PSEUDO-STATIC THRUST ON CANTILEVER WALLS

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ABSTRACT

This technical note extends the application field of the method proposed by the author (Greco, 1999) to walls subject to seismic conditions analyzed by the pseudo-static method of Mononobe and Okabe. The active thrust is calculated maximizing its horizontal component (or minimizing its vertical one) with respect to the inclination angles of the planes bounding the thrust wedge. This also corresponds to minimizing the safety factor against sliding along the base. Cantilever walls with both elongated and short heels are studied in this note.

Key words: active earth pressure, (Coulomb's method), earthquake, (Rankine's method), retaining walls, stability analysis (IGC: E5/E13)

INTRODUCTION

The pseudo-static approach is a simple and reliable tool for solving earth thrust problems, when seismic action is present. This method was introduced by Okabe (1926) and Mononobe and Matsuo (1929) as an extension of the method of Coulomb (1773), and is still customarily used (Motta, 1994; Koseki et al., 1998). According to this approach, static supplementary forces F_h and F_v , conventionally due to earthquakes, are added to the weight of the thrust wedge W (Fig. 1),

$$F_h = k_h W, \quad (1a)$$

$$F_v = k_v W, \quad (1b)$$

where k_h and k_v are the seismic coefficients, horizontal and vertical, respectively.

The active thrust S_a , obtained through the equilibrium conditions of forces acting on the wedge ABC, is given by:

$$S_a = \frac{1}{2} \gamma h^2 \frac{1+k_v}{\cos \theta} K_a, \quad (2)$$

where γ is the soil unit weight, h the height of the retaining wall, $\theta = \tan^{-1} [k_h / (1+k_v)]$ and K_a is the thrust coefficient in seismic conditions, given by:

$$K_a = \left(\frac{\frac{\sin(\beta + \theta - \phi')}{\sin \beta}}{\sqrt{\sin(\beta + \theta + \delta)} + \sqrt{\frac{\sin(\phi' + \delta) \sin(\phi' - \varepsilon - \theta)}{\sin(\beta - \varepsilon)}}} \right)^2. \quad (3)$$

Angles β and ε are shown in Fig. 1, while δ is the inclination angle of the thrust with respect to the normal to the plane BC and ϕ' is the shear strength angle of the soil.

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Fig. 1. Forces acting on thrust wedge in method of Mononobe and Okabe

For cantilever walls Rankine's method is generally preferred, because the wedge is not in direct contact with the wall and the angle δ is more difficult to fix. In this method, the thrust on a generic plane inclined at β has the same shape as Eq. (2), with a different thrust coefficient, given by:

$$K_a = \frac{\sqrt{1 + \sin^2 \phi' + 2 \sin \phi' \cos(2\beta - \varepsilon + \Delta + \theta)}}{\cos(\varepsilon + \theta) + \sqrt{\cos^2(\varepsilon + \theta) + \cos^2 \phi'}} \times \frac{\sin(\beta - \varepsilon)}{\sin^2 \beta}, \quad (4)$$

with $\Delta = \sin^{-1} [\sin(\varepsilon + \theta) / \sin \phi']$.

Although formulas (3) and (4) are formally different

$$\frac{\partial S_{ah}}{\partial \alpha} = \sin(\beta + \phi') \frac{\partial P_a}{\partial \alpha} = -\tan(\beta + \phi') \frac{\partial S_{av}}{\partial \alpha}. \quad (8)$$

The safety factor against sliding along the base, F_s , is a function of these thrust components. Neglecting the passive resistance of the soil before the wall, F_s is indeed given by:

$$F_s = \frac{R_n}{R_t} \tan \phi_b = \frac{(S_{av} + \bar{V}) \cos \nu - (S_{ah} + \bar{H}) \sin \nu}{(S_{ah} + \bar{H}) \cos \nu + (S_{av} + \bar{V}) \sin \nu} \tan \phi_b, \quad (9)$$

where ϕ_b is the friction angle between wall and soil along the base, while $\bar{V}$ and $\bar{H}$ are, respectively, vertical and horizontal components of mass forces acting on wall and soil between planes CE and CP, and surface forces acting on EP. The partial derivatives of F_s and P_a with respect to α are linked by:

$$\frac{\partial F_s}{\partial \alpha} = -\frac{\partial P_a}{\partial \alpha} \frac{\tan \phi_b}{R_t} \frac{\cos(\beta + \phi' - \lambda)}{\cos(\lambda - \nu)}. \quad (10)$$

Because $\beta + \phi' < \pi - \nu$ and $\lambda < \pi/2 - \nu$ (Fig. 2), so $\lambda > \beta + \phi' - \pi/2$ and then we obtain $\cos(\beta + \phi' - \lambda) > 0$. Therefore, maximizing P_a with respect to α , for an assigned value of β , corresponds to minimizing F_s with respect to the same variable. Consequently, the angle α must be selected so that the function P_a is maximized, as in the original method of Coulomb. The function $\alpha_c(\beta)$ is obtainable, in implicit terms, by maximizing the thrust P_a with respect to α .

Concerning the partial derivatives of the functions S_{ah} and S_{av} with respect to β , we have:

$$\begin{aligned} \frac{\partial S_{ah}}{\partial \beta} &= -\tan(\alpha - \phi') \frac{\partial S_{av}}{\partial \beta} \\ &= -\frac{\partial F_s}{\partial \beta} R_t \frac{\cos(\lambda - \nu)}{\tan \phi_b} \frac{\sin(\alpha - \phi')}{\cos(\alpha - \phi' - \lambda)}. \end{aligned} \quad (11)$$

Thus, for an assigned value of α (and in particular for $\alpha = \alpha_c$) the critical condition for wall stability against sliding is achieved for a value β_c of β , which simultaneously maximizes S_{ah} and minimizes S_{av} . The same critical condition can also be obtained by minimizing the inclination angle ψ of the thrust S_a (Fig. 2).

$$\psi = \tan^{-1}(S_{av}/S_{ah}) \quad (12)$$

In conclusion, the values of the angles β_c and α_c for which S_{ah} and S_{av} attain their extrema, and F_s attains its minimum, are given by the conditions:

$$\begin{aligned} \max_{\alpha} P_a(\alpha, \beta) &\rightarrow \alpha_c(\beta), \\ \min_{\beta} \psi(\alpha_c, \beta) &\rightarrow \beta_c. \end{aligned} \quad (13)$$

Since no hypothesis has been formulated about the shape of the backfill profile and charge distribution on the backfill, the functions S_{ah} and S_{av} attain their extrema for the same couple of angles β_c and α_c , independently of the way static and seismic loads are distributed.

WALLS WITH SHORT HEEL

In cantilever walls with a short heel, the point B of the plane inclined at β , is located on the wall (Fig. 3(a)). Barghouthi (1990) proposed calculating thrusts on these walls using Coulomb's approach, calculating first the thrust S_1 relative to the wedge TBD (Fig. 3(b)), and then the thrust S_2 through the equilibrium conditions of forces acting on the wedge TBCA (Fig. 3(c)). The critical condition for the wall stability was assumed by maximizing the thrust S_a acting on a prefixed reference plane CE, which may be vertical (Fig. 3(d)).

However, maximizing S_a does not necessarily correspond to the critical condition for wall stability, as in Coulomb's method where the ratio between the thrust components S_{ah} and S_{av} is fixed. On the contrary, in Barghouthi's methods this ratio depends on β , thus the wall stability criterion should be based on the components S_{ah} and S_{av} . Numerical results showed, however, that the maximum of S_{ah} is attained for the same values of the angles α and β minimizing S_{av} (Greco, 1992). Successively an analytical proof was also carried out (Greco, 2001). Here that proof will be extended to the pseudo-static condition.

With reference to Fig. 3(a), the thrust S_1 is given by the equilibrium condition of forces acting on the wedge TBD:

$$S_1 = \frac{V_1 \sin(\eta - \phi') + H_1 \cos(\eta - \phi')}{\cos(\phi' + \delta - \eta)}, \quad (14)$$

where V_1 is the vertical component of the resultant of the mass forces acting on the wedge TBD and surface forces acting on TD, while H_1 is the horizontal component of the same resultant. The thrust S_1 vanishes for $\eta = \eta'' = \pi/2$, and also for the value $\eta = \eta'$ given by:

$$\eta' = \phi' - \tan^{-1}(H_1/V_1), \quad (15)$$

which is solvable by a trial and error procedure. In the interval $[\eta', \eta'']$, the function S_1 is positive and, if differentiable, has a maximum for a value $\eta = \eta_c$.

Equilibrium conditions of forces acting on the wedge TBCA (Fig. 3(c)) give the thrust S_2 :

$$S_2 = \frac{V \sin(\alpha - \phi') + H \cos(\alpha - \phi') - S_1 \cos(\phi' + \delta - \alpha)}{\sin(\beta + 2\phi' - \alpha)}, \quad (16)$$

where V is the vertical component of the resultant of mass forces acting on the wedge TBCA and surface forces acting on TA, while H is the horizontal component of the resultant of the same forces. In order to make S_2 a positive value, the angle β must be less than $\pi - 2\phi' + \alpha$. Forces V and H can also be devised in:

$$V(\beta, \alpha) = V'(\beta) + V''(\alpha), \quad (17a)$$

$$H(\beta, \alpha) = H'(\beta) + H''(\alpha), \quad (17b)$$

where V' and H' are the components, vertical and horizontal respectively, of the resultant of mass forces acting on the wedge TBCE and surface forces acting on

Table 1. Values of the angles β , α and η for which the functions S_{ah} , S_{av} , ψ and F_s attain their extrema

Condition	β_c ($^\circ$)	α_c ($^\circ$)	η_c ($^\circ$)	S_{ah} (kN/m)	S_{av} (kN/m)	ψ ($^\circ$)	F_s
Static $k_h=0, k_v=0$	96.7	65.30	—	111.0	53.7	25.81	2.19
	110.0	65.30	60.31	115.3	46.1	21.79	2.06
Seismic $k_h=0.1, k_v=0$	95.6	60.29	—	127.8	72.9	29.71	1.65
	104.7	59.94	—	126.4	75.9	31.00	1.68
	118.5	58.05	42.34	118.9	93.6	38.22	1.85

Fig. 5. Safety factor against sliding in terms of angle β for Fig. 4 example

Both static and seismic cases have been analyzed and compared. In the seismic case, a horizontal strip of charge with modulus 8 kPa, due to earthquake and applied together with the vertical one, has also been considered. The determination of the angles for which the functions S_{ah} , S_{av} , ψ and F_s attain their extrema requires an implementation of the proposed method by means of a small computer code. The search for these extrema can be performed by direct search. To this end, due to the small number of variables, even the grid method can be used. In the analysis, the wall is treated as a wall with an elongated heel when $\beta < \beta_T = 106.7^\circ$ (Figs. 2 and 4), and as a wall with an short heel when $\beta > \beta_T$.

Figure 5 shows the safety factor F_s , both for static and seismic conditions, in terms of angle β , for $\alpha = \alpha_c$ and $\eta = \eta_c$. As can be seen in this figure, a discontinuity is present for $\beta = 102.53^\circ$, where stepping from $\beta = 102.5^\circ$ to 102.6° the weight of the thrust wedge is increased by the triangle UVS. A discontinuity is also present in the range of α . Indeed, stepping from $\alpha = 65.29^\circ$ to 65.30° the weight of the triangle PQR disappears (Fig. 4). Moreover the function F_s has more local extrema: in the static condition two local extrema are present respectively for $\beta = 96.7^\circ$ and $\beta = 110^\circ$; while three local extrema characterize the seismic condition: for $\beta = 95.6^\circ$, 104.7° and 118.5° (Table 1).

Nevertheless the local maxima of S_{ah} are achieved for

Fig. 6. Thrust components in terms of angle β , for Fig. 4 example

the same values of α_c , β_c and η_c for which S_{ah} and F_s attain their local minima; and, moreover, the global maximum of S_{ah} and the global minima of S_{av} and F_s are obtained for the same values of α_c , β_c and η_c (Fig. 6).

Finally it is interesting to note that in static analysis the critical condition for the wall of Fig. 4 is given by the short heel case, while in seismic conditions the critical condition is connected to the elongated heel case. Therefore, a wall which in static analysis is characterized by a short heel, under seismic action can show behavior typical of walls with an elongated heel. As a result, it is not possible to state a priori, on the basis of wall size and backfill only, and without considering seismic action, if a cantilever wall has a short or elongated heel.

CONCLUSIONS

This note has illustrated a method, based on Mononobe and Okabe's approach, for calculating thrusts on the cantilever walls subject to seismic action.

In this method the thrust on cantilever wall is determined by minimizing the safety factor of the wall against sliding. Alternatively, the same results can be obtained minimizing the angle ψ or S_{av} , or maximizing S_{ah} , with respect to a fixed thrust plane of reference.

The proposed method is very easy and can be implemented by means of a small computer code.

The method's main shortcoming, in the author's opinion, is as in Coulomb's general approach, that concerning the assumption of planarity of the surfaces bounding the thrust wedge.

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