NUMERICAL PREDICTION OF THE DYNAMIC BEHAVIORS OF AN RC GROUP-PILE FOUNDATION

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ABSTRACT

In a major earthquake, it is reasonable to assume that the mechanical behavior of a pile foundation and the surrounding ground is nonlinear. It is commonly accepted that the dynamic behavior of a group-pile foundation is not only related to its own mechanical properties, but also to the upper structure supported by the foundation, and to the surrounding ground. It is recommended, therefore, that a full system, which consists of superstructures, foundation and ground, be considered in the dynamic analysis because of the merit that relatively few assumptions are adopted. The most important thing in the dynamic analysis, however, is that the individual nonlinear behavior of soils and piles should be properly evaluated. In this paper, a new beam theory is proposed for a reinforced concrete material (RC material). The theory is based on a weak form in which the axial-force dependency in the nonlinear moment-curvature relation is considered. Its validity is verified by experimental results of an RC cantilever beam. Then, an elevated bridge with a 12-pile foundation is analyzed using a three-dimensional elastoplastic finite element analysis (DGPILE-3D). The piles are cast-in-place reinforced concrete and have a diameter of 1.2 m. Meanwhile, the ground soil is simulated with a *tij* subloading model in which the concepts of kinematic hardening and subloading are adopted. The purpose of the paper is to provide an accurate numerical way of evaluating the dynamic behavior of a pile foundation during an earthquake.

Key words: dynamic, elasto-plasticity, finite element method, foundation, pile (IGC: E4)

INTRODUCTION

It is known that during a strong earthquake, the dynamic behavior of a group-pile foundation is not only related to the inertial force coming from the superstructures but also to the deformation of the surrounding ground. During the Hyogoken-Nambu earthquake, it was found from field observations (Horikoshi et al., 1996) that even in the absence of a superstructure, piles failed because of the deformation of the surrounding ground. The moments developed due to the deformation of ground are usually referred to as kinematic moments. Some research has been done in dealing with the dynamic interaction between piles and soils in the frequency domain; this can be found in works by Nikolaou et al. (1995), Guin and Banerjee (1998), and Gazetas and Mylonakis (1998). In these works, only a single pile is considered and its nonlinearity is not considered because of the restriction of the analysis in the frequency domain.

On the other hand, dealing with a full system, which consists of superstructures, foundation and ground, in a numerical dynamic analysis in the time domain is usually thought to be effective and executable nowadays when the nonlinearity of the superstructure, piles and soil is considered. A few studies have been done in this field through

both experiments and numerical analyses. Matsuda et al. (1994) performed a shaking table model test and conducted a three-dimensional dynamic effective-stress analysis of a light structure buried in saturated sandy ground. Taji et al. (1997) conducted a large-scale shaking-table test and a centrifuge model test to investigate the soil-pile-structure interaction in potentially liquefying sand. Murono et al. (1997) conducted a dynamic model test on a structure-foundation-ground system to investigate the seismic behavior of a group-pile foundation. Fukutake (1997) conducted a seismic evaluation of a group-pile foundation surrounded by a frame wall using a 3-D nonlinear dynamic analysis. Wakai et al. (1997) conducted a seismic analysis of a bridge-ground system using a 3-D nonlinear dynamic analysis. Taguchi et al. (1997) conducted a numerical simulation of soil-foundation interaction with a 3-D nonlinear dynamic analysis considering subsoil liquefaction.

Zhang et al. (2000b) simulated numerically a field test of a real-scale 2-pile foundation subjected to lateral cyclic loading up to the ultimate state with a 3-D elastoplastic finite element analysis, considering the influence of different constitutive models adopted for soils. Kimura and Zhang (2000) conducted a series of static and dynamic 3-D elastoplastic finite element analyses on a simplified

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sway-rocking model (S-R model) and on a full system to investigate the dynamic behavior of a group-pile foundation during an earthquake. It is, however, well known that the bending strength and the load-sharing ratio of the piles in a pile group are totally different in different positions, e.g. front, back and middle position, when subjected to lateral loadings. In the aforementioned works, a very important fact, that greatly affects the nonlinearity of piles, the influence of axial force on the stiffness and the bending strength of RC piles, was neglected. The reason why this influence was not considered is that it is difficult to model the influence under cyclic loading condition within the framework of common beam theory.

In structural engineering, a few axial-force dependent nonlinear models for RC materials have been proposed. Among them, the multi-spring model and the fiber model (Lai et al., 1984; Li and Kubo, 1999) are rather popular. They are mainly used in structural analyses in which soils are not considered. The equilibrium equations of these models are established in a strong form, that is, differential equations of structural analysis. If the interaction between piles and soils is considered in finite element analysis, the compatibility of the deformation prescribed for FEM should be satisfied at every node. However, these two models do not satisfy the condition. For this reason, by introducing a new weak form of the equilibrium equation for beams in this paper, the interaction between the bending moment and the axial force can be properly evaluated under generalized loading conditions. In calculating the stiffness matrix of an RC beam, the concept of the discretization of the RC material proposed by Li and Kubo (1999) is introduced to the new beam theory proposed in this paper, in which the compatibility of deformation is satisfied.

Zhang et al. (2000c) conducted a 3-D FEM analysis of a laterally cyclic-loaded, real-scale 9-pile foundation based on an axial-force dependent hysteretic model for RC. In the analysis, the field measurements can be simulated to a quite accurate level.

As to the nonlinearity of soil, engineers prefer a simple model whose material parameters can be determined based on some easily conducted and commonly used field test results, e.g., the N -value of standard penetration test (SPT N -value), without losing too much accuracy of the description for soils. It is obvious, however, that to describe the general behavior of soils with a four-parameter model, for instance, the Drucker-Prager model or Mohr-Coulomb model, is too ambitious. It is known that in order to simulate the mechanical behavior of soils under generalized stress condition, a more sophisticated constitutive model is usually necessary, which often means that the determination of the parameters based on laboratory tests and in-situ measurements is needed. For this reason, a kinematic hardening elastoplastic constitutive model using the concept of subloading, known as a tij subloading model (Chowdhury et al., 1999), is adopted for soils in the 3-D dynamic analysis conducted in this paper.

The purpose of the paper is to provide an accurate numerical method of evaluating the dynamic behavior of a group-pile foundation based on 3-D finite element analysis, in which the nonlinearity of a pile is described by the new beam theory proposed in this paper and a ground is described by the tij subloading model. In order to understand the accuracy of the analysis in which simple constitutive models are adopted for soils and piles, analyses using the Drucker-Prager model (D-P model) and trilinear model (see Appendix I and II) are also conducted and their results are compared with the results from the new beam theory and tij subloading model analyses.

NEW FORMULATION OF THE BEAM THEORY CONSIDERING THE AXIAL-FORCE DEPENDENCY

In the multi-spring model and the fiber model (Lai et al., 1984; Li and Kubo, 1999), the RC member material is modeled by a number of springs or fibers whose mechanical behavior is strictly determined according to the member material and the geometric properties. In the models, each steel bar is represented by a steel spring or fiber, and the concrete area of the RC member section is discretized and represented by a large number of concrete springs or fibers. By taking the plane-section assumption, that is, when an RC member is subjected to biaxial bending, axial force, and shear forces, any section along the member longitudinal axis is kept as a plane, the models can properly take into consideration the interactions among biaxial bending and axial forces. In the models, however, the equilibrium equations are not derived strictly from a weak form of an integration that satisfies the compatibility of deformation. It is, therefore, difficult to be directly used in finite element analysis. For this reason, a new weak form of the equilibrium equation for a beam, which satisfies the compatibility of the deformation, is proposed here. In the theory, the plane-section assumption is still kept valid and the stress-strain relations of reinforcement and concrete are shown in Fig. 1.

As shown in Fig., ϵ_a , the strain at an arbitrary point $P(x, y)$ at the sectional plane of a beam, can be divided into three parts, that is, the bending strain ϵ_{m1} due to M_x , the bending strain ϵ_{m2} due to M_y and axial strain ϵ_0 due to axial force, as shown in following equation,

$$\epsilon_a = \epsilon_{m1} + \epsilon_{m2} + \epsilon_0 = (x\{H_u''(z)\}^T + y\{H_v''(z)\}^T - \{H_w'(z)\}^T) \cdot [A]\{\delta\} = \{F(z)\}^T \cdot [A]\{\delta\} \quad (1)$$

where $\{\delta\} = \{u_i \ v_i \ w_i \ \theta_{xi} \ \theta_{yi} \ u_j \ v_j \ w_j \ \theta_{xj} \ \theta_{yj}\}^T$ is the nodal displacement vector.

$$\{F(z)\}^T = (x\{H_u''(z)\}^T + y\{H_v''(z)\}^T - \{H_w'(z)\}^T) \quad (2)$$

$$\{H_u''(z)\}^T = \{0 \ 0 \ 0 \ 0 \ 0 \ 2 \ 0 \ 0 \ 0 \ \sigma_z\}$$

$$\{H_v''(z)\}^T = \{0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 2 \ 0 \ \sigma_z \ 0\} \quad (3)$$

$$\{H_w'(z)\}^T = \{0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0\}$$

Fig. 1. Nonlinear properties of reinforcement and concrete

$$[A] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -3/l^2 & 0 & 0 & 0 & -2/l & 3/l^2 & 0 & 0 & 0 & -1/l \\ 0 & -3/l^2 & 0 & -2/l & 0 & 0 & 3/l^2 & 0 & -1/l & 0 \\ 0 & 0 & -1/l & 0 & 0 & 0 & 0 & 1/l & 0 & 0 \\ 0 & 2/l^3 & 0 & 1/l^2 & 0 & 0 & -2/l^3 & 0 & 1/l^2 & 0 \\ 2/l^3 & 0 & 0 & 0 & 1/l^2 & -2/l^3 & 0 & 0 & 0 & 1/l^2 \end{pmatrix} \quad (4)$$

The virtual energy stored in the beam element due to a virtual strain can be expressed as,

$$U = \iiint_V \sigma_a \cdot d\epsilon_a dv = \{d\delta\}^T \int \int \int E \times [A]^T \{F(z)\} \{F(z)\}^T [A] dv \cdot \{\delta\} \quad (5)$$

On the other hand, virtual energy W brought about by the external force due to a virtual displacement, is $W = \{d\delta\}^T \{F\}$. Therefore, the virtual energy theory ($W = U$) can be obtained as

$$\{F\} = \iiint E \cdot [A]^T \{F(z)\} \{F(z)\}^T [A] dv \cdot \{\delta\} = [K] \cdot \{\delta\} \quad (6)$$

where $[K]$ is the stiffness matrix of the beam element and can be rewritten as

$$[K] = \iiint E [A]^T [I] [A] dv \quad (7)$$

$$[I] = \{F(z)\} \cdot \{F(z)\}^T = (x\{H_u''(z)\} + y\{H_v''(z)\} - \{H_w''(z)\}) \cdot (x\{H_u''(z)\}^T + y\{H_v''(z)\}^T - \{H_w''(z)\}^T) \quad (8)$$

In the classical beam theory, $[I]$ is defined by Eq. (9), in which the influence on the $M-\Phi$ relation due to the axial force is not considered, namely,

$$[I] = [I_1] = x^2 \{H_u''(z)\} \cdot \{H_u''(z)\}^T + y^2 \{H_v''(z)\}$$

$$\times \{H_v''(z)\}^T + \{H_w''(z)\} \cdot \{H_w''(z)\}^T \quad (9)$$

Based on Eqs. (8) and (9), $[I]$ can be rewritten as

$$[I] = [I_1] + [I_2] \quad (10)$$

where $[I_2]$ is evaluated by the following equation:

$$[I_2] = xy \cdot (\{H_u''(z)\} \cdot \{H_v''(z)\}^T + \{H_v''(z)\} \cdot \{H_u''(z)\}^T) - x(\{H_u''(z)\} \cdot \{H_w''(z)\}^T + \{H_w''(z)\} \cdot \{H_u''(z)\}^T) - y(\{H_v''(z)\} \cdot \{H_w''(z)\}^T + \{H_w''(z)\} \cdot \{H_v''(z)\}^T) \quad (11)$$

$[I_2]$ is the newly added item which takes into consideration the influence on the $M-\Phi$ relation due to the axial force. Based on the above equations, $[K]$ can be expressed as

$$[K] = [A]^T \cdot [D] \cdot [A], \quad [D] = \begin{bmatrix} 0_{5 \times 5} & 0_{5 \times 5} \\ 0_{5 \times 5} & D_1 \end{bmatrix} \quad (12)$$

where

$$[D_1] = \begin{bmatrix} 4EI_y & 4EI_{xy} & -2EI_x & 6I^2EI_{xy} & 6I^2EI_y \\ 4EI_{xy} & 4EI_x & -2EI_y & 6I^2EI_x & 6I^2EI_{xy} \\ -2EI_x & -2EI_y & IEA & -3IE_y & -3I^2E_x \\ 6I^2EI_{xy} & 6I^2EI_x & -3IE_y & 12I^3EI_x & 12I^3EI_{xy} \\ 6I^2EI_y & 6I^2EI_{xy} & -3I^2E_x & 12I^3EI_{xy} & 12I^3EI_y \end{bmatrix} \quad (13)$$

$$EA = \iint E \cdot dA, \quad EI_y = \iint E \cdot x^2 \cdot dA \\ EI_x = \iint E \cdot y^2 \cdot dA, \quad EI_{xy} = \iint E \cdot x \cdot y \cdot dA \\ E_x = \iint E \cdot x \cdot dA, \quad E_y = \iint E \cdot y \cdot dA \quad (14)$$

The integration in Eq. (14) can be evaluated by discretizing area A into N small areas A_i and taking the sum as

$$EI_y = \iint E \cdot x^2 \cdot dA = \sum_{i=1}^N E_i x_i^2 A_i \quad (15)$$

For instance, a circular section can be discretized to m -concentric rings with equal thickness of $r = r_j - r_{j-1} = R/m$, as shown in Fig. 2(b) (Li and Kubo, 1999). Each ring is divided again into $8j-4$ elements for the j -th ring ($j = 1, 2, \dots, m$). It makes a total of $4m^2$ divided elements with equal areas of $a = \delta R^2 / 4m^2$. The elements of the j -th ring are located on the circumference with radius

$$\bar{r}_j = \frac{2(r_{j-1}^2 + r_{j-1}r_j + r_j^2)}{3(r_{j-1} + r_j)} \quad (16)$$

In order to confirm the validity of the new beam theory proposed in this paper, a cantilever (Cantilever I) with a circular section, whose material parameters are listed in Table 1, subject to monotonic and cyclic loads, is first calculated with a finite element beam analysis based on the proposed theory. Cantilever I considered here is an RC material with a length of 8 m and a diameter of 1.2 m. In the calculations, the cantilever is simulated with eleven beam elements and the circular section is divided into 100 elements in the form shown in Fig. 2. Monotonic and cyclic axial and/or bending loads are applied at the top of the cantilever. Figure 3(a) shows the $M-\Phi$ relations at different axial loads. Figure 3(b) shows the relation between the strength (peak value) of bending moment and the axial force. In the figure, N_c and N_t represent the compressive and extensive axial strengths respectively, under the condition that no bending moment acts on the beam. The strengths of the bending moment at different axial loads are also compared with the results given by the Design Codes for Concrete Structures of Japan Railway (1992). The moment and the curvature in the figure are those from the center of the beam. It is found from the analysis that the axial-force dependent $M-\Phi$ relation is well simulated under monotonic loading condition.

Figure 4 shows the nonlinear behavior of Cantilever I under cyclic loading. Cyclic loads are applied in two directions, that is, axial force and horizontal force. The axial force is a 4-cycle sin function whose maximum and minimum values are 4 MN and zero. The horizontal loading is applied by a cyclic given displacement δ_i expressed by the following equation as,

$$\delta_i (\text{unit:m}) = \left(\sum_{k=1}^i 0.15k\omega \cos(k\omega) \right) / 2000, \quad i = 1, 2, \dots, 2000, \quad \omega = (8\pi/2000) \quad (17)$$

It is interesting that the maximum bending moment always occurs at the time when the maximum axial load is applied to the beam. It is found from the analysis that the axial-force dependent $M-\Phi$ relation can also be qualitatively well simulated under cyclic loading condition.

Then, the test of an RC beam with a circular section subjected to a one-direction cyclic horizontal load under constant axial force is simulated. The material parameters of the cantilever (Cantilever II) are listed in Table 2. The height from the loading position to the fixed point of the cantilever is 244 cm. The constant axial force is 645 kN. The cyclic given horizontal displacement shown in Fig. 5(a) is applied to the cantilever. It is found

Table 1. Parameters of Cantilever I

Compressive strength of concrete: $\sigma_c = 36.0$ MPa
Tensile strength of concrete: $\sigma_t = 3.0$ MPa
Young's modulus of concrete: $E_c = 2.5 \times 10^4$ MPa
Young's modulus of steel: $E_s = 2.1 \times 10^4$ MPa
Yielding strength of steel: $\sigma_y = 3.8 \times 10^2$ MPa
reinforcement: $\phi 29 \times 24$
Overburden of reinforcement: 15 cm
Diameter: 120 cm

Table 2. Parameters of Cantilever II

Compressive strength of concrete: $\sigma_c = 24.0$ MPa
Tensile strength of concrete: $\sigma_t = 3.5$ MPa
Young's modulus of concrete: $E_c = 2.4 \times 10^4$ MPa
Young's modulus of steel: $E_s = 2.0 \times 10^4$ MPa
Yielding strength of steel: $\sigma_y = 3.0 \times 10^2$ MPa
reinforcement: $\phi 19 \times 20$
Overburden of reinforcement: 3.5 cm
Diameter: 61 cm

Fig. 2. Discretization of circular section of RC pile

Fig. 3. Nonlinear behaviors of Cantilever I under monotonic loading

Fig. 4. Nonlinear behavior of Cantilever I under cyclic loading

Fig. 6. Theoretical and tested stress-strain relations in generalized stress paths under monotonic loading (After Nakai and Matsuoka, 1986)

residual state. In present case, such adjustment was not conducted.

NONLINEAR PROPERTIES OF SOILS

Nakai and Matsuoka (1986) proposed a *tij* clay model based on the concept of *SMP* (spatially mobilized plane), in which the influence of the intermediate principal stress can be properly evaluated. The model has been verified through many true triaxial tests on normally consolidated clay in generalized stress paths. Chowdhury et al. (1999) extended the *tij* clay model to a kinematic model using the concept of subloading so that it can not only describe a monotonic loading, but also a cyclic loading. Figure 6 shows the stress-strain relation and the stress path in the monotonic true triaxial tests under constant mean stress, in which θ represents the angle between the σ_1 -axis (axial stress) and the corresponding radial stress (σ_2) on the deviatoric plane (π plane). The angle $\theta = 0^\circ$ and $\theta = 180^\circ$ denote the stress path of triaxial compression and triaxial extension respectively. Stress paths $\theta = 15^\circ$, 30° and 45° denote three different principal stress paths. Figure 7 shows the stress-strain relation in cyclic conventional triaxial tests under constant mean stress. From the figures, it is clear that the model can describe the stress-strain-dilatancy relation of soft clay well in monotonic and cyclic loadings. The model consists of seven parameters that can be determined with conventional triaxial compression tests. Detailed discussion about how to determine these parameters can be referred to in the corresponding reference.

On the other hand, it is known that the D-P model has

Fig. 5. Comparison of cantilever under cyclic loading

from the analysis that the calculation simulates the test results to a very satisfactory accuracy, as shown in Fig. 5. For simplicity, the new beam theory is called the axial-force dependent model (AFD model) in the following discussion. In final three-cycle loading, a softening behavior was observed in the test. In the calculation the behavior can also be simulated, but the degree of the softening is not so big as the tested one. This might be improved by reducing $\lambda_c \sigma_c$, the compressive strength at

Fig. 7. Theoretical and tested stress-strain relations in generalized stress paths under cyclic loading (After Chowdhury et al., 1999)

the advantage that it is very simple and only four parameters need to be determined (Zhang et al., 2000b). In geotechnical engineering, Japanese engineers prefer to determine these parameters with the SPT N -value, instead of with the laboratory tests. The disadvantage of the model is that it cannot describe precisely the stress-strain-dilatancy relation of soil under monotonic and cyclic loading conditions. Therefore, the accuracy of a calculation based on the D-P model should be checked by a calculation based on a precise model.

SIMULATION OF DYNAMIC BEHAVIOR OF AN ELEVATED BRIDGE WITH A GROUP-PILE FOUNDATION

Based on the discussion in previous sections, an elevated highway bridge with a group-pile foundation is considered in a seismic evaluation. The bridge is supported by a group-pile foundation made of 3×4 cast-in-place reinforced concrete piles, 1.2 meters in diameter (D) and 30 meters in length, as shown in Fig. 8. The distance between the centers of the two piles is $2.5 D$. The ground is composed of five layers. The surface layer of the ground is a sandy reclaimed soil, followed by a very soft alluvial clayed layer, 10 meters in thickness, with a small SPT N -value. The third layer is also alluvial clayed soil, and the fourth layer is an alluvial sandy soil. The pile group is laid on the fifth layer, diluvial gravel. The bottom layer of the ground is supposed to be an elastic material in the numerical analysis. Figure 9 shows the setup of the group-pile foundation and the geologic profile of the ground. In the figure, $b_1 \sim b_{12}$ represent the numbers of the beam elements whose results are discussed in detail.

A dynamic analysis of the elevated highway bridge using a full system is conducted to simulate the mechanical behavior in a major earthquake. The direct integration method of Newmark- β is adopted. The finite element mesh for the ground is shown in Fig. 10, in which the mass of the super structure is represented by a nodal mass located at the position of inertial force shown in Fig. 8. The pier is simulated by beam elements with consistency

Fig. 8. Geometry of elevated highway bridge

(a) Plane view of piles and footing

(b) Geologic profile of ground and the position of the elements under consideration

Fig. 9. Setup of group-pile foundation and ground

Fig. 10. Finite element mesh

Fig. 11. Input earthquake wave

mass. The boundary condition is that: (a) the bottom of the ground is fixed; (b) the vertical boundaries parallel to the XOZ plane are fixed in the y direction and free in the x and z directions; (c) an equal-displacement-boundary condition is used between the two side boundaries whose normal direction is parallel to the x -axis to simulate the infinite boundary in real situation (Kimura and Zhang, 2000). The boundary condition of the piles in the calculation is that the head of the pile is fixed with the footing and the toe of the pile is free. Figure 11 shows the input earthquake wave used in the analysis, which is an earthquake wave recorded on Kobe's Port-island during the Hyogoken-Nambu earthquake. It has a maximum acceleration of 687 gal in the horizontal direction. A Rayleigh type of damping is adopted and the values of the structures and the ground are assumed as 2% and 10%, respectively, in the dynamic analysis of the full system. It is known that damping that consists of hysteretic damping and viscous damping is very difficult to evaluate for a full system, and its value may greatly affect the elastic dynamic response. In the plastic region, however, the hysteretic damping from the elastoplastic behaviors of soils and piles may become a dominant fact. In engineering design, the hysteretic damping is usually equalized to an equivalent viscous damping, by taking a large value of damping for strong elastoplastic materials. For example, the value of damping for soft clay under strong earthquake movement may take a value of 20%. Therefore, the value of damping assumed here is not determined by a strict analysis and is only an empirical one. Although the stiffness of the ground, the piles, and the pier may change because of the nonlinearity of these materials, the viscous matrix calculated from the Rayleigh type of damp is assumed to be constant irrespective of the changes in the stiffness matrix.

In calculating the viscous matrix, an eigenvalue analysis for the full system is conducted to evaluate the first two eigenvalues. The eigenvalue analysis shows that the first two eigen periods are 1.155 and 1.067 sec, respectively. The eigenvalue analysis is conducted with a hybrid of Jacobian and subspace methods. In the dynamic analysis, the time interval of the integration is 0.01 sec. It should be pointed out that, in the case of nonlinear analyses, eigenvalue analysis is only related to the initial stiffness.

The parameters of the piles described by AFD model are listed in Table 3. The nonlinearity of the pier described by a trilinear model (Kimura and Zhang, 2000) is

Table 3. Parameters of pile

Compressive strength of concrete: $\sigma_c = 2.4 \times 10^4$ kPa
Tensile strength of concrete: $\sigma_t = 3.5 \times 10^3$ kPa
Young's modulus of concrete: $E_c = 3.0 \times 10^7$ kPa
Yielding strength of steel: $\sigma_y = 3.0 \times 10^5$ kPa
Arrangement of the reinforcement:
Part A (0.00–2.40) main: $\phi 29 \times 28$, hoop: $\phi 16$ ctc 150, OB: 15 cm
Part B (2.40–10.0) main: $\phi 29 \times 28$, hoop: $\phi 16$ ctc 300, OB: 15 cm
Part C (10.0–30.0) main: $\phi 29 \times 14$, hoop: $\phi 16$ ctc 300, OB: 15 cm

OB: Overburden of reinforcement; ctc: center-to-center distance

Table 4. Nonlinearity of pier (trilinear model)

M_x^y (MN $\cdot$ m)	Φ_c (1/cm)	M_y^x (MN $\cdot$ m)	Φ_y (1/cm)	M_z^x (MN $\cdot$ m)	Φ_u (1/cm)
30.13	0.00934	79.84	0.0785	103.37	0.681

$E_{\text{concrete}} = 300 \times 10^4$ (MPa)

Table 5. Material parameters of soil
(a) Clays described by tij subloading model

Soil	SPT- N	γ (kN/m 3)	λ	κ	ϕ' ($^\circ$)	a	X_r	k_r	e_0	ν
Ac1	2	17.0	0.0553	0.0081	34.6	1.0	0.3	5.0	0.88	0.42
Ac2	8	18.0	0.0088	0.0017	34.6	1.0	0.3	5.0	0.88	0.40

(b) Sandy soils described by D-P model (See Appendix I)

Soil	SPT- N	γ (kN/m 3)	E (MPa)	ν	K_0	c (KPa)	ϕ' ($^\circ$)
B	4	18.0	10	0.30	1.00	0.0	23
As1	16	19.0	40	0.30	0.50	0.0	35
Dg	50	20.0	125	0.30	0.50	elastic	elastic

listed in Table 4. Because the parameters involved in the tij subloading model should be evaluated by laboratory tests which are not available in the present case, these values are estimated with the SPT N -value in such a way that the initial stiffness for the tij subloading and D-P models is the same at small strain level, and that the shear stress ratio q/σ'_m at critical state for both models is the same, if noticing the fact that four parameters involved in D-P model can be easily determined with the SPT N -value (Zhang et al., 2000b). The corresponding values of the parameters of the tij subloading model for clays are listed in Table 5(a). The sandy soils of the ground are described by D-P model and their values are listed in Table 5(b).

Dynamic Behaviors of Soils and RC Piles in Earthquake

Figure 12 shows the distribution of the maximum difference of shear strain ($\gamma_{zx}^{\max} - \gamma_{zx}^{\min}$) occurring at different positions. Here, $\gamma_{zx}^{\max}$, $\gamma_{zx}^{\min}$ represent the maximum

Fig. 12. Distribution of shear strain in ground at different positions

positive and maximum negative shear strain of γ_{zx} . It can be seen that the shear strains of the soils between the piles (soils in Column 1 and Column 2) are almost the same as those further afield (30 m away from the footing). Moreover the shear strain of the soils in front of the piles are much larger than the others, showing that large shear strains occur at the interface between the soil and outside piles of a pile group. The soils within the piles, however, behave in the same way as the soils further afield. This phenomenon can also be observed in the hysteresis of the shear stress-strain relations at different soil layers, as shown in Fig. 13. In Fig. 13, the numbers e1 ~ e28 represent the positions of the soil elements shown on the left of Fig. 12. The maximum shear strain at the interface is about 12% while further afield its value is less than 4%. The history loops of the shear stress-strain relations are much bigger at the interface between the soil and outside piles of a pile group than those at other places.

Figure 14 shows the distribution of the sectional forces in the piles at the time of 3.1 second, when a maximum deformation of the surface ground in the left direction happened. In the figure, the sectional forces M , N and S are normalized with M_{max} , N_{max} and S_{max} that represent the maximum values of the bending moment, the axial force and the shear force existed in all piles, respectively. In this case, Pile 1 is a front pile and Pile 4 is a back pile. Due to the different axial force occurring in Pile 1 to Pile 4, the distributions of the bending moments and the shear forces in different piles are quite different and show a very clear tendency; that is, the larger the axial force is, the larger the bending moment and shear force will be. In the figure, the distributions of the sectional forces are unified by dividing the corresponding maximum forces as shown in the figure. It is known from the calculation that the difference of the maximum bending moment in four piles, which occurs at the heads of piles, is about 30%, while the difference of the maximum shear force is about 150%. Without introducing AFD model, the difference cannot be clearly identified

The same phenomenon shown in Fig. 14 can also be observed in the distribution of the sectional forces in the piles at the time of 3.6 second, when a maximum defor-

mation of the surface ground in the right direction occurred, as shown in Fig. 15. In this case, Pile 4 is a front pile and Pile 1 is a back pile.

Figure 16 shows the load sharing ratios of the piles at the depths of 5 m, 7 m and 15 m. In the figure, the sectional forces N and S are normalized with the maximum values of N_{max} and S_{max} respectively. It is clear that the shear force in the piles changes with the axial force in seismic loading at all depths, a fact that has been verified under static loading condition in a full-scale cast-in-place RC 9-pile foundation subjected to a cyclic lateral loading up to ultimate state (Kimura et al., 1993).

From Figs. 14~16, it can be concluded that the phenomenon that the bending strength and the load-sharing ratio of the piles in a pile group subjected to lateral loading can be properly simulated with the analysis.

Figure 17 shows the hysteretic loop of the $M-\Phi$ relation of the pile at different depths, as shown in Fig. 9(b). It is known from the figure that if the bending moment reaches a certain large value, a large hysteretic loop will occur in the $M-\Phi$ relation, which means that hysteretic damping will be dominant in the total damping of the beam materials.

Figure 18 shows the histories of the moments in the pile at the corresponding positions. It is known from the figure that due to the plastic deformation of the pile, residual moments remains after the earthquake has finished. Because the time when the maximum bending moment occurs is about 3.1 seconds and is the same as the time when the maximum horizontal displacement occurs, it is reasonable to select the time as a representative used in Figures 14, 19 and 20.

Influence of the Nonlinearity of Soil and RC Material Described by Different Models

In order to investigate the influence of the nonlinearity of the soil and RC material described by different models, the analytical results obtained from the following 4 types of analyses are compared and discussed in detail:

- Case I: soils and RC material are all elastic (elastic);
- Case II: soils are described by D-P model and RC material is simulated with a trilinear model (dp-tri);
- Case III: soils are described by D-P model and RC material is simulated with an AFD model (dp-afd);
- Case IV: soils are described by tij subloading model and RC material is simulated with an AFD model (tij-afd).

The nonlinearity of the piles described by the trilinear model is listed in Table 6. The material properties of the grounds described by the elastic model are listed in Table 7, in which the velocities of shear waves are measured by seismic prospecting. Table 8 shows the material parameters in the D-P model, whose values are determined based on the SPT N -value (Zhang et al., 2000b).

Figure 19 shows a comparison of the distribution of the moments in different piles obtained from the aforementioned four types of analyses. The distribution of the mo-

Fig. 13. Stress-strain relations of ground at different positions

ment depicted here is a simultaneous one at the time when a maximum bending moment occurs at the pile. The times for different cases are also shown in the figure. In Cases I and II, in which the piles are described by elastic and trilinear modes, no outstanding difference can be observed in the distributions, showing that in the present analyses, without introducing the AFD model, it is im-

possible to describe the axial-force dependent behavior of RC piles. The same conclusion can be given in the analysis in which the soils are described with the t_{ij} subloading model (Zhang et al., 2000a). On the other hand, if the AFD model is introduced to the analysis, the difference of the bending moment due to axial force can be properly simulated irrespective of the models adopted for soils.

Fig. 14. Distribution of sectional forces of group piles when bending occurs in left direction ($t = 3.10$ second)

Fig. 15. Distribution of sectional forces of group piles when bending occurs in left direction ($t = 3.60$ second)

Figure 20 shows the comparisons of the moments obtained by different analyses. The shapes of the distribution of the moments in piles are similar in all cases. Generally speaking, the results from elastic analysis are totally different from those obtained from nonlinear analysis. In the present analysis, the result obtained from elastic analysis are occasionally of the same order as the results from nonlinear analyses. This is thought to be the reason that the initial Young's Modulus evaluated from the velocity of shear wave and SPT N -value are different. In the present case, the former is much larger than the later. Besides, the maximum bending moment happens at the pile head in all cases. In the cases of introducing AFD model, different constitutive model adopted for soils may greatly affect the maximum bending moment. This is due to the fact that the stiffness of the soil evaluated from D-P model (elasto-perfect plasticity) is much smaller than those of tij subloading model at yielding state, resulting in a large deformation of the ground.

Figure 21 shows the comparison of the responding dis-

placement and acceleration at the surface of the ground and the top of pier obtained from the analyses aforementioned. The responding acceleration in the case of the tij subloading model is larger than those of the D-P model, while the displacement in the case of the tij subloading model is only about the third of that in the D-P model. It is also known from the figure that the deformation of the ground is mainly dependent on the constitutive model adopted for soils irrespective of the constitutive model for piles. It is clear from the figure that the acceleration obtained from elastic analysis is much larger than those from nonlinear analyses. On the other hand, the displacement is of the same order for all analyses, which explains the reason why the bending moment is of the same order for all cases. The spectrum of the responded accelerations at the surface of the ground and the top of the pier are shown in Fig. 22, in which the eigen period of the responded wave becomes longer in the case of the D-P model. The reason for the phenomenon is that at larger strain level, the stiffness of the ground evaluated from the

Fig. 16. Load sharing ratio of group piles

Fig. 17. $M-\phi$ relations of pile

tij subloading model is larger than that of the D-P model (elasto-perfect plasticity). As a result, the maximum bending moment at the bottom of the pier in the case of the tij subloading model is larger than that of the D-P model, as shown in Fig. 23.

CONCLUSIONS

Based on a new proposed beam theory that can properly describe the axial-force dependency of RC materials,

a series of three-dimensional elastoplastic finite element dynamic analyses are conducted on a superstructure-foundation-ground system. By considering different constitutive models for soils and RC piles, the influence of the nonlinearity of the ground and the piles is carefully investigated. The conclusions can be given as follow:

An axial-force dependent nonlinear model for the $M-\phi$ relation of RC material is proposed based on a new weak form of the equilibrium equation for RC material. It can be applied to finite element analyses and satisfies the com-

Fig. 18. Time history of bending moment of pile

Table 6. Nonlinearity of piles (trilinear model)

Position (m)	M_c (MN*m)	ϕ_c (1/cm)	M_y (MN*m)	ϕ_y (1/cm)	M_u (MN*m)	ϕ_u (1/cm)
0.0~2.4	0.545	0.0186	1.588	0.2126	2.476	2.639
2.4~10.0	0.545	0.0186	1.604	0.2102	2.426	1.585
10.0~30.0	0.509	0.0186	0.867	0.1914	1.319	2.122

Table 7. Material properties of soils (elastic)

Soil	SPT-N	Thickness (m)	γ (kN/m ³)	G (MPa)	ν	K_0	v_s (m/s)
B	4	4	18.0	29.6	0.49	1.00	127
Ac1	2	10	17.0	27.5	0.49	0.80	126
Ac2	8	14	18.0	73.5	0.49	0.67	200
As1	16	4	19.0	79.1	0.49	0.50	202
Dg	50	2	20.0	177.6	0.49	0.50	295

Table 8. Material properties of soils (D-P model)

Soil	SPT-N	Thickness s(m)	γ (kN/m ³)	E (MPa)	ν	K_0	c (KPa)	ϕ' (°)
B	4	4	18.0	10	0.30	1.00	0.0	23
Ac1	2	10	17.0	5	0.42	0.80	12.5	0
Ac2	8	14	18.0	20	0.40	0.67	50.0	0
As1	16	4	19.0	40	0.30	0.50	0.0	35
Dg	50	2	20.0	125	0.30	0.50	elastic	elastic

patibility of deformation.

The new model can well simulate the axial-force dependency of RC materials under monotonic and cyclic loadings. Its validity is verified by experimental results with a very high accuracy.

By introducing an axial-force dependent nonlinear model for the M - ϕ relation of RC material, the difference among the distribution in bending moments of front, back and middle piles in a group-pile foundation subjected to seismic loading can be well simulated. In the present

analyses, without using a suitable axial-force dependent model for piles, such as the AFD model, it is impossible to simulate well the difference aforementioned.

A different constitutive model of soil may give a quite different prediction of the deformation and the acceleration of the ground, resulting in a different response of the superstructure. The acceleration transferred from the ground to upperstructure is underestimated by the D-P model due to the overestimated absorption of earthquake energy by the deformation of the ground.

Due to the difference of the deformation predicted by different constitutive models, the distributions of the moment in the piles are quite different. The time history of the moment is also different if the soil is described with different models.

A large moment occurs not only at the head of piles, usually caused by the inertial force from the upperstructure, but also within the ground. Therefore, it is worth emphasizing that the influence of the deformation of a ground on the piles must be considered carefully.

Fig. 19. Comparison of distribution of bending moment in group piles

Fig. 20. Comparison of distribution of bending moment in each pile obtained from different analyses

The D-P model is simple and only four parameters are needed. These parameters can be determined with the SPT N -value, which is quite familiar to engineers. On the other hand, because the mechanical behavior of soil is complicated, the four-parameter constitutive model has its limitations. A precise model such as the kinematic tij subloading model is preferred if the ground information from laboratory tests is available in a seismic evaluation of a pile foundation.

The validity of the numerical analyses proposed in this paper, by which the dynamic behaviors of a group-pile

foundation is predicted, should be verified with model tests or centrifuge tests in future research.

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Fig. 21. Responded acceleration and displacement obtained from different analyses

Fig. 22. Spectrum ($h=0.02$) of acceleration at top and ground

calculation conducted in this paper.

NOTATIONS

$\sigma_1, \sigma_2, \sigma_3$	Principal stresses of soil
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	Principal strain of soil
J_2	Second invariant of the deviatoric stress tensor
σ_m	Mean stress of the deviatoric stress tensor
ν	Poisson's ratio of soil
λ	Compression index
κ	Swelling index
γ_s	Unit weight of soil
e_0	Initial void ratio
ϕ'	Internal frictional angle
a, X_r, k_r	Material parameters involved in tij subloading model
α, κ_s	Parameters involved in D-P model
E_c	Young's modulus of concrete
E_s	Young's modulus of steel
σ_c	Compressive strength of concrete
σ_t	Tensile strength of concrete
σ_y	Yielding strength of steel
M_c, M_y, M_u	Cracking, yielding and failure moments of RC beam
Φ_c, Φ_y, Φ_u	Curvatures of RC beam corresponding to cracking, yielding and failure moments
ε_{m1}	Bending strain due to M_x
ε_{m2}	Bending strain due to M_y
ε_0	Axial strain due to axial force
ν_s, β_s, γ	Parameters involved in nonlinear stress-strain relation of steel
$\nu_c, \tau, \mu, \lambda_c$	Parameters involved in nonlinear stress-strain relation of concrete
ε_c	Compressive strain of concrete when stress reaches compressive strength
ε_t	Tensile strain of concrete when stress reaches tensile strength
ε_m	Maximum tensile strain of concrete
E_r	Compressive stiffness of concrete at post-peak loading process

Fig. 23. M - ϕ relations of pier at fixed point with foundation obtained from different analyses

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APPENDIX I

It is known that the yielding function of the D-P model is expressed as follows:

$$f_y = (J_2)^{1/2} - 3\alpha\sigma_m - \kappa_s = 0 \quad (A1)$$

where J_2 is the second invariant of the deviatoric stress tensor and σ_m is the mean stress. α and κ_s are material constants which can be determined from ϕ' , the internal frictional angle of soil, and c , the cohesion of soil. It is known that there are several ways of evaluating the parameters α and κ_s . If the Mohr-Coulomb failure sur-

Fig. A1. Tri-linear model for RC beam

face is circumscribed by the cone, which represents the failure surface described by the D-P model, then the following relation is valid:

$$\alpha = \frac{2 \sin \phi'}{\sqrt{3}(3 + \sin \phi')}, \quad \kappa_s = \frac{6 c \cos \phi'}{\sqrt{3}(3 + \sin \phi')} \quad (\text{A2})$$

In this paper, Eq. (A2) is used to evaluate α and κ_s from ϕ' and c .

APPENDIX II

The trilinear model of RC material is described by Fig. A1, in which the degrading effect in the unloading process is not considered.