency dependent properties of one-dimensional waves propagating in saturated soil are investigated based on the closed form solution derived from Biot's equation for poro-elastic material. Various soils with typical physical and mechanical properties are examined in order to grasp the effect of the frequency dependent nature on engineering practice and to clarify the limitations of Biot's equation. It is concluded that, under the elastic wave propagation, coupled treatment between solid and

fluid phases may not be necessary in most earthquake engineering practice in which frequencies less than several tens Hz are involved; the undrained assumption usually holds. The only exception may be gravel, which shows a frequency dependent nature from about 15 Hz. If the frequency range involved increases to several hundreds of Hz, such as in suspension method PS logging, the frequency dependent nature cannot be neglected for gravel, but it need not be considered for other soils. If the frequency range increases to several thousands of Hz such as in acoustic emission, frequency dependent characteristics appear in loose sand, and possibly in dense sand.

Since the characteristic frequency increases inversely proportional to the square of the grain size, and the maximum frequency below which the assumption of continuum holds increases inversely proportional to grain size, the frequency dependent nature, especially properties at a high frequency state, predicted by the Biot's theory becomes invalid as the grain size decreases. The boundary exists between silt and clay. In summary, the theory derived here is valid from silt to gravel.

One of the important features of the frequency dependent property shown here is that it makes it possible to measure in situ the mechanical properties of the soil. In the ordinary ground exploration with elastic wave

Fig. 13(a-d). Effect of permeability on dilatational wave of first kind propagation properties: (a) wave velocity, (b) amplitude ratio, (c) phase lag and (d) logarithmic damping ratio

propagation, only a few mechanical properties of soils such as shear modulus and one-dimensional compressibility are calculated from the measured velocities of shear and dilatational waves. If the measurement of some other wave properties such as wave attenuation, amplitude ratio and phase lag becomes possible in the relationship with varying wave frequency, it becomes possible to evaluate more mechanical properties of soils, such as densities, porosity, permeability, saturation of pore, and volume compressibility of pore fluid based on the formulation presented here.

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NOTATION

α_s : amplification factor of shear wave
 α_{d1}, α_{d2} : amplitude factors of dilatational waves
 δ_s : exponential attenuation distance of shear wave [L]

δ_{d1}, δ_{d2} : exponential attenuation distances of dilatational waves [L]
 Δe_{sij} : strain increment tensor of solid phase
 Δe_{fij} : strain increment tensor of fluid phase
 $\Delta \theta_s$: phase lag of shear wave
 $\Delta \theta_{d1}, \Delta \theta_{d2}$: phase lags of dilatational waves
 λ_s : Lamé's constant of solid phase [$L^{-1}MT^{-2}$]; $\lambda_s = 2\nu_s G_s / (1 - 2\nu_s)$
 ρ_a : mass density of pore air [$L^{-3}M$]; $\rho_a = 0.0$ (kg/m³)
 ρ_d : bulk density of dry soil [$L^{-3}M$]; $\rho_d = (1 - n)\rho_s$
 ρ_f : bulk density of fluid [$L^{-3}M$]; $\rho_f = S_f \rho_w + (1 - S_f)\rho_s$
 ρ_s : mass density of soil particles [$L^{-3}M$]
 ρ_t : bulk density of wet soil [$L^{-3}M$]; $\rho_t = n\rho_f + (1 - n)\rho_s$
 ρ_w : mass density of pore water [$L^{-3}M$]; $\rho_w = 1000.0$ (kg/m³)
 σ_{sij} : effective stress tensor of solid phase [$L^{-1}MT^{-2}$]
 σ_{fij} : stress tensor of fluid phase [$L^{-1}MT^{-2}$]; $\sigma_{fij} = p\delta_{ij}$
 ν_s : Poisson's ratio of solid phase
 ω : angular frequency [T^{-1}]
 ω_0 : characteristic angular frequency [T^{-1}]; $\omega_0 = ng\rho_w/k\rho_f$
 Ω : normalized angular frequency; $\Omega = \omega/\omega_0$
 ψ_s : logarithmic damping ratio of shear wave
 ψ_{d1}, ψ_{d2} : logarithmic damping ratios of dilatational waves
 b_i : body force vector per unit mass [$L T^{-2}$]; $b_i = \{0, 0, g\}$ under gravitation in z-axis direction
 B' : Skempton's B -value in one dimensional deformation;
 $B' = K_f / (K_f + nE_{us})$
 B_f : averaged bulk modulus of fluid phase [$L^{-1}MT^{-2}$]
 E_s : Young's modulus of solid phase [$L^{-1}MT^{-2}$]; $E_s = 2(1 + \nu_s)G_s$

Fig. 14(a-d). Effect of permeability on dilatational wave of second kind propagation properties: (a) wave velocity, (b) amplitude ratio, (c) phase lag and (d) logarithmic damping ratio

- E_{us} : one-dimensional stiffness of solid phase [$L^{-1}MT^{-2}$]; $E_{us} = \lambda_s + 2G_s = E_s(1 - \nu_s)/(1 + \nu_s)(1 - 2\nu_s)$
- g : gravity acceleration [LT^{-2}]
- G_s : shear modulus of solid phase [$L^{-1}MT^{-2}$]; $G_s = E_s/2(1 + \nu_s)$
- h_s : critical damping ratio of shear wave
- h_{d1}, h_{d2} : critical damping ratios of dilatational waves
- k : coefficient of permeability [LT^{-2}]
- K_a : bulk modulus of air [$L^{-1}MT^{-2}$]
- K_f : bulk modulus of fluid phase [$L^{-1}MT^{-2}$]; $1/K_f = S_r/K_w + (1 - S_r)/K_a$
- K_s : bulk modulus of soil particles [$L^{-1}MT^{-2}$]
- K_w : bulk modulus of water [$L^{-1}MT^{-2}$]
- L_s : wave length of shear wave [L]
- L_{d1}, L_{d2} : wave lengths of dilatational waves [L]
- n : porosity
- p : pore fluid pressure [$L^{-1}MT^{-2}$]
- s_{ij} : Stokes' drag tensor [$L^{-3}MT^{-1}$]; $s_{ij} = (n\rho_w g/k)\delta_{ij}$ for isotropic permeability
- S_r : degree of saturation of pore
- Δu_n : displacement increment vector of fluid phase [L]
- Δu_s : displacement increment vector of solid phase [L]
- V_s : velocity of shear wave [LT^{-1}]
- V_{d1}, V_{d2} : velocity of dilatational waves [LT^{-1}]

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