ONE-DIMENSIONAL COMPRESSION BEHAVIOUR OF UNIFORMLY GRADED SAND RELATED TO SINGLE PARTICLE CRUSHING STRENGTH

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ABSTRACT

In order to compare one-dimensional compression behaviour with the crushing strength characteristics of single particles, one-dimensional compression and single particle crushing tests were carried out on various granular materials with uniform gradings. The average characteristic tensile stress acting on a particle in a sample was calculated using a simplified approach. The one-dimensional compression yield stress was related to the particle size and this was also related to the single particle crushing strength. The decrease of the vertical yield stress with increasing initial void ratio can be explained by the increase in the particle characteristic stress as the void ratio increased and a corresponding decrease in co-ordination number. The single particle strengths were compared with the characteristic tensile stress for a particle embedded in the soil matrix. The logarithm of the characteristic tensile stress at maximum compression index for a particle embedded in a granular matrix could be related to the logarithm of the single particle strength multiplied by a proportional factor. The introduction of a factor was necessary because the calculated average characteristic tensile stress did not take into account the non-uniform distribution of inter-particle stresses. The ratio of the single particle strengths to the average characteristic tensile stress for a particle embedded in the soil matrix was considered to be an indicator of the non-uniform distribution. This ratio decreased and hence uniformity of inter-particle stress distribution increased with angularity and surface roughness.

Key words: compression, (microscopic stress), particle crushing, sand, (single particle crushing strength), yield (IGC: D5)

INTRODUCTION

A yield stress can be defined for soil subjected to one-dimensional compression at the point where the relation of the deformation to the stress increment changes rapidly on a semi-logarithmic plot. This stress is an important parameter in geotechnical engineering when considering settlements. It is well known that this yield stress is related to the point of initiation of particle crushing and that after yielding, the degree of particle crushing increases rapidly. High pressure one-dimensional compression tests on sand have been carried out by many researchers (e.g. Terzaghi and Peck, 1948; De Souza, 1958; Hendron, 1963; Hagerty et al., 1993; Yamamuro et al., 1996; Nakata et al., 2001). The degree of particle crushing has been evaluated from the change in the particle size distribution curves prior to and after testing. From these tests it has already been established that particle crushing is affected by particle size, initial density, parti-

cle shape and mineral composition. However, little is known about yield characteristics in the context of the strength for individual particle crushing.

Particle crushing occurs when the force acting on a grain exceeds the crushing strength, even if the breakage of the grain is in a soil matrix (which might be termed collective crushing). The relationship between single particle and collective crushing behaviours is not straightforward because the loading of a particle in a matrix cannot be exactly modelled by single particle crushing and the relationship between macroscopic and microscopic stresses is complex. It is important however to discuss the mechanical behaviour of a soil in the context of the crushing characteristics of individual particles in order to understand the effects of particle crushing on the mechanical behaviour of soils.

Nakata et al. (1999b) first examined the statistics of crushing of individual particles in a soil matrix by seeding marked particles in triaxial samples subjected to

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shearing and compression. This work was later developed by Nakata et al. (1999a and 2001) who presented more detailed statistical data related to the degree of crushing of individual particles inserted in a silica sand matrix and subjected to one-dimensional compression. In the case of uniformly graded sand it was found that major splitting of particles mainly occurred between the yield stress and the point at which the compression index reached a maximum. It was observed that for this sand, immediately after yielding, 50% of the particles had undergone major splitting. For well-graded sand there was a distinct difference in the crushing behaviour depending on the particle size. In the zone between the yield stress and the stress at the maximum compression index, an increase in the breaking of asperities and surface grinding occurred for the large seeded particles. For smaller particles on the other hand, the particle crushing in this region was characterised by a clear increase in major particle splitting rather than the breaking of asperities. These microscopic data indicated that the one-dimensional compression behaviour could be related to single particle crushing.

A series of one-dimensional compression tests was therefore carried out on several different uniformly graded granular materials in order to examine the influence of particle size, initial void ratio, angularity and mineralogy on the compression and yield characteristics of the materials tested. Single particle crushing tests were also performed on the same materials to investigate the individual particle crushing characteristics and the relationship between one-dimensional compression and single particle crushing behaviour.

ONE-DIMENSIONAL COMPRESSION BEHAVIOUR

Characteristic Tensile Stress Acting on a Single Particle during a Test

In order to investigate the particle crushing behaviour in more detail, it is necessary to examine the average force or stress acting on a single grain. The average force acting on a particle embedded in a sample has been calculated using the following simplified approach (Nakata et al., 1999b). The volume of solids V_s in a unit volume of soil is given by:

$$V_s = \frac{1}{1+e} \quad (1)$$

where e is the void ratio. The number of sand particles per unit volume is then given below by making the simplifying assumption of a soil with single size particles:

$$N = \frac{V_s}{V_{sp}} \quad (2)$$

where V_{sp} is the volume of a single particle.

In a unit cubic volume containing N particles, any straight line of unit length will cut through $N^{1/3}$ particles. Thus the number of particles per unit cross sectional area can be then calculated as $(N^{1/3})^2$. The force F_{sp} acting on

a single particle in the specimen is then given by dividing the normal stress σ by the number of particles across a plane of unit cross section.

$$F_{sp} = \frac{\sigma}{N^{2/3}} \quad (3)$$

In order to calculate the particle tensile stress, the average volume of a single particle is taken as:

$$V_{sp} = \frac{1}{6} \pi \bar{d}^3 \quad (4)$$

where $\bar{d}$ is the mean particle diameter.

The tensile stress in a sphere subject to point loads F_{sp} across the diameter d can be shown to be:

$$\sigma_t = \frac{\chi F_{sp}}{d^2} \quad (5)$$

where it was suggested by Billam (1972) that $\chi = 2.0/\pi$ and by Hiramatsu and Oka (1966) that $\chi = 0.9$.

The authors have therefore defined the tensile stress in terms of a characteristic stress, σ_{sp} , following Jaeger (1967), Lee (1992), Shipway and Hutchings (1993), McDowell et al. (1996) and McDowell and Bolton (1998).

$$\sigma_{sp} = \frac{F_{sp}}{d^2} \quad (6)$$

Substituting Eqs. (1), (2) and (4) into (3) gives the force on a single particle embedded in a soil matrix:

$$F_{sp} = \sigma \left(3 \sqrt{\frac{(1+e)\pi}{6}} \right)^2 \bar{d}^2 \quad (7)$$

and the corresponding characteristic tensile stress

$$\sigma_{sp} = \frac{F_{sp}}{\bar{d}^2} = \sigma \left(3 \sqrt{\frac{(1+e)\pi}{6}} \right)^2. \quad (8)$$

It can be seen in Eq. (8) that, for a given set of applied stresses, the average characteristic stress is not dependent on the particle size but instead is a function of the void ratio.

Effect of Particle Size

One-dimensional compression tests were carried out on dry sand samples 50 mm in diameter and 10 mm in height at a constant strain rate of 0.01 mm/min. Initial tests had been carried out on samples 20 mm in height but it was found that the results hardly differed from those obtained from 10 mm samples. Thus a height of 10 mm was adopted for the test programme. A full list of the materials used is shown in Table 1. Samples of each material were prepared to various void ratios by lightly tamping with a 30 mm diameter tamper until the required void ratio was reached.

Figure 1 shows the e -log σ_v curve for uniformly graded samples of Silica sand having an initial void ratio of 0.666 and particle sizes of 1.4–1.7 mm, 0.6–0.71 mm, and 0.25–0.3 mm. Each sand has a clear yield point. The yield stress for Silica sand depends on the particle size, increasing with decreasing particle size. It has been estab-

Fig. 1. Effect of particle size on e -log σ_v curve Fig. 2. Effect of initial void ratio on e -log σ_v curve

Table 1. List of materials used

Material	Grains size (mm)	G_s	e_{max}	e_{min}	D_{50}
Silica _{1.4-1.7}	1.4-1.7	2.639	0.881	0.632	1.550
Silica _{0.6-0.71}	0.6-0.71	2.659	0.968	0.659	0.655
Silica _{0.25-0.3}	0.25-0.3	2.648	1.088	0.666	0.275
Toyouura	0.106-0.25	2.636	0.977	0.609	0.202
Aio	0.85-2.0	2.631	0.971	0.699	1.138
Masado	1.4-1.7	2.600	1.216	0.847	1.550
Glass ballotini (G.B.)	0.85-1.0	2.490	0.699	0.600	0.925
Angular glass (A.G.)	0.85-1.0	2.505	1.145	0.746	0.925

lished that the compressibility of sands increases with particle crushing and that the yield point is related to the threshold stress for particle crushing when subjected not only to one-dimensional compression but also to isotropic pressures (e.g. Miura and Yamanouchi, 1972 and 1977; Miura et al., 1984; Yasufuku et al., 1991; Miura and Yagi, 1997). It has been shown by Nakata et al. (2001) that the compression of a granular material at the point of maximum compression index is strongly characterised by major splitting of grains. Examination of Fig. 1 shows that the yield stress increases as the particle size decreases. Thus it can be inferred that the single particle strength increases as the particle size decreases. Evidence to support this inference will be presented later in this paper.

Effect of Initial Void Ratio

The e -log σ_v curves for three samples of Toyoura sand prepared to different void ratios are shown in Fig. 2. In this case, the yield stress decreased with increase in initial void ratio as has been found by many other researchers, e.g. Hagerty et al. (1993) and Pestana and Whittle (1995). It can also be seen that the lines merge after yielding, approaching a common e -log σ_v curve. Examination of Eq. (8) shows that the characteristic tensile stress σ_{sp} increases as the void ratio increases. Field (1963) and Oda (1977) both suggested that the average number of contact points per particle, often defined as the co-ordination number

Fig. 3. Effect of particle angularity on e -log σ_v curve

C_a increases with decreasing void ratio. Field (1963) showed that the average co-ordination number is:

$$C_a = \frac{12}{1+e} \quad (9)$$

An analysis by Jaeger (1967) suggested that the tensile stress for a particle in a matrix decreases with increasing co-ordination number, thus further explaining why the yield stress increases with decreasing void ratio.

Effect of Particle Angularity

The results for samples of glass ballotini and crushed glass particles each having a particle size range of 0.85-1.0 mm are shown in Fig. 3. In each case the samples were prepared to the same initial relative density of 100% giving an initial void ratio of 0.74 for the angular glass (AG) and 0.6 for the glass ballotini (GB). It can be seen that the yield stress for angular glass is 6 MPa while for glass ballotini it reaches 20 MPa. The e -log σ_v for angular glass after yielding is smooth while for glass ballotini it is discontinuous at the yield point with a very high cur-

vature (Nakata et al., 1999a). The void ratio decreases sharply by 0.25 at a constant σ_v value. Hagerty et al. (1993) showed that there was a higher degree of crushing for angular glass of the same size and concomitantly showed that the angular material developed larger compressive strains in the early stages of loading. This can be seen reflected in our results in Fig. 3, where the angular glass has a steeper initial e -log σ_v curve. It can also be seen in this figure that, following yield, the curves for both types of glass merge into a single line.

Effect of Mineral Composition

Samples of three different sands, viz. Silica sand, Aio sand and Masado, were each prepared to 100% relative density and similar particle size ranges. The mineral composition of individual particles was as follows: Silica sand, 100% quartz; Aio sand, 70% quartz, 30% feldspar; Masado, 30% quartz, 40% feldspar and 30% mixed mineral particles (quartz and feldspar). The sample with the most angular particles was Masado while the least angular was Aio sand. Figure 4 shows the e -log σ_v curves for the three different sands. Notwithstanding the variation in initial void ratio and angularity of each of the sands the yield stress seemed to increase as the quartz content increased. It can also be seen that each of the sands merged onto a single post-yield curve. It would seem that there was no influence due to the mineral composition in the post-yield region.

Figure 5 shows the data for an additional series of tests carried out on the same sands but this time with the same initial void ratio. In this case although the yield points lie closer together, the yield stress still increases with quartz content. It could be argued that the quartz content influenced the curvature at the yield point; however, this effect cannot be verified since each of the sands had a different degree of angularity.

Compression Index C_c

Figures 6(a)–(e) are derived from the same data as shown in Figs. 1–5. Figure 6(a) shows the effect of particle size on the variation of compression index. The compression index for Silica_{1.4-1.7} was 0.02 for a vertical stress of 0.4 MPa. As the vertical stress increased, the index increased gradually until point B (4 MPa). After this, the compression index increased markedly until it reached a peak of 0.7 at $\sigma_v = 18$ MPa (point A). The gradients are related to the curvature of the e -log σ_v curve, with the higher gradients being for higher curvatures. The yielding region occurs prior to the peak compression index. Thus for a higher gradient, this yielding region, which is the region between point B and point A, becomes narrower. After this peak, the index dropped to 0.4. Here although the peak value of C_c was similar for each particle size, the peak value occurred at a lower vertical stress for the larger grain sized materials. Figure 6(b) shows the influence of the initial void ratio. In this case the maximum value of C_c is independent of initial void ratio. But the value of C_c in the initial stages is higher as the initial void ratio is higher. Thus the gradients during yield are

Fig. 4. Effect of mineral composition on e -log σ_v curve (initial relative density = 100%)

Fig. 5. Effect of mineral composition on e -log σ_v curve (initial void ratio = 0.75–0.84)

steeper for a lower initial void ratio. Figure 6(c) shows the effect of particle angularity. The stress at maximum compression index and peak gradient C_c are both lower for the angular glass while the data, although scattered for the glass ballotini, suggests they have a higher yield stress and narrower yield region. Finally, Figs. 6(d) and (e) show the effect of mineralogy with the silica sand having a much steeper curve and higher peak than the Masado or Aio sands, which have a lower quartz content. In Figs. 6(c)–(e) the compression index C_c at the end of the test was always 0.4.

Yield Characteristics during One-Dimensional Compression

In Fig. 7(a) the vertical yield stress $(\sigma_v)_y$ is plotted against the median particle size D_{50} while in Fig. 7(b) the stress at the maximum compression index $(\sigma_v)_{C_c=\max}$ is plotted against the median particle size D_{50} . The yield stress is defined as the point of maximum curvature. A list of the one-dimensional compression test results is

Fig. 6. Variation of compression index during one-dimensional compression: (a) effect of particle size, (b) effect of initial void ratio, (c) effect of particle angularity, (d) effect of mineral composition (initial relative density=100%) and (e) effect of mineral composition (initial void ratio=0.75-0.84)

shown in Table 2. In Figs. 7(a) and (b) the dashed lines are the least squares regressions for the silica sand which was 100% quartz. In both cases the stress reduces with increasing median particle size. It can be seen that the median particle size has a greater effect on the level of stress at peak compression index than on the yield stress $(\sigma_v)_y$. The different gradients indicate that the different particle crushing characteristics influence the two stress levels (Nakata et al., 2001).

In Figs. 8(a) and (b) the logarithms of $(\sigma_v)_y$ and

$(\sigma_v)_{C_c=\max}$ are plotted against the logarithm of initial void ratio. The vertical yield stress increases with decreasing initial void ratio, as indicated by the linear relation on the figure. On the other hand, the stress at the maximum compression index only slightly increases with decreasing initial void ratio. Thus for a given soil, the stress at the peak compression index is almost the same irrespective of the initial void ratio.

As was seen in Figs. 1-5, the curvature of the e -log σ_v curves is dependent on the initial void ratio, grain shape

Fig. 7. Effect of average grain size on yield stress: (a) $\log(\sigma_v)_y$ - $\log D_{50}$ relationship and (b) $\log(\sigma_v)_{C_c=\max}$ - $\log D_{50}$ relationship

and mineral content. A parameter has been defined which is a measure of the curvature of the e - $\log \sigma_v$ curve in the yield zone.

$$R = \log \left[\frac{(\sigma_v)_{C_c=\max}}{(\sigma_v)_y} \right] \quad (10)$$

This parameter R decreases with increasing curvature with a limiting value of zero. It can be seen in Figs. 9(a) and (b) that R generally increases with increasing initial void ratio and also with the range of void ratios between maximum and minimum, $e_{\max}-e_{\min}$. This latter parameter according to Yoshimura and Ogawa (1993) is related to particle shape, increasing with increasing particle angularity. This parameter will be examined in more detail later in this paper.

Pestana and Whittle (1995) have developed a constitutive model for compression using linear $\log e$ - $\log \sigma_v$ relationships and a sub-loading (boundary) surface for plasticity originally developed by Hashiguchi and Ueno (1977) and Dafalias (1979). One of the parameters used in this model is a curve fitting parameter for the curvature of the elasto-plastic yielding zone. This parameter increases with decreasing curvature of the e - $\log \sigma_v$ curve and increasing uniformity coefficient. Pestana and

Table 2. One-dimensional compression test results

Material	Grains size (mm)	D_{r0} (%)	e_0	$(\sigma_v)_y$ (MPa)	$(\sigma_v)_{C_c=\max}$ (MPa)	R
Silica _{1.4-1.7}	1.4-1.7	100	0.632	11.94	17.69	0.39
		87.6	0.666	9.63	17.86	0.62
		45	0.769	8.23	15.71	0.65
Silica _{0.6-0.71}	0.6-0.71	100	0.659	11.62	31.75	1.01
		96.6	0.666	13.44	34.86	0.95
		45	0.829	8.22	22.46	1.01
Silica _{0.25-0.3}	0.25-0.3	100	0.666	16.91	60.23	1.27
		45	0.898	10.41	44.73	1.46
Toyoura	0.106-0.25	100	0.617	17.46	49.89	1.05
		80	0.683	13.01	39.95	1.12
		45	0.812	8.73	42.21	1.58
Aio	0.85-2.0	100	0.695	5.26	13.34	0.93
		80	0.753	4.79	10.81	0.81
		45	0.845	2.64	9.58	1.29
Masado	1.4-1.7	100	0.838	2.23	7.44	1.20
		80	0.918	1.82	7.11	1.36
		45	1.045	1.68	9.12	1.69
Glass ballotini (G.B.)	0.85-1.0	100	0.600	26.49	27.33	0.03
		93	0.607	26.94	32.26	0.18
		45	0.648	27.00	27.33	0.01
Angular glass (A.G.)	0.85-1.0	100	0.744	4.78	10.46	0.78
		86	0.801	3.68	9.55	0.95
		45	0.956	2.08	8.06	1.36

Whittle (1995) hinted at a qualitative relation between the curvature and the particle shape. Figure 9(b) is a qualitative evaluation of this relation. Nakata et al. (2001) have also shown that the curvature is related to the microscopic particle crushing characteristics. A full list of R values for the materials used is shown in Table 2.

Particle Size Distributions

After testing, samples were removed, broken up under finger pressure and spread in a tray. There was no evidence of any cementation under the high stress regimes. Figures 10(a)-(c) show the particle size distributions before and after testing. It can be seen that although the original particle size distributions were quite different, following the compression tests the particle grading curves evolved such that they converged in the smaller particle size region. In all cases the fines content (<0.074 mm) exceeded 10%. The values of D_{40}/D_{10} and D_{30}/D_{10} were approximately 4.5 and 3.0 respectively, excepting Silica_{0.025-0.3} and Toyoura sand. These values are similar to those obtained by Nakata et al. (2001) for final particle size grading curves after one-dimensional compression testing on samples with various initial particle size distribution curves. It would appear reasonable to hypothesize a critical particle size distribution for one-dimensional compression towards which the grading curves evolve as they merge (Nakata et al., 2001). Indeed McDowell et al. (1996) pointed out that the theoretical evolution of particle size distributions for crushing under high pressures is self-similar or fractal. Thus the particle size distribution

Fig. 8. Effect of initial void ratio on yield stress: (a) $\log(\sigma_v)_y - \log e_0$ relationship and (b) $\log(\sigma_v)_{C_c=\max} - \log e_0$ relationship

curve should become linear on a log-log scale. It can be seen in Fig. 10(c) that although the evolved distributions on a log-log scale are generally non-linear, it is interesting to note that an almost linear particle size distribution has begun to develop for particles of less than 0.1 mm.

SINGLE PARTICLE CRUSHING STRENGTH

Figure 11 shows a schematic diagram of the apparatus used for particle crushing (Nakata et al., 1998, 1999a, 1999b). The test was carried out by placing a particle between hardened platens and then moving the lower platen at a constant rate of displacement to crush the particle. The diameter of the removable hardened platens was 30 mm. The load measuring capacity was 500 N with a resolution of 0.01 N. A displacement rate of 0.1 mm/minute was used. During the tests, force and displacement were measured.

Figure 12 shows a typical sawtooth load displacement relationship for a silica particle. A consistent definition of the force required to crush a particle is not straightforward. Two levels are marked on the figure. F_c is the first peak and represents the breakage of an asperity while F_f is the maximum peak loading where major split-

Fig. 9. Curvature of one-dimensional curve during yielding: (a) $R - e_0$ relationship and (b) $R - (e_{\max} - e_{\min})$ relationship

ting of the particle has occurred. A single particle crushing strength σ_f was defined similar to Eq. (6):

$$\sigma_f = \frac{F_f}{d_0^2} \quad (11)$$

where d_0 is the distance between the platens at the start of the single particle crushing test and F_f is the load at failure, as defined in Fig. 12. Figure 13 shows the relationship between the single particle crushing strength σ_f and the initial particle diameter. A regression line drawn through this data on a log-log scale has a slope of -0.79 . The grain size dependency of strength has already been well established by Braddick (1963), Marsal (1969), Billam (1972), Lee (1992), Fukumoto and Hara (1998) and Nakata et al. (1999b). The gradient of this line is similar to the value of -0.75 reported by Braddick (1963) based on tensile tests by Reinkoker on quartz fibres. The dependency of the one-dimensional compression yield stress on the particle size is directly related to the single particle crushing strength as already shown in Figs. 1 and 7.

Weibull (1951) proposed a distribution function for the breaking of blocks in tension such that:

Fig. 10. Grain size distribution curve: (a) initial curve, (b) after test, semi-logarithmic scale and (c) after test, logarithmic scale

$$P_s = \exp \left[- \left(\frac{\sigma}{\sigma_0} \right)^m \right] \quad (12)$$

where P_s is the probability of survival of a particle under a characteristic stress defined as:

$$P_s = \frac{\text{number of particles crushing at } \sigma \geq \sigma_f}{\text{total number of particles tested}} \quad (13)$$

Fig. 11. Schematic diagram of particle crushing apparatus

Fig. 12. Typical load-displacement relationship (Silica_{1.4-1.7})

Fig. 13. Effect of particle size on single particle crushing strength (Silica and Toyoura sands)

Fig. 14. Frequency distributions of single particle crushing strengths for Silica_{1.4-1.7} and Masado: (a) survival probability curve and (b) normalised survival probability curves

In addition, σ_0 is the characteristic tensile stress where 37% of the particles survive and m is the Weibull modulus. The Weibull modulus defines the variability of the particle crushing strength, which increases with decreasing modulus. For brick, pottery and cement, m is about 5 and for engineering ceramics whose variability is less, m is about 10 (Ashby and Jones, 1986).

Figures 14(a) and (b) show the frequency distributions for single particle crushing strengths σ_f expressed as probabilities of surviving grain splitting. In Fig. 14(a) the stress σ_{f0} at which the probability of survival is 37%, is 31 MPa for Silica_{1.4-1.7} and 24.2 MPa for Masado. Figure 14(b) shows the survival probability curves normalised by the appropriate values of σ_{f0} . In this figure, the Weibull curves are drawn using the following equation:

$$P_s = \exp \left[- \left(\frac{\sigma_f}{\sigma_{f0}} \right)^{m_f} \right] \quad (12')$$

where m_f is the Weibull modulus for the characteristic single particle crushing strength.

The modulus for Silica sand was almost 3 while for Masado it was between 1.2 and 1.4. As might be expected

Fig. 15. Relationship between roundness coefficient and $e_{\max} - e_{\min}$

Fig. 16. Relationship between Weibull modulus m_f and roundness coefficient

Table 3. Single particle crushing test results

Material	Grains size (mm)	σ_{f0} (MPa)	m_f
Silica _{1.4-1.7}	1.4-1.7	30.96	3.04
Silica _{0.6-0.71}	0.6-0.71	72.87	2.17
Silica _{0.25-0.3}	0.25-0.3	110.87	1.82
Toyouura	0.106-0.25	147.40	2.17
Aio	0.85-2.0	38.85	1.93
Masado	1.4-1.7	24.18	1.23
Glass ballotini (G.B.)	0.85-1.0	365.80	5.90
Angular glass (A.G.)	0.85-1.0	62.10	2.10

the variability for the Masado, a decomposed granite, was much greater than the single mineral Silica sand. Table 3 gives a full listing of the single particle crushing test results.

Figure 15 shows the relation between the range of void ratios, $e_{\max} - e_{\min}$, and the roundness coefficient defined as:

$$\text{Roundness coefficient} = \frac{(\text{Particle perimeter})^2}{4\pi A} \quad (14)$$

Fig. 17. Relationship between single particle characteristic strengths and characteristic tensile stress at maximum compression index: (a) mean strength, (b) 37% survival strength, (c) 50% survival strength, (d) 70% survival strength and (e) 90% survival strength

where A is the cross-sectional area of the particle.

The minimum value of the roundness parameter is unity for a perfectly spherical particle. The roundness coefficient increases with particle angularity and corresponding increases can be seen for $e_{\max}$ - $e_{\min}$, as pointed out by Yoshimura and Ogawa (1993). The relationship between the Weibull modulus m_f and the roundness coefficient is shown in Fig. 16. There appears to be a relation such that as the roundness coefficient increases so

the variability of the crushing strength decreases. In other words, as the angularity of the grains increases so does the variability of the crushing strength. Therefore, the dependency of the curvature of the e -log σ_v curve in the yield zone on $e_{\max}$ - $e_{\min}$ as shown in Fig. 9(b) is also partly related to the angularity of grains and the variability of the crushing strength.

YIELD STRESS RELATED TO SINGLE PARTICLE CRUSHING STRENGTH

Figure 17(a) shows the relationship between the mean single particle crushing strength $(\sigma_f)_{\text{mean}}$ and the characteristic tensile stress $(\sigma_{sp})_{C_c=\text{max}}$ at the maximum compression index of a particle embedded in the granular matrix defined as:

$$(\sigma_{sp})_{C_c=\text{max}} = \frac{F_{sp}}{d^2} = (\sigma_v)_{C_c=\text{max}} \left(3 \sqrt{\frac{(1+e)\pi}{6}} \right)^2. \quad (15)$$

The dashed line in the figure is where the mean single particle crushing strength would be equal to the characteristic tensile stress of a particle embedded in the soil matrix. It can be seen in this figure that the mean single particle crushing strength is greater than the characteristic tensile stress for a particle embedded in the granular matrix. The logarithm of the characteristic tensile stress at maximum compression index of a particle embedded in a granular matrix can be evaluated from the logarithm of the mean single particle strength multiplied by a proportional factor. This is true for the strength with a 37%, 50% and 70% probability of survival as shown in Figs. 17(b)–(d). The proportional factor decreases with increasing probability of survival. For most of the geomaterials in Fig. 17(e), the single particle stress for a 90% probability of survival coincides with the matrix stresses at maximum compression index. If at the point of the maximum compression index only the criterion of single particle crushing strength is considered then Fig. 17(e) suggests that 90% of the particles would survive. There is evidence from visual examination of seeded particles that at the point of maximum compression index 50% of particles were split (Nakata et al., 2001). Clearly therefore other criteria also have an important influence on the breaking of particles. One criterion that could be considered is the effect of the co-ordination number. In Fig. 17 for a given soil type there is little difference in the value of $(\sigma_{sp})_{C_c=\text{max}}$. The small differences were thought to be due mainly to the effects of co-ordination number. The characteristic tensile stress of a particle embedded in a granular matrix decreases with increasing co-ordination number. Therefore, the overall effect of the co-ordination number on the proportional factor is minor. The main reason for the difference between observed strength and calculated stress is considered to be due to the assumptions made in the calculation of the average characteristic tensile stress acting on a single particle in the soil matrix.

Oda and Konishi (1974), Cundall and Strack (1979), Rothenburg and Bathurst (1992) and Nakase et al. (1992) showed that the force on each particle in the matrix is not the same, such that there are two types of particle loading in the matrix. A proportion of particles will carry large forces while others will only carry small loads or even none at all. Therefore when calculating the characteristic tensile stress acting on a single particle in the matrix, the non-uniform distribution of inter-particle stress should be taken into account. Figure 18 shows the ratio of $(\sigma_f)_{50}$ (the single particle crushing strength at

Fig. 18. Relationship between ratio of 50% survival strength to characteristic tensile stress at maximum compression index and $e_{\text{max}} - e_{\text{min}}$

which 50% of the particles would survive splitting) to $(\sigma_{sp})_{C_c=\text{max}}$ (the calculated matrix particle stress) against $e_{\text{max}} - e_{\text{min}}$. In this figure, the ratio is 13–15 for glass ballotini, 6–7 for angular glass and 2–4 for other geomaterials. The ratio of $(\sigma_f)_{50} / (\sigma_{sp})_{C_c=\text{max}}$ is an indicator of the ratio of active to non-active particles. Thus, in a uniform material such as the glass ballotini, a large proportion of the particles are not carrying any load or only a small load. As the shape of the grain becomes more angular and the surface roughness increases, then the load distribution through the matrix becomes more uniform.

CONCLUSIONS

One-dimensional compression tests were carried out on various granular materials with single sized particles to examine the influence of particle size, initial void ratio, angularity and mineralogy. Single particle crushing tests were also carried out on the same materials to investigate the individual particle crushing characteristics and the relationship between one-dimensional compression and single particle crushing behaviour. The following conclusions were obtained.

- (1) The stresses at the yield point, $(\sigma_v)_y$ and at the maximum compression index, $(\sigma_v)_{C_c=\text{max}}$ reduced with increasing median particle size. Linear log-log relationships were drawn for the 100% quartz silica sand. The median particle size had a greater effect on the level of stress at the peak compression index than on the yield stress.
- (2) The average characteristic tensile stress acting on a particle in a sample was calculated using a simplified theoretical approach which showed that, for a given set of applied stresses, the characteristic stress was not dependent on the particle size but instead was a function of the void ratio.
- (3) The vertical yield stress increased with decreasing initial void ratio. This is because the characteristic ten-

sile stress increases with increasing void ratio and with decreasing co-ordination number. On the other hand, the stress at the peak compression index was almost the same irrespective of the initial void ratio.

- (4) Comparison of samples containing various proportions of quartz and feldspar showed that the stronger mineral, quartz, dominated the compression characteristics with the yield stress increasing as the quartz content increased and each of the compression curves merging with that for a 100% quartzitic sand at high stresses.
- (5) A parameter has been defined which is a measure of the curvature of the e -log σ_v curve in the yield zone. The parameter generally increased with increasing initial void ratio and also with the range of void ratios between maximum and minimum.
- (6) The relationship between single particle strength and particle size on a log-log scale had a slope of -0.79 . The dependency of the one-dimensional compression yield stress on the particle size mentioned in (1) was directly related to the single particle crushing strength.
- (7) There was a relation between the range of void ratios, $e_{\max}-e_{\min}$, and the roundness coefficient of the particle. As the angularity of the grains increased, so did the variability of the crushing strength. The dependency of the curvature for the e -log σ_v curve in the yield zone on $e_{\max}-e_{\min}$ described in (3) was partly related to the angularity of grains or the variability of the crushing strength.
- (8) The single particle strength was compared with the characteristic tensile stress for a particle embedded in the soil matrix. The logarithm of the characteristic tensile stress at maximum compression index of a particle embedded in a granular matrix could be evaluated from the logarithm of the single particle strength multiplied by a proportional factor. The main reason for the factor was that the average characteristic tensile stress did not take into account the non-uniform distribution of inter-particle stresses.
- (9) The ratio of the single particle strength to the characteristic tensile stress for a particle embedded in the soil matrix was an indicator of the ratio of active to non-active particles. In a uniform material such as the glass ballotini, a large proportion of the particles were not carrying any load or only a small load. As the shape of the grain became more angular and the surface roughness increased, then the load distribution through the matrix became more uniform.

NOTATION

C_a :	co-ordination number
C_c :	compression index
d :	diameter of grain under a diametral force
$\bar{d}$:	mean particle diameter
d_0 :	distance between platens at start of single particle crushing test
D_{10} , D_{30} , D_{40} :	largest particle size in the smallest 10%, 30% and 40% of particles

D_{50} :	median particle size
e :	void ratio
$(e_{\max}-e_{\min})$:	range of void ratios between maximum and minimum
F_c :	load at first peak point of single particle crushing test
F_f :	load at maximum peak point where major splitting of single particle has occurred
F_{sp} :	force acting on a single particle in specimen
m :	Weibull modulus (variability) for characteristic tensile strength
m_f :	Weibull modulus for single particle crushing strength
N :	number of sand particles per unit volume
P_s :	probability of survival of a particle
R :	measure of the curvature of the e -log σ_v curve in the yield zone; $\log [(\sigma_v)_{C_c=\max} / (\sigma_v)_y]$
V_s :	volume of solids in unit volume of soil
V_{sp} :	volume of a single particle
σ_0 :	characteristic tensile stress where 37% of the particles survive
σ_f :	single particle crushing strength (major splitting strength of single particle crushing test)
σ_{f0} :	37% survival probability strength (single particle crushing strength where 37% of the particles survive)
$(\sigma_f)_{50}$:	50% survival probability strength
$(\sigma_f)_{70}$:	70% survival probability strength
$(\sigma_f)_{90}$:	90% survival probability strength
$(\sigma_f)_{\text{mean}}$:	mean single particle crushing strength
σ_i :	tensile stress in a sphere subject to point loads
σ_{sp} :	characteristic tensile stress acting on a single particle in specimen
$(\sigma_{sp})_{C_c=\max}$:	characteristic tensile stress acting on a single particle in specimen at the maximum compression index
σ_v :	vertical stress
$(\sigma_v)_y$:	vertical stress at yield point
$(\sigma_v)_{C_c=\max}$:	vertical stress at the maximum compression index

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