THE EFFECTS OF SOIL YIELDING ON SEISMIC RESPONSE OF SINGLE PILES

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ABSTRACT

In the seismic analysis of pile foundations the soil is often assumed to be an elastic material and the pressure at the soil pile interface is not limited during the analysis. This may result in a considerable error, as the computed pressure from an elastic analysis may go well beyond the ultimate lateral pressure of real soil. In fact, significant yielding at the soil-pile interface has been observed during real earthquakes, and also in laboratory tests. The yield zone is usually near the ground surface where the effect of inertial force due to the superstructure is higher. This yielding redefines the pile response, and in general cannot be ignored.

In order to examine the effects of soil yielding on the internal pile response during earthquakes an approximate analysis is described in this paper which is an extension of a static method developed by the second author (1982) for the analysis of piles subjected to lateral soil movement. This method is then used to investigate the effects of soil yielding on the internal response of piles through a comparative study in which real earthquakes are used. It is shown that for strong earthquakes and heavily loaded piles the soil yielding may considerably increase the amount of maximum pile moment developed in the pile. A marked difference in the effects of yielding on the pile moment and shear is observed and discussed.

Key words: earthquake, elasto-plastic analysis, pile foundation, pile internal forces, seismic response, soil yielding (IGC: E4/E8)

INTRODUCTION

In most of the elastic methods available, the pile-soil-superstructure interaction analysis is performed in two phases. In the first phase the superstructure is ignored and the response of the pile head along with its stiffness and damping in different directions are calculated. In this phase the amount of pressure developed at the soil-pile interface is usually smaller than the ultimate lateral soil pressure and the elastic assumption for the soil is largely valid. In the second phase, the superstructure is assumed to be mounted on the springs and dashpots whose coefficients are equal to the head stiffness and damping values obtained in the previous phase and is subjected to the head response also calculated in the first phase. The underlying assumption is that even when the superstructure is included, the pile head impedance remains equal to that for the elastic case and the inevitable yielding of the soil near the surface due to superstructure inertia has no effect on pile head impedance. Although inevitable in an elastic analysis, such an assumption is far from what really happens at the pile-soil interface and a nonlinear methodology is required to model the real case.

Parmelee et al. (1964) and Penzien (1970), who developed the earliest organised methods for dynamic analysis of pile foundations, employed a nonlinear discrete model and assumed that static stress-strain relationships were adequate to explain pile dynamics. Penzien used the Mindlin solution and determined the nonlinear spring constants of a Winkler model. The pile inertial effects were modelled by lumped masses and the radiation damping was accounted for by viscous damping. The major advantage of this method was its ability to model nonlinear behaviour of a homogeneous or non-homogeneous soil mass. This methodology was later extended by Matlock, Reese, Prakash and their co-workers and resulted in the development of the so-called p-y curves. Kagawa (1980) and Kagawa and Kraft (1980, 1981) studied soil-pile-structure interaction via a finite element method which could represent a nonlinear soil response and examined the effect of pile, soil and loading conditions on the p-y curves. They also developed a simple Winkler model for seismic analysis of single piles in which p-y curves are used. In recent years several researchers have developed approximate nonlinear methods for the seismic analysis of single piles and pile groups. Norris (1994) presented an

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“equivalent linear subgrade modulus profile approach” in which the modulus obtained from p-y curves is replaced by an equivalent linear modulus profile. With the soil modulus known, Norris used the available closed form formulae, or static methods, to obtain the dynamic lateral pile head stiffness. El Naggar and Novak (1996, 1995) developed a nonlinear Winkler model for dynamic lateral analysis of pile response. In this model the soil has been divided into two parts; the first part is an inner nonlinear field, and the second part is a linear far field which accounts for wave propagation away from the pile. This method is, in essence, similar to a Winkler method developed by Nogami and his co-workers for nonlinear analysis of pile foundations. Nogami and Konagai (1988, 1987) developed a time-domain Winkler model which for flexural response consists of three Voigt models of springs and dashpots connected in series. The Finite Element Method, a versatile tool in dealing with nonlinear soil behaviour is, in general, computationally very expensive for seismic analysis of pile foundations. Recently Finn and Wu (1996) developed a quasi-3D method which permits dynamic nonlinear effective stress analysis of pile groups in layered soils. In this method the boundary conditions associated with a full 3D analysis have been relaxed through simplified assumptions for the generation of waves at the soil-pile interface.

The available simplified nonlinear methods for dynamic analysis of pile foundations lack the simplicity of equivalent linear methods, and generally do not help in drawing general conclusions about the seismic behaviour of pile foundations. In this paper a simplified method based on engineering judgement is presented which is especially useful when the effect of soil yielding is to be determined. This method, which is applicable to single piles, is an extension of a static methodology developed by Poulos (1973, 1982) for piles subjected to lateral soil movement.

The need for such a study was felt warranted due to the fact that in recent years simple dynamic linear Winkler methods have become available and popular amongst designers. As noted by Pender (1994) these methods do not need a lot of computer coding and can be easily used via simple spread sheet calculations. The danger is that the attractive simplicity of these methods make designers complacent about their limitations. The most significant limitation is of course the soil yielding which is often inevitable in any lateral pile loading.

**METHODOLOGY**

The pile is assumed to be a thin vertical strip of width $d$, length $L$, and constant flexibility $E_sI_s$, and is divided into $n+1$ elements, all elements being of equal length $\delta$, except those at the top and tip, which are of length $\delta/2$ (Fig. 1). The soil is first assumed to be an ideal isotropic elastic material, having a Young's modulus $E$, and Poisson's ratio $\nu$, that are unaffected by the presence of the pile. The stresses developed between the pile and the soil are assumed to act normal to the face of the pile and no account is taken of possible shear stresses developed between the soil and the sides of the pile. Each pile element is acted upon by a uniform horizontal stress $p$, which is assumed constant across the width of the pile. It is also assumed that the soil at the back of the pile near the surface adheres to the pile.

If purely elastic conditions prevail within the soil, the horizontal displacements of the soil and the pile are equal. In this analysis, these displacements are equated at the element centres, except for the two extreme elements for which displacements are equated at the top and the tip of the pile.

In determining the pile displacements, use is made of the differential equation for bending of a thin beam. This equation can be written in finite-difference form as:

$$\frac{E_sI_s}{\delta^2} [D]\{u\} + [M]\{\ddot{u}\} + [C]\{\dot{u} - \dot{u}_s\} = -d\{p\} \tag{1}$$

in which $\{p\}$ is the vector of pressure, $\{u\}$ is the vector of pile displacements, $\{u_s\}$ is the vector of external soil movement, $d$ is pile diameter or width, $[D]$ is the matrix of finite difference coefficients and $[M]$, and $[C]$ are mass and damping matrices and a dot superscript means differentiation with time.

In the static analysis the soil displacements can be calculated based on the Mindlin equation which gives the displacements within a semi-infinite elastic isotropic homogeneous mass caused by a horizontal point load. Douglas and Davis (1964) integrated this equation over a rectangular area. Their result can be directly used in the discretization scheme of the current analysis. The soil displacements for all points along the pile, which arise both from the external source of movement and the pressure caused by the soil-pile interaction, may be expressed as

$$\{u_s\} = \frac{d}{E_s} [I_s]\{p\} + \{u_s\} \tag{2}$$
where \( \{u_t\} \) is the column vector of horizontal soil displacement and \( [I_t] \) is the \( n+1 \) by \( n+1 \) matrix of soil-displacement-influence factors, elements of which are evaluated by integration over a rectangular area of the Mindlin equation for the horizontal displacement of a point load within a semi-infinite mass.

A solution to the problem is obtained by imposing displacement compatibility between the pile and the adjacent soil, by combining Eqs. (1) and (2), which leads to the following incremental equation:

\[
D \frac{II}{E_p I_p} \cdot E_i \cdot \delta^4 \{\Delta u\} + \frac{[M]}{E_p I_p} \delta^4 \{\Delta \bar{u}\} + \frac{[C]}{E_p I_p} \{\Delta \bar{u}\} - \frac{[II]}{E_p I_p} \cdot E_i \cdot \delta^4 \{\Delta u_e\} = \frac{[II]}{E_p I_p} \cdot E_i \cdot \delta^4 \{\Delta u_e\} \quad (3)
\]

where \([II] = [I_t]^{-1}\), the inverted soil-displacement-factor matrix, \(\{\Delta u\}\) is the incremental displacement at soil pile interface, and \(\{\Delta u_e\}\) is the incremental external soil movement.

Equation (3) leads to \(n+1\) equations for \(n+1\) unknown displacements. Application of this equation to the end nodes, however, requires 2 auxiliary points beyond the ends of the two ends of the pile; the total unknowns are therefore \(n+5\). Four other equations can be obtained from 4 boundary conditions at the pile ends.

The use of Mindlin Theory in Eq. (3) is valid only for the case of a homogeneous soil, as the Mindlin equation is strictly valid for this case. However, in the static analysis, it has been shown that for non-homogeneous soil by assuming that the deflection at a point is given by the Mindlin equation, using the average value of soil modulus, \(E_s\), at the point of application of the load and at the point where this load is received (influencing and influenced points), has given satisfactory results (Poulos and Davis, 1982).

In general, the Mindlin equation is not valid for dynamic loading, and its dynamic equivalent does not exist in closed form. However, as noted by Penzione (1970), if the characteristic wave length (in the current analysis, the shear wave) in the soil medium is long compared with the horizontal distance across the zone of major influence resulting from interaction, the elastic displacement and stress fields can still be adequately defined by Mindlin’s theory.

The Mindlin hypothesis, of course, does not automatically satisfy the condition of soil radiation damping, and this should be accounted for separately. Berger et al. (1975) assumed that a horizontally-moving pile cross section generates solely one dimensional P-waves travelling in the direction of shaking, and one dimensional SH-waves travelling in the direction perpendicular to shaking (Fig. 2b)) and computed the radiation damping per unit length of the pile as:

\[
c_{r} = 2d \rho_s V_t \left( 1 + \frac{V_s}{V_t} \right) \quad (4)
\]

in which \(V_p\) and \(V_s\) are the velocity of P and S waves respectively and \(\rho_s\) is soil mass density. This simple frequency-independent viscous damping coefficient was used by Kagawa and Kraft (1980) to model pile radiation damping. Gazetas and Dobry (1984) assumed that radiated S waves propagate with velocity \(V_t\) in two quarter planes and compression-extension waves propagate with the so-called Lysmer’s analog “wave velocity”, \(V_{la}\) in the other two (Fig. 2a) and derived the radiation damp coefficient associated with a circular cross section with diameter \(d\) as:

\[
c_r = \frac{2d \rho_s V_t}{\omega} \left( 1 + \frac{3.4}{\pi(1-\nu)} \right)^{4/3} \left( \frac{\pi}{4} \right)^{-3/4} a_0^{1/4} \quad (5)
\]

where \(a_0 = ad / V_t\) is the non-dimensional frequency with \(\omega\) being the angular frequency of the vibration. If \(V_p\) in the Berger et al. formulation is replaced with \(V_{la}\), the value of radiation damping will be equal to \(4\rho_s V_{la}d\), which is the value used in the pile computer program PAR (PMB, 1994). Kaynia (1988) proposed the value of \(5\rho_s V_{la}d\) for radiation damping in his Winkler method. This value was obtained by matching the head response of a single pile modelled by the Finite Element and Winkler models. Several other simple approximations for the radiation damping of a vibrating pile are available in the literature. All of these formulations provide relatively close values for the radiation damping. In the current analysis, Kaynia’s coefficients are used, mainly because they are rather conservative and are also frequency-independent. In pile design, extreme care should be taken where radiation damping is considered. This is because soil layers are not generally horizontally infinite, and soil is not completely elastic, and its behaviour changes seasonally with change in water level, nearby excavations or inclusions, etc.

The vector of external soil movement \(\{u_e\}\) in Eq. (3) may be obtained from free-field ground analysis. By assuming that the earthquake consists of vertically incident SH waves the site response can be obtained using the concept of wave propagation in a layered medium as used in the development of the well-known SHAKE program (Schnabel et al., 1972). However, as the main aim of this paper is to deal with the soil yielding a nonlinear methodology used in the development of the ERLS program (Poulos, 1992) has been employed here. In this method the soil layers are modelled as a multi degree of freedom system of mass-spring-dashpots (Fig. 3). This system, in which the spring and dashpot coefficients are...

\[\text{Fig. 2. Horizontal propagation of waves away from a vibrating pile:} \]

\[\text{a) Gazetas and Dobry (1984) model, b) Berger et al. (1975) model}\]
TREATMENT OF THE SOIL YIELDING

The assumption that the soil and pile will have the same displacement during the earthquake and imposition of displacement compatibility between the soil and the pile is not correct when soil yielding occurs.

A solution scheme can be introduced in which the value of the pressure at the soil-pile interface is monitored at each element and at every time step. As long as this value is less than ultimate lateral soil pressure, \( P_y \), the elastic compatibility Eq. (3) is enforced at that element. If this value is larger than \( P_y \), the compatibility equation is replaced by the condition that the pressure at that element is equal to \( P_y \). This violates the equilibrium of forces applied to the pile, therefore, the pressure at all pile elements is recalculated and it is ensured by iteration that at no element does the pressure exceed \( P_y \). This is illustrated in the flow chart in Fig. 4.

Modeled as an ideal elasto-plastic material, the soil is assumed linearly elastic as long as the pressure is below \( P_y \), and is yielded with no ability to bear load when it is equal to or more than \( P_y \). This has been shown in Fig. 5. When one element yields the pressure increases on the adjacent elastic elements, changing the pressure distribution along the pile and this continues as long as the direction of loading has not changed. When the direction of loading changes the yielded elements exert no resistance, but the elastic elements continue to resist the loading in the other direction. There are no differences between the loading and unloading behavior of the elastic elements.

The earlier discussion on the radiation damping was only for the elastic case, and in principle is not applicable when the soil yields. The amount of radiation damping after soil yield is not known, but several tests have shown that it is far less than the value obtained by the elastic assumption. Nogami (1987) showed that when a gap develops between pile and soil, the radiation damping is overestimated by elastic methods. He compared the results of his model with the field tests on a pile subjected to dynamic harmonic loads at its head. He concluded that neglecting the gaps which form at the soil-pile interface due to soil yield resulted in overestimation of damping and underestimation of pile deflection. Chako (1995) performed a number of centrifuge tests on different piles and concluded that the radiation damping based on the
elastic formulation is important during small amplitude shaking, but is excessive when larger displacements occur.

In the absence of any well-developed guidelines, and based on the results of such tests, in this analysis the radiation damping is ignored wherever soil yields; therefore, for the elements that have remained elastic, the radiation damping is taken into account in the same way as discussed earlier, but is ignored at the elements that have undergone yielding.

The ultimate lateral pressure of soil is usually estimated based on plasticity theory. For piles in clay it is generally accepted that:

\[ P_y = N_c \cdot c_u \]  
(6)

in which \( N_c \) is the bearing capacity factor, and \( c_u \) is undrained shear strength. Depending on the theory used and the roughness condition of the pile, \( N_c \) can vary between about 8 and 12, but the most commonly used value is 9 (Broms, 1964a). Near the surface, \( N_c \) tends to decrease, from a value of 9 below about 3 to 4 diameters depth, to a value of 2 at the surface (Poulos, 1992). These values are in conformity with those obtained by Randolph and Houlshby (1984).

For piles in sand, Broms (1964b) suggests:

\[ P_y = N_p \cdot P_p \]  
(7)

where \( N_p \) is a factor which appears to range between about 3 and 5, and \( P_p \) is the Rankine passive pressure, i.e.

\[ P_p = \bar{\sigma}_v \cdot \tan^2 (45 + \phi / 2) \]  
(8)

in which \( \bar{\sigma}_v \) is the effective vertical stress and \( \phi' \) is effective angle of internal friction of the soil.

These formulae, which depend only on the common shear strength parameters, provide a simple way for the estimation of soil ultimate lateral pressure and are employed in this paper. Typical distribution of ultimate lateral pressure along the pile has been shown in Fig. 6. Alternatively the ultimate lateral soil pressure can be approximated based on the formulae proposed by the American Petroleum Institute (API, 1991).

**COMPARISON WITH OTHER METHODS**

The methodology explained in previous sections has been incorporated in a FORTRAN code named SEPAP (Seismic Elasto-Plastic Analysis of Piles) and verified by extensive comparisons with other methods, laboratory tests and field measurements (see Tabesh, 1997). Some of the results of these comparisons are presented herein. The first comparison is made with a Winkler formulation for the frequency-domain dynamic linear analysis of pile foundations developed by Kavvadas and Gazetas (1993).
An end-bearing pile embedded in a two layer soil system as shown in Fig. 8 and subjected to earthquake excitation at its tip is considered. The response of the pile for several ratios of the shear wave velocities of the upper and lower layers, $V_s/V_o$, is obtained by SEAP and is compared with the Kavvadas and Gazetas (K&G) formulation. As the K&G method is a frequency domain formulation, the time domain response has been obtained by Fourier transformation. In SEAP, the analysis is sensitive to element size which should be small enough to ensure unhindered wave propagation through the discretized model and accurate evaluation of pile curvature. For the current comparison with Kavvadas and Gazetas method an element size of 75 cm was found adequate.

Figures 9 to 11 show the profile of moment along the pile at the time step of maximum moment for several soil conditions and the 1987 Whittier and the 1994 Newcastle earthquakes (Table 1). The acceleration time history of the earthquakes may be found in (Tabesh, 1997). As can be seen the agreement between the two methods is good even when the soil medium is strongly nonhomogeneous, and is very close indeed for the homogeneous case.

Figure 12 shows the time history of the head response of a 0.3 m diameter, 12 m long concrete pile carrying a cap-mass of 41.8 tonnes and embedded in a homogeneous clay soil with medium stiffness. The agreement is very good.

### Table 1. Earthquakes' characteristics

<table>
<thead>
<tr>
<th>Earthquake record</th>
<th>Peak acc. (g)</th>
<th>Duration (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newcastle 1994</td>
<td>0.092</td>
<td>22</td>
</tr>
<tr>
<td>Taft 1969</td>
<td>0.046</td>
<td>20</td>
</tr>
<tr>
<td>Whittier Narrows 1987</td>
<td>0.18</td>
<td>20</td>
</tr>
</tbody>
</table>

Fig. 9. The profile of the moment at the time step of the largest moment

Fig. 10. The profile of the moment at the time step of the largest moment

Fig. 11. The profile of the moment at the time step of the largest moment

Fig. 12. The time history of the pile head displacement

Fig. 13. The envelope of the positive and negative moment along the pile
Another comparison has been made with the measurements carried out by Japan's Shimizu corporation at the site of Ohba-Ohashi pile foundation in Japan. The details of the foundation and measurements can be found elsewhere (Tazoh et al., 1988; Fan, 1992; Tabesh, 1997). Figure 13 compares the envelope of the positive and negative moment along an instrumented pile at the Ohba-Ohashi foundation obtained from SEAP analysis and the measurements of the 8th earthquake which has struck this foundation during the measurement period. The soil parameters used in the SEAP analysis are the same as the measured values reported by Tazoh et al. (1988). The measured and computed moment-time histories at the location of strain meter No. 4, some 20 meters below the pile head, are compared in Fig. 14 and their frequency content is compared in Fig. 15. Considering the complexities of the Ohba-Ohashi pile foundation, being a closely spaced group pile consisting of battered and vertical piles, the agreement between the measured and computed response is good.

The elasto-plastic methodology developed in this paper has been further compared with the results of a number of centrifuge tests performed in the California Institute of Technology. The details of the tests can be found in Finn and Gohl (1987). As an example the computed and measured pile head response time histories in Test No. 12 have been compared in Fig. 16, and their frequency content have been depicted in Fig. 17. Again the agreement is reasonable. The soil parameters used in the computations are the same as measured by Finn and Gohl (1987).

THE EFFECTS OF SOIL YIELDING

In this section the results of a comparative study performed on the effects of soil yielding on the internal pile response is presented. A pile embedded in a clay layer with a uniform soil modulus is considered and it is assumed that it is carrying a cap-mass equal to its allowable vertical load capacity divided by gravity. In other words, it is assumed that the pile has been loaded to its allowable capacity by a concentrated weight at its head. This assumption is required in order to be able to compare the performance of piles with different lengths and diameters. In calculating the allowable load capacity a safety factor of 2.5 has been used.

In order to see the typical effects of soil yielding the response of a 20 metre long pile with a diameter of 0.6 m in a soil with a $c_v$ equal to 80 kPa is obtained. The cap-mass is calculated (as explained above) to be 94.4 tones. The earthquake is Taft 1969 (Fig. 7), scaled to a maximum bedrock acceleration of 0.2 g. The time history of
the soil pressure at the head of this fixed head pile is shown in Fig. 18 from both an elastic and elasto-plastic analysis. This figure clearly shows the smoothing effect of the soil yielding on the pressure. It should be noted that ultimate lateral soil pressure at the pile head is 2 times \( c_u \), i.e. 0.16 MPa; this value increases linearly to 0.72 MPa at 2.4 metres below the surface. Figure 19 shows the envelope of positive and negative pressure developed along the pile in the elastic and elasto-plastic analyses. It is obvious that yielding is concentrated near the soil surface. This apparently small concentrated region of yielding, nevertheless, has a profound effect on the value of maximum moment and, to a lesser extent, on maximum shear. This can be seen in Fig. 20 in which the maximum developed moment, shear and relative head displacement in the elastic and elasto-plastic analyses are compared. This figure shows that the effect of yielding on relative head displacement is minimal, suggesting that the yield has a far greater effect on pile curvature.

The Taft earthquake has been again considered, and 100 analyses involving different diameters, lengths and soil cohesion factors have been carried out. The analyses have been performed both for the Taft earthquake scaled to a peak bedrock acceleration of 0.1 g (Taft. 1) and for the same earthquake scaled to 0.6 g (Taft. 6). This is to see the effect of very strong earthquakes. The values of cap-mass and soil modulus are estimated based on \( c_u \) values. The pile head boundary condition is fixed and the cap-mass is assumed to be carried by the pile. In these analyses the soil profiles are assumed homogeneous, thus, the maximum moment always occur at the pile head. In a layered soil medium significant bending moment may develop at the interfaces of the soil layers with different stiffnesses.

The ratio of the maximum values of moment, shear and relative displacement in the elasto-plastic and elastic analyses both for Taft. 1 and Taft. 6 are shown in Fig. 21. There are four blocks of columns in each of these graphs, which from left to right represent the pile lengths of 12, 20, 30 and 45 metres. Each block has 5 sub-blocks representing pile diameters of 0.3, 0.6, 0.9, 1.2, and 1.5 metres. Each of these sub-blocks have 5 columns, which are for soil moduli of, 25, 50, 75, 100 and 160 MPa (associated to \( c_u \) values of 25, 50, 75, 100 and 160 kPa) from left to right.

As expected the effects of yielding is far more severe in the Taft. 6 case. It is obvious that the yielding has caused an increase in maximum relative head displacement and an even sharper increase in maximum moment. In some cases the yielding has almost tripled the value of moment. Interestingly, this is not the case for the maximum shear. In some cases this value has been significantly decreased by yielding, but in some cases it has been increased.

The shear behaviour is directly related to the cap-mass acceleration. In the previous analyses where the pile head was assumed to be fixed, the maximum shear occurred at the head level. The value of the shear at this point is equal to the cap-mass multiplied by the acceleration. In fact, charts exactly like those for shear in Fig. 21 are obtained when the ratio of the maximum pile head acceleration in the elasto-plastic and elastic cases are drawn. The shear behaviour, and the fact that it is sometimes less and sometimes more in elasto-plastic case, means that yielding in some occasions causes an increase in cap-mass acceleration, and sometimes lessens it. The amount of augmentation in the cases considered rarely reaches 10 percent, but the decrease which happens far more frequently is sometimes about 60 percent.

A close look at Fig. 21 shows that smaller diameter
Fig. 21. There are four blocks of columns in each of these graphs, which from left to right represent the pile lengths of 12, 20, 30 and 45 metres. Each block has 5 sub-blocks representing pile diameters of 0.3, 0.6, 0.9, 1.2 and 1.5 metres. Each of these sub-blocks have 5 columns, which are for soil moduli of 25, 50, 75, 100 and 160 MPa (associated to $c_v$ values of 25, 50, 75, 100 and 160 kPa) from left to right.

piles, in the examples considered, are more likely to experience a reduction in acceleration in the elasto-plastic case. There is a tendency for the reduction of acceleration in the elasto-plastic case when the soil modulus, and as a result the cap-mass, increases. It is recalled that the cap-mass in each case has been calculated based on the ultimate load capacity of the pile.

Even in the cases where the maximum acceleration has been reduced due to the soil yielding, the value of maximum relative head displacement and the maximum moment have increased substantially. This suggests that yielding has caused extra, and sometimes substantial, pure bending in the pile.

Another interesting fact illustrated by Fig. 21 is that an increase in the soil modulus is often accompanied by an increase in relative head displacement and moment. This is because the cap-mass, which has been obtained from load capacity of the pile, is larger for a higher soil modu-
lus. In fact a higher soil modulus (a stiffer soil) offers higher load capacity, which if used, may make the pile more vulnerable to the effects of soil yielding. In other words, the extra soil modulus allows an extra mass to be carried by the pile and an extra mass exerts a larger pressure on the soil causing more yielding and pile bending.

In order to gain a better insight, the fourth pile configuration (Taft 6, \(L = 12\) m, \(d = 0.3\) m, \(Cap = 34.8\) tonnes, \(E_s = 100\) MPa, and \(c_s = 100\) kPa) is considered in more detail. Figure 22 shows the envelope of positive and negative relative displacement along the pile. It is obvious that in the elasto-plastic case the deflection near the pile head is significantly larger. Figure 23, which depicts the envelope of positive and negative moment along the pile, shows that yielding has significantly increased the amount of maximum moment in top 4 metre length of the pile. The time history of the moment at the pile head is depicted in Fig. 24.

In this case, the envelope of positive and negative shear along the pile, depicted in Fig. 25 shows a trend very similar to Fig. 23 which was for the moment. This means that, in this case, the yielding has increased the maximum pile shear or head acceleration.

Figure 26 shows the envelope of positive and negative pressure developed at the soil-pile interface, and demonstrates the fact that the yielding assumption causes redistribution of pressure along the pile. With this assumption, the pressure near the head has been substantially reduced, but it has been increased in interface elements between 1.5 to 5 metre depth below the pile head. This figure also shows how unrealistic the assumption of linear elastic soil behaviour near the pile cap-mass can be; the value of soil pressure at the pile head in elastic case has reached 2.5 MPa, some three times the value of ultimate lateral soil pressure deep inside the soil mass.

The other example which will be considered in more detail is the 80th pile configuration (\(L = 45\) m, \(d = 0.3\) m, \(Cap = 197.8\) tonnes, \(E_s = 160\) MPa, \(c_s = 160\) kPa). Due to the long pile length and the large soil undrained shear
strength, the value of cap-mass obtained based on geotechnical capacity is a relatively large value of 197.8 tonnes. This however exceeds the structural capacity of a 0.3 metre diameter pile. In fact the maximum value of allowable cap-mass, based on an average yield limit of concrete is hardly more than 100 to 120 tonnes. However, there is some merit in looking at this configuration in more detail. The large cap-mass has reduced the pile natural frequency, and this configuration has resulted in a significant reduction in maximum shear in the elasto-plastic analysis compared to the elastic analysis.

Figures 27 and 28 show the time history of the shear and moment developed at the pile head during the earthquake. It is obvious that the yielding has damped out some of the high frequency vibrations present in the elastic analysis. Interestingly, neither in the elastic nor in the elasto-plastic analysis are these vibrations present in the time history of head displacement. This can be seen in Fig. 29, which instead shows a delay in response in the elasto-plastic analysis compared with the elastic analysis. This means that in the elastic analysis the pile curvature at the head level changes rapidly, causing high frequency vibration of moment and shear. These vibrations are not present in the elasto-plastic analysis, as yielding “smooths” the curvature significantly.

As can be seen in Fig. 27 to Fig. 29 the yielding has caused significant increases in moment and relative head displacement, but it has brought about a similarly significant decrease in shear. Figure 30 shows that in the elastic analysis the pressure at soil-pile interface can become unreasonably high. Almost any soil, under such a pressure, would yield.

In order to gain a better insight into the significant effect of yielding, a 15 metre long pile with a diameter of 0.3 metres is considered. The maximum mass, against the yield value of concrete that this pile can carry is about 100 tonnes. 100 analyses involving different values of cap mass from 1 tonne to 100 tonnes have been performed; in each analysis the cap-mass is only 1 tonne larger than in the previous analysis. This is to capture any possible resonance effect. The analyses have been carried out for both the elastic and elasto-plastic cases. Figures 31(a), to 31(c) depicts the maximum values of relative head displacement, moment and shear against the natural period of pile-soil-cap-mass system. These figures confirm the previous conclusion that yielding causes a higher value of relative displacement and moment, but a smaller value of shear. The other fact illustrated by these figures is that yielding “softens” the peaks and troughs associated with resonance. These figures show that for some values of mass, namely when the mass is quite large, the moment in the elasto-plastic case is smaller than in the elastic case. This is contrary to what was observed earlier, where moment was always larger in the elasto-plastic case. The rela-
tive displacement, however is, as before, much larger in the elasto-plastic case than in the elastic case. In order to understand this behaviour better, the last configuration of the previous analyses, namely, the one with a mass of 100 tones, is examined more closely.

Figure 32(a) shows the envelope of maximum positive and negative pressure developed along the pile during earthquake. As can be seen the soil around the top two thirds of the pile length has yielded. This yield has brought about a significant change in the behaviour of relative head displacement, as seen in Fig. 32(b). In this figure, it can be observed that at the start of yield, about 3.5 seconds into the earthquake, pile head has distanced itself from its elastic position some 15 centimetres. This has caused a significant delay and change of behaviour during the remaining duration of the earthquake. The pile has missed several peaks and troughs of the elastic analysis, and, in effect, has moved much more slowly than in the elastic analysis.

The head moment and shear versus time histories of the pile are shown in Figs. 32(c) and 32(d). These figures show how significantly the yielding has “softened” the curvature of the pile and how this softening has reduced the maximum values of moment and shear.

What can be concluded from this analysis is that if the loading, soil conditions, and earthquake are such that the yielding penetrates well below the surface level, for small diameter piles, it is quite possible that the pile head displacement becomes much larger than in the elastic analysis. This increase is due to bending of the pile, which is distributed along the portion of the pile adjacent to the yielded soil. In other words, in contrast to the elastic analysis, or in the cases where the yielding is concentrated over a small portion of the pile length, there is a larger length of the pile to accommodate bending due to pile head displacement, resulting in an overall decrease in the amount of moment at a typical local point, especially at and near the cap-mass. This can be best seen in Figs. 32(e) and 32(f), in which the distribution of relative displacement and moment along the pile are depicted for the time step, where maximum relative head displacement has occurred.

Similar analyses have been performed for piles embedded in dry sand with a linearly increasing modulus with depth. The ratio of the maximum relative displacement, shear and moment in the elasto-plastic and elastic cases obtained for a hundred different pile configurations are shown in Fig. 33. The results are akin to those explained previously for clay. One noticeable difference is that the effect of yielding has been less pronounced in the sand case. A reason for this seems to be the fact that yielding is usually concentrated near the pile head. With the assump-
Fig. 32. The case of a pile carrying a large cap-mass embedded in a soft soil and subjected to a very strong earthquake.

CONCLUSIONS

In this paper a time domain methodology has been presented for the elasto-plastic analysis of single piles. In this method the pressure at the soil-pile interface is constantly monitored, and the analysis ensures that it never goes beyond the ultimate lateral soil pressure. Whenever soil yielding occurs, the displacement compatibility condi-
Fig. 33. There are four blocks of columns in each of these graphs, which from left to right represent the pile lengths of 12, 20, 30 and 45 metres. Each block has 5 sub-blocks representing pile diameters of 0.3, 0.6, 0.9, 1.2 and 1.5 metres. Each of these sub-blocks have 5 columns, which are for $E_I (L=12)$ equal to 73.5, 88.8, 108.1, 132.9 and 165.7 MPa (associated to friction angles of 20, 25, 30, 35 and 40 degrees) from left to right.

tion between pile and soil is replaced with an iterative procedure which ensures the pressures along the pile remain at or below the ultimate lateral soil pressure. Based on the developed methodology scores of analyses involving different pile and soil configurations embedded in a homogeneous clay were performed and the results of the elastic and elasto-plastic analyses were compared. The aim of these analyses was to see how the yielding affects the maximum pile moment, shear and relative head displacement.

It was found that when the cap-mass was included in the analysis, significant soil yielding was likely to develop. The yielding was often concentrated near the pile head within a depth of 4 diameters below the surface. However, for flexible piles with large cap-masses, embedded in soft soil and subjected to strong earthquakes, the
yielding was likely to go beyond a 4-diameter depth, resulting in a strong change in the pile dynamic behaviour.

It was noted that in the elastic analysis, when cap-mass is included, the pressure at the pile-soil interface may go well beyond the ultimate lateral soil pressure, for average piles carrying average loads. In such cases, the elastic analysis is to a large extent inaccurate.

In the overwhelming majority of the cases considered, it was noted that yielding is accompanied by an increase in maximum pile relative displacement, and usually, a more profound increase in maximum pile moment. The value of moment in some occasions was several times the value obtained in the elastic analysis.

In most of the cases, however, the yielding caused a reduction in the maximum value of pile shear, and pile head acceleration. The yielding was found to cause extra pure bending of the pile, which means larger moment, but not necessarily larger shear.

For very strong earthquakes, and for flexible piles carrying large cap-masses and embedded in soft soil, the yielding was likely to extend well below the surface. In such cases very significant changes in the response were observed. In such cases, many peaks and troughs of the elastic response were not present in the elasto-plastic analysis, and both values of moment and shear were often smaller.

Smaller diameter piles, or relatively flexible piles, and long piles, which usually carry larger cap-masses, are more likely to experience soil yielding. Stiffer clay layers also offer more load capacity and consequently permit a greater cap-mass. They at the same time offer a larger ultimate lateral soil pressure. It was observed that, if the extra cap-mass capacity is used, it usually outweighs the extra lateral soil resistance and can make the effect of yielding worse.

A clay soil with a uniform modulus was compared with a sand soil with a linearly increasing modulus with depth. It was observed that, in general, the yielding effects were less pronounced in the second case. This was attributed to the fact that a low soil modulus near the pile head resulted in smaller lateral interface pressures and a reduced possibility of soil yielding.

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