APPLICATION OF MICROMECHANICS MODEL TO STUDY ANISOTROPY OF SOILS AT SMALL STRAINS

S. Yimsiri\textsuperscript{1)} and K. Soga\textsuperscript{2)}

ABSTRACT

A micromechanics model is used to analyse the stiffness anisotropy of soils at small strains. Five material constants for a cross-anisotropic elastic material are related to micromechanics variables such as fabric anisotropy, contact stiffness, particle radius, and the number of contacts in a given volume of particulate assembly. The analytical results from the model are compared with the published experimental data on small-strain stiffness anisotropy in order to estimate typical soil fabric conditions of sands and clays. The relationship between the small-strain shear modulus obtained from triaxial tests and shear tests is examined using the micromechanics model. The analysis shows that, when a soil is stiffer in the horizontal direction, the shear modulus evaluated from the conventional triaxial drained tests underestimates \( G_{sh} \) and \( G_{bh} \). The opposite is true when a soil is stiffer in the vertical direction. When a soil is sheared in undrained condition, the measured shear modulus is closer to \( G_{sh} \) than \( G_{bh} \), especially when the soil is stiffer in the horizontal direction. The effect of soil anisotropy on the stiffness measured from different stress paths in triaxial condition is investigated.

Key words: anisotropy, elasticity, shear modulus, small strain, soil structure, triaxial test, Young's modulus (IGC: D6)

INTRODUCTION

Natural soils are believed to possess cross-anisotropic behaviour in the pre-failure strain range due to their mode of deposition, which is one dimensional in nature. Anisotropic consolidation stresses align platy particles and particle groups with their long axes perpendicular to the major principal stress. An anisotropic fabric and an anisotropic stress state result in anisotropic mechanical behaviour. Unfortunately, it is rare that cross-anisotropy is properly taken into account when interpreting experimental data. If a soil is anisotropic, the small-strain modulus obtained from in-situ tests (e.g. pressuremeter, cross-hole, and down-hole techniques) and from laboratory tests (e.g. triaxial, torsional shear, resonant column, and ultrasonic tests) can be different because of the difference in the modes of shear deformation. Hence, when the measured values are compared, it is important to take the effects of soil anisotropy into consideration.

Using the continuum mechanics approach, the ordinary cross-anisotropic elastic model requires five independent parameters to define its compliance matrix (Love, 1944).

\[
\begin{bmatrix}
\delta e_v \\
\delta e_h \\
\delta e_h \\
\delta y_v \\
\delta y_h
\end{bmatrix}
\begin{bmatrix}
1 \\
\nu_{vh} \\
\nu_{hh} \\
\nu_{hv} \\
\nu_{vh}
\end{bmatrix}
\begin{bmatrix}
E_v \\
E_h \\
E_h \\
E_v \\
E_h
\end{bmatrix}
\begin{bmatrix}
\sigma_v' \\
\sigma_h' \\
\sigma_h' \\
\tau_{vh} \\
\tau_{vh}
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{G_{sh}} \\
\frac{1}{G_{bh}} \\
\frac{1}{G_{bh}} \\
\frac{1}{G_{sh}} \\
\frac{1}{G_{sh}}
\end{bmatrix}
\begin{bmatrix}
\delta e_v \\
\delta e_h \\
\delta e_h \\
\delta y_v \\
\delta y_h
\end{bmatrix}
\]

where subscripts \( v \) and \( h \) denote the vertical and horizontal directions, and \( \nu_{vh}/E_h = \nu_{hv}/E_v \) from the symmetry of the compliance matrix, \( G_{sh} = (1/2)E_v/(1 + \nu_{vh}) \) due to isotropy in the horizontal plane, and \( G_{bh} = G_{hv} \).

The use of micromechanics models is an alternative approach to model the small-strain behaviour of soils. A macroscopic compliance matrix similar to Eq. (1) can be derived from modelling the interaction between discrete particles in an assembly. This type of modelling requires parameters that are based on the particulate level (e.g. contact stiffness and particle orientation), which are difficult to quantify in practice. However, it can provide

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Manuscript was received for review on August 6, 2001.

Written discussions on this paper should be submitted before May 1, 2003 to the Japanese Geotechnical Society, Sugayama Bldg. 4F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101-0063, Japan. Upon request the closing date may be extended one month.
some insights into the fundamental mechanism of soil behaviour in question. Some researchers have derived micromechanics models according to some specific hypotheses and assumptions (e.g. Chang and Liao, 1994; Chang and Gao, 1996). In this paper, the micromechanics model derived by Yimsiri and Soga (2000) is used for the investigation because this model explicitly includes the fabric anisotropy while other models concentrate on isotropic fabric. Soil fabric conditions of various soils are evaluated from the published experimental data on small-strain stiffness anisotropy via the micromechanics model. The small-strain stiffness measured in triaxial tests is discussed in relation to soil anisotropy. The effect of soil anisotropy on the stiffness measured at various stress paths is examined.

The anisotropic stiffness of clay and sand is discussed collectively in this paper. Although clay and sand are different in several aspects such as particle shape, minerals, and the degree of electro-magnetic forces between particles, they have, to some extent, similar empirical equations to characterise their small-strain behaviour (e.g. Hardin and Black, 1966, 1968, 1969). Some researchers have suggested that the aggregate clusters of clay particles are the minimum units controlling the macroscopic mechanical behaviour of clayey soils and it would be appropriate to consider the aggregate clusters of clays having granular behaviour (e.g. Barden, 1972; Collins and McGown, 1974; Matsuo and Kamon, 1977).

MICROMECHANICS MODEL

The stress-strain behaviour of soil at small strains is governed by the state of the soil fabric and the force conditions at the particle contacts. By considering the interactions among the discrete particles in the assembly, micromechanics models relate the small-strain modulus of soil to the soil fabric and the applied stress state. In other words, by fitting the model to the experimental data, it is possible to gain some understanding of the conditions of the soil, such as the contact conditions and the soil fabric characteristics.

The cross-anisotropic elastic parameters obtained from the micromechanics model are the following. The detailed derivation of the model can be found in Yimsiri and Soga (2000).

\[
E_r = \frac{4r^2NK_N}{3V(3-a)} \left[ \frac{21(5+a)^2\chi}{15(14-2a+21\chi+9a\chi)} \right] \tag{2a}
\]

\[
E_h = \frac{4r^2NK_N}{3V(3-a)} \left[ \frac{21(5-3a)^2\chi}{15(14-6a+21\chi-15a\chi)} \right] \tag{2b}
\]

\[
G_{rh} = G_{hv} = \frac{4r^2NK_N}{3V(3-a)} \times \left[ \frac{21(5+a)(5-3a)\chi}{10(5-a)(105-46a+23a^2+70\chi-24a\chi+2a^2\chi)} \right] \tag{2c}
\]

\[
G_{nh} = \frac{4r^2NK_N}{3V(3-a)} \left[ \frac{21(5-3a)^2\chi}{10(21-11a+14\chi-10a\chi)} \right] \tag{2d}
\]

where \(r\) is the radius of the spherical soil particles, \(N\) is the number of the contacts in the assembly, \(V\) is the volume of the assembly, \(a\) is the degree of fabric anisotropy, \(K_N\) is the normal contact stiffness, \(K_h\) is the tangential contact stiffness, and \(\chi\) is equal to \(K_h/K_N\).

The fabric of naturally deposited soil is assumed to be cross-anisotropic, and the degree of fabric anisotropy is represented by a single parameter \(a\). The degree of fabric anisotropy, \(a\), relates to the contact normals distribution of the particles according to the Fourier series proposed by Chang et al. (1989) as shown in Eq. (3).

\[
E(\gamma, \beta) = \frac{3(1+a \cos 2\gamma)}{4\pi(3-a)} \tag{3}
\]

where \(a\) is the degree of fabric anisotropy \((-1 < a < 1)\) and \(\gamma, \beta\) angles are defined in Fig. 1. Equation (3) has the symmetry \(E(\pi + \gamma) = E(\gamma)\) and is independent of \(\beta\).

The contact normal distribution functions according to Eq. (3) for different \(a\) values are shown in Fig. 2. The contact normals concentrate more in the vertical direction when \(a\) is greater than zero, whereas they concentrate more in the horizontal direction when \(a\) is less than zero.

The soil fabric is isotropic when \(a\) equals zero and Eq. (2) becomes identical to those derived earlier by Chang and Liao (1994) for isotropic fabric.

Using the micromechanics model, the elastic parameters are directly related to the soil fabric condition (degree of fabric anisotropy \(a\)) and the interaction between particles (\(\chi\)). It is important to note here that the relationship among the normal modes of deformation \((E_r, E_h, v_{rh}, v_{hv}\text{ and } v_{nh})\) and the shear modes \((G_{rh} \text{ and } G_{nh})\) can be found through Eq. (2) for given micro-
mechanics variables ($a$ and $\chi$). This makes possible the comparison of the small-strain stiffness of soils measured in different shear modes (e.g. triaxial vs. torsional shear).

**Contact Conditions**

The contact model is represented by normal contact stiffness $K_N$ and tangential contact stiffness $K_\theta$. However, these parameters are at the contact level and they are difficult to quantify in the laboratory.

The non-linear elastic Hertz-Mindlin contact model represents the contact condition between elastic, perfectly rounded, and smooth-surface spheres. The normal and tangential contact stiffness of spheres between two equal-sized spheres are defined as follows:

- **Normal contact stiffness** (Mindlin and Deresiewicz, 1953):
  \[ K_N = \left( \frac{3 \pi G^2}{(1 - \nu^2)} \right)^{1/3} f_N^{1/3}. \]  

- **Tangential contact stiffness** (Bowden and Tabor, 1964; Johnson, 1985):
  \[ K_\theta = \frac{2(1 - \nu)}{(2 - \nu)} K_N \left( \frac{f_N}{f_N \tan \phi_\theta} \right)^{1/3} \]  

where $G$ and $\nu$ are the shear modulus and Poisson's ratio of the solid particles, $\phi_\theta$ is the inter-particle friction angle, $f_N$ is the normal contact force, and $f_\theta$ is the tangential contact force.

In general, $K_N$ and $K_\theta$ of the Hertz-Mindlin contact model cannot explicitly be described since they are functions of $f_N$ and $f_\theta$, which change with contact directions. For isotropic fabric assembly under isotropic stress condition ($a = 0$ and $f_\theta = 0$), the value of $\chi (= K_\theta / K_N)$ is 0.82 for a typical Poisson's ratio of 0.3. However, the contact condition of two smooth elastic spheres assumed in the model renders the anomalous pressure dependence of small-strain stiffness (e.g. slope of $1/3$ in $\log G_{nh}$ versus $\log \sigma$ for isotropic fabric assembly under isotropic stress condition, where $\sigma$ is the confining pressure), which is the limitation of the model as discussed by Yimsiri and Soga (2000).

Greenwood and Tripp (1967) performed mathematical analysis of the contact between a smooth sphere and rough surface, of which the asperities heights are distributed according to the Gaussian probability function. Their result showed that the Hertz contact theory represents the large load limit for a rough surface. At a smaller load, the contact pressure along the rough surface is much lower and the contact spreads over a very much larger area than that obtained from the Hertz contact model. This analysis was confirmed experimentally by Johnson (1985). The experimental results by O'Connor and Johnson (1963) suggested that the tangential contact stiffness between rough surfaces can still be described by Eq. (5). Based on these results, Yimsiri and Soga (2000) proposed the Rough-Surface contact model, which is derived for the contact conditions between two rough elastic spheres. It was found that this model can better simulate the pressure dependency of small-strain stiffness measured in laboratory (for details see Yimsiri and Soga, 2000). According to the Rough-Surface contact model, the value of $\chi$ depends on the normal contact force and the surface roughness properties. The ratio is usually greater than one at low normal contact forces and approaches to the Hertz-Mindlin contact behaviour at high normal contact forces.

The numerical analyses results from the Hertz-Mindlin and the Rough-Surface contact models at various stress ratios ($\sigma' : \sigma'' = 4:1$ to $1:4$) at constant $\rho' = 1$ MPa with various degrees of fabric anisotropy ($a = 0.6$ to $-0.6$) are plotted in Fig. 3. These numerical analyses include the effects of confining pressure (low pressure = Rough-Surface model, high pressure = Hertz-Mindlin model) and anisotropic stress state on non-linear contact characteristics. The results show that the relationship between **Fig. 3. Relationship between $E_i/E_{ih}$ and $G_{oh}/G_{oh}$ from micromechanics models**
\(E_s/E_h\) and \(G_{sh}/G_{hh}\) from the Hertz-Mindlin and the Rough-Surface contact models are similar, and that they are also close to the results from the values of \(\chi\) between 0.5 and 1.0. Consequently, in this paper, the range of \(\chi\) between 0.5 and 1.0 is taken as a typical contact condition of a representative soil for simplicity. The analytical results from the micromechanics model, as shown in Fig. 3, indicates that, if \(E_s/E_h\) is less than 1.0, \(G_{sh}/G_{hh}\) also has to be less than 1.0 and the soil is stiffer in the horizontal direction (\(a\) is less than zero), and vice versa.

**TYPICAL FABRIC CONDITIONS OF SOILS**

The small-strain stiffness anisotropy reported by various researchers are summarised in Tables 1 and 2 for sands and clays, respectively. The proposed micromechanics model can be used to back-calculate the degree of anisotropy \(a\) from \(G_{sh}/G_{hh}\) (Eq. (6)) or \(E_s/E_h\) (Eq. (7)). The degrees of fabric anisotropy \(a\) shown in the tables are calculated from the average of the values obtained by assuming \(\chi = 0.5\) and 1.0 in the micromechanics model. The values of \(a\) at small strain are typically between \(-0.64\) and \(-0.62\) for sand and between less than \(-1.0\) and \(-0.53\) for clay, for both isotropic and \(K_0\) stress conditions. It is also worth noting that the data of clays indicate that they are consistently stiffer in the horizontal direction than in the vertical direction, and they tend to have stronger anisotropic characteristics than sands.

\[
\frac{G_{hh}}{G_{sh}} = \frac{(5-a)(105-46a-23a^2+70\chi-24a\chi+2a^2\chi)}{(5+a)^2(21-11a+14\chi-10a\chi)}\]

(Eq. (2d)/Eq. (2c))

(6)

\[
E_s = \frac{(5-3a)^2(14-2a+21\chi+9a\chi)}{(5+a)^2(14-6a+21\chi-15a\chi)}\]

(Eq. (2b)/Eq. (2a))

(7)

It is important to note that there is inconsistency in the experimental results of the small-strain stiffness anisotropy of sands obtained from static and dynamic measurements under the isotropic stress condition (see Table 1). All reported cases of the anisotropy of small-strain Young's modulus measured by small cyclic loading in triaxial tests give \(E_s/E_h < 1.0\), which suggests that the soils are stiffer in the vertical direction (e.g. Hoque and Tatsuoka, 1998; Kuwano, 1999). On the other hand, all reported cases of the anisotropy of small-strain shear modulus measured by wave propagation techniques give \(G_{sh}/G_{hh} > 1.0\), which suggests that the soils are stiffer in the horizontal direction (e.g. Lo Presti and O'Neill, 1991; Stokoe et al., 1991; Bellotti et al., 1996). This difference may be attributable to the specific assumptions employed by the researchers to derive their small-strain modulus. For instance, Hoque and Tatsuoka (1998) assumed that \(v_{sh}\) equals to \(v_{nh}\) in the isotropic stress state when the small-strain stiffness parameters are derived from small-cyclic triaxial tests. Kuwano (1999) assumed that the modulus from small-cyle triaxial tests and bender element tests are compatible and used a combination of them to calculate the small-strain Young's modulus. Lo Presti and O'Neill (1991), Stokoe et al. (1991), and Bellotti et al. (1996) derived their small-strain stiffness parameters based on wave propagation theory. This inconsistency has not yet been fully explained and further investigation is needed.

**INTERPRETATION OF ANISOTROPIC BEHAVIOUR IN TRIAXIAL TESTS BY MICROMECHANICS MODEL**

In recent years, the triaxial testing apparatus with a local small-strain measuring device has become a popular tool to investigate the small-strain behaviour of soils. Different local small-strain measuring devices are now available (e.g. Scholey et al., 1995; Yimsiri and Soga, 2002). The small-strain stiffnesses obtained from triaxial tests are often compared with the results from other laboratory tests (e.g. torsional shear, resonant column, and ultrasonic tests) and in-situ tests (e.g. pressuremeter, cross-hole, and down-hole techniques). When doing this comparison, extra consideration is necessary when the modes of shear deformation are different. The measured small-strain stiffness can be different due to the cross-anisotropic nature of the soil. In this section, the small-strain stiffness measured in triaxial tests is evaluated using the micromechanics model.

**Small-Strain Stiffness Derived from Triaxial Tests**

From the cross-anisotropic elastic model (Eq. (1)), the incremental vertical and horizontal stresses \((\delta \sigma_v, \text{ and } \delta \sigma_h, \text{ respectively})\) by the incremental vertical and horizontal strains \((\delta \sigma_v', \text{ and } \delta \sigma_h', \text{ respectively})\) measured in triaxial tests can be calculated as:

\[
\delta \sigma_v = \frac{\delta \sigma_v'}{E_v} = -2\frac{v_{sh}\delta \sigma_h'}{E_h}
\]

(8)

\[
\delta \sigma_h = \frac{\delta \sigma_h'}{E_h} + \frac{\delta \sigma_v'}{E_v}
\]

(9)

Substituting the cross-anisotropic elastic constants obtained from the micromechanics model (Eq. (2)) into Eqs. (8) and (9) yields;

\[
\delta \sigma_v = \frac{5V_3(3-a)}{14r^2N\chi(5+a)} \left[ \frac{(14-2a+21\chi+9a\chi)\delta \sigma_v'}{5} \right]
\]

(10)

\[
\delta \sigma_h = \frac{5V_3(3-a)}{28r^2N\chi(5-3a)} \left[ \frac{(7-a)(1-\chi)\delta \sigma_v'}{5} \right] + \frac{(7-a+28\chi-20a\chi)\delta \sigma_h'}{(5-3a)}
\]

(11)

Finally, the small-strain 'triaxial' shear modulus \(G_{sh}\), often calculated from triaxial test data, can be obtained as shown in Eq. (12). Note that this calculation inherently assumes isotropic elastic behaviour, which is often done in practice but not the case in reality.
<table>
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<th>Soil type</th>
<th>Reconstituting method</th>
<th>Test to determine small-strain stiffness</th>
<th>Stress condition</th>
<th>Small-strain stiffness anisotropy</th>
<th>Degree of fabric anisotropy $a$</th>
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<td>Dry medium to coarse Ticino sand</td>
<td>Dry pluviation through travelling sand spreader $D_r = 40%$ to $85%$</td>
<td>Wave propagation in calibration chamber</td>
<td>Isotropic at $p' = 30$–$600$ kPa</td>
<td>$M_y/M_\sigma = 1.2$</td>
<td>$G_{th}/G_{th} = 1.2$</td>
<td>$-0.44$</td>
</tr>
<tr>
<td>Dry medium dense, washed mortar sand</td>
<td>Dry pluviation through dispersing screen $\rho_m = 13.1$ kN/m$^3$</td>
<td>Wave propagation in calibration chamber</td>
<td>Isotropic at $p' = 85$ kPa</td>
<td></td>
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<tr>
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<td></td>
<td>Axis-symmetric at $\alpha_\sigma = 85$ kPa and $\sigma_\varepsilon = 50$ kPa</td>
<td></td>
<td>$E_y/E_C = 1.35$</td>
<td>$G_{th}/G_{th} = 1.22$</td>
<td>$-0.37$</td>
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<tr>
<td>Dry Ticino sand</td>
<td>Dry pluviation through travelling sand spreader $D_r = 45%$</td>
<td>Wave propagation in calibration chamber</td>
<td>Isotropic at $p' = 200$ kPa</td>
<td>$E_y/E_C = 1.21$</td>
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<td>$-0.23$</td>
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<tr>
<td>Dry Toyoura sand</td>
<td>Dry pluviation $D_r = 90%$</td>
<td>Triaxial test with very small amplitude vertical and horizontal cyclic loading and local strain measurement by LDTs</td>
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<td>$E_y/E_C = 0.90$</td>
<td>$E_y/E_C = 0.95$</td>
<td>$0.06$</td>
</tr>
<tr>
<td>Dry Ticino sand</td>
<td>Dry pluviation $D_r = 100%$</td>
<td>Wave propagation in calibration chamber</td>
<td>Isotropic at $p' = 98$ kPa</td>
<td>$E_y/E_C = 0.55$</td>
<td>$E_y/E_C = 0.60$</td>
<td>$0.52$</td>
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<tr>
<td>Dry Silver Leighton Buzzard sand</td>
<td>Vertical compaction using a vibrator $D_r = 90%$</td>
<td>Wave propagation in oedometer</td>
<td>$K_s (\alpha_\sigma = 0.45)$ at $\sigma_\varepsilon = 0$ to $300$ kPa</td>
<td>$G_{th}/G_{th} = 0.80$</td>
<td>$E_y/E_C = 0.60$</td>
<td>$0.51$</td>
</tr>
<tr>
<td>Dry Hime gravel</td>
<td>Dry pluviation $D_r = 90%$</td>
<td>Wave propagation in oedometer</td>
<td>$K_s (\alpha_\sigma = 0.45)$ at $\sigma_\varepsilon = 0$ to $300$ kPa</td>
<td>$G_{th}/G_{th} = 0.76$</td>
<td>$E_y/E_C = 0.60$</td>
<td>$0.52$</td>
</tr>
<tr>
<td>Dry Pool Filter sand</td>
<td>Dry pluviation $D_r = 70%$</td>
<td>Wave propagation in oedometer</td>
<td>$K_s (\alpha_\sigma = 0.45)$ at $\sigma_\varepsilon = 0$ to $300$ kPa</td>
<td>$G_{th}/G_{th} = 1.1$</td>
<td>$E_y/E_C = 0.60$</td>
<td>$0.51$</td>
</tr>
<tr>
<td>Dry Ottawa 20–30 sand</td>
<td>Dry pluviation $D_r = 70%$</td>
<td>Wave propagation in oedometer</td>
<td>$K_s (\alpha_\sigma = 0.45)$ at $\sigma_\varepsilon = 0$ to $300$ kPa</td>
<td>$G_{th}/G_{th} = 1.1$</td>
<td>$E_y/E_C = 0.60$</td>
<td>$0.52$</td>
</tr>
<tr>
<td>Saturated Ham River sand</td>
<td>Dry pluviation + tapping $D_r = 33%$</td>
<td>Bender element in triaxial apparatus</td>
<td>Isotropic at $p' = 400$ kPa</td>
<td>$G_{th}/G_{th} = 1.03$</td>
<td>$E_y/E_C = 0.60$</td>
<td>$0.52$</td>
</tr>
<tr>
<td>Saturated Dunkerque sand</td>
<td>Dry pluviation + tapping $D_r = 57%$</td>
<td>Bender element in triaxial apparatus</td>
<td>Isotropic at $p' = 400$ kPa</td>
<td>$G_{th}/G_{th} = 1.03$</td>
<td>$E_y/E_C = 0.60$</td>
<td>$0.52$</td>
</tr>
<tr>
<td>Saturated glass bead #10</td>
<td>Dry pluviation + tapping $e_0 = 0.705$</td>
<td>Bender element in triaxial apparatus</td>
<td>Isotropic at $p' = 400$ kPa</td>
<td>$G_{th}/G_{th} = 1.03$</td>
<td>$E_y/E_C = 0.60$</td>
<td>$0.52$</td>
</tr>
</tbody>
</table>

(Cont.)
where $\lambda = \delta \sigma_z / \delta \sigma_z$

The following additional variables describing the stress path direction are introduced for the following discussion.

$$\frac{\delta q}{\delta p'} = \frac{3(1 - \lambda)}{1 + 2\lambda} \theta = \tan^{-1} \left( \frac{\delta q}{\delta p'} \right)$$

The direction of the stress path measured counter-clockwise from $p'$ axis

Equation (12) shows that $G_x$ is related to the stress path direction ($\lambda$ or $\theta$) and to the micromechanics variables such as the soil fabric condition ($a$) and the contact condition ($\chi$).

**Conventional Drained Triaxial Tests**

When conventional drained triaxial compression and extension tests ($\lambda = 0$ or $\theta = 72^\circ$) are performed, the comparison between the measured drained small-strain triaxial shear modulus ($G_{xh}$) from Eq. (12) with $\lambda = 0$ and $G_{h,\text{micromech}}$ (Eq. (2c)) or $G_{h,\text{micromech}}$ (Eq. (2d)) becomes;

$$\begin{align*}
\frac{G_{xh}}{G_{h,\text{micromech}}} &= \frac{(5 - a)(105 - 46a - 23a^2 + 70\chi - 24a\chi + 2a^2\chi)}{(5 - 3a)(105 - 50a + 5a^2 + 70\chi - 20a\chi - 26a^2\chi)} \\
\frac{G_{xh}}{G_{h,\text{micromech}}} &= \frac{(5 + a)^2(21 - 11a + 14\chi - 10a\chi)}{(5 - 3a)(105 - 50a + 5a^2 + 70\chi - 20a\chi - 26a^2\chi)}
\end{align*}$$

where subscript $d$ denotes drained condition.

Figures 4 and 5 show the ratios of ($G_{xh}$) to $G_{h,\text{micromech}}$ and $G_{h,\text{micromech}}$ as a function of $a$ and $K_R/K_N$. The ranges of typical values of the degree of fabric anisotropy $a$ of sands and clays derived from the previous section are also included. When a soil is stiffer in the horizontal direction ($a < 0$), both $G_{xh}$ and $G_{h}$ will be underestimated in drained triaxial tests. On the other hand, $G_{xh}$ and $G_{h}$ will be overestimated when a soil is stiffer in the vertical direction ($a > 0$). The small-strain shear modulus derived from drained triaxial compression and extension tests is within ± 10% of $G_{h,\text{micromech}}$, when $-0.35 < a < 0.2$, and within ± 10% of $G_{h,\text{micromech}}$ when $-0.15 < a < 0.1$, for the cases of $K_R/K_N = 0.5$ and 1.0.
<table>
<thead>
<tr>
<th>Soil type</th>
<th>Soil condition</th>
<th>Test to determine small-strain stiffness</th>
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<th>Small-strain stiffness anisotropy</th>
<th>Degree of fabric anisotropy α</th>
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<tr>
<td>Panigaglia clay</td>
<td>Undisturbed</td>
<td>Bender element in Oedometer test</td>
<td>One dimensional consolidation</td>
<td>$G_{th}/G_{nh} = 1.4$ (NC)</td>
<td>$-0.83$</td>
<td>Jamiołkowski et al. (1995)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$G_{th}/G_{nh} = 1.5$ (OCR = 1.7 to 5)</td>
<td>Less than $-1.0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$G_{th}/G_{nh} = 2.0$ (OCR = 12 to 27)</td>
<td>Less than $-1.0$</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>$G_{th}/G_{nh} = 1.7$ (at K&lt;sub&gt;n&lt;/sub&gt; = 1.0)</td>
<td>Less than $-1.0$</td>
<td></td>
</tr>
<tr>
<td>Pisa clay</td>
<td>Undisturbed</td>
<td></td>
<td>One dimensional consolidation</td>
<td>$G_{th}/G_{nh} = 1.3$ (NC)</td>
<td>$-0.71$</td>
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<td></td>
<td>$G_{th}/G_{nh} = 1.4$ (OCR = 1.5 to 4)</td>
<td>$-0.83$</td>
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<td>$G_{th}/G_{nh} = 1.6$ (OCR = 8 to 16)</td>
<td>Less than $-1.0$</td>
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<td></td>
<td>$G_{th}/G_{nh} = 1.4$ (at K&lt;sub&gt;n&lt;/sub&gt; = 1.0)</td>
<td>$-0.83$</td>
<td></td>
</tr>
<tr>
<td>Gault clay</td>
<td>Reconstituted</td>
<td>Bender element in triaxial test</td>
<td>Isotropic at $p' = 50$–$500$ kPa</td>
<td>$G_{th}/G_{nh} = 1.5$ (NC and OC at OCR &lt; 10)</td>
<td>Less than $-1.0$</td>
<td>Pennington et al. (1997)</td>
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<tr>
<td></td>
<td>Undisturbed</td>
<td></td>
<td>Consolidated to in situ stress state</td>
<td>$G_{th}/G_{nh} = 2.0$</td>
<td>Less than $-1.0$</td>
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</tr>
<tr>
<td></td>
<td>OCR &gt; 30</td>
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<tr>
<td>Sperwhite kaolin clay</td>
<td>Reconstituted (NC)</td>
<td>Bender element in triaxial test</td>
<td>Isotropic at $p' = \sigma';$ during $K_0$ consolidation</td>
<td>$G_{th}/G_{nh} = 1.7$</td>
<td>Less than $-1.0$</td>
<td>Jovicic and Coop (1998)</td>
</tr>
<tr>
<td>London clay</td>
<td>Undisturbed</td>
<td>Bender element in triaxial test</td>
<td>Isotropic at $p' = \text{in situ } p'$</td>
<td>$G_{th}/G_{nh} = 1.5$</td>
<td>Less than $-1.0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(heavily OC)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Reconstituted</td>
<td>Bender element in triaxial test</td>
<td>Isotropic at $p' = \sigma'$ during $K_0$ consolidation</td>
<td>$G_{th}/G_{nh} = 1.24$</td>
<td>$-0.53$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(NC)</td>
<td></td>
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<tr>
<td></td>
<td>Reconstituted</td>
<td>Bender element in triaxial test</td>
<td>Isotropic at $p' = \sigma'$ of OCR = 3.75 during $K_0$ consolidation</td>
<td>$G_{th}/G_{nh} = 1.5$</td>
<td>Less than $-1.0$</td>
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</tr>
<tr>
<td></td>
<td>(OCR = 3.75)</td>
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<tr>
<td>Gault clay</td>
<td>Undisturbed</td>
<td>Bender element in triaxial test</td>
<td>Consolidated to in situ stress state</td>
<td>$G_{th}/G_{nh} = 2.25$</td>
<td>Less than $-1.0$</td>
<td>Ling et al. (2000)</td>
</tr>
<tr>
<td></td>
<td>(heavily OC)</td>
<td>Multiple drained triaxial stress path</td>
<td></td>
<td>$E_s/E_s = 4.0$</td>
<td>Less than $-1.0$</td>
<td></td>
</tr>
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</table>
Conventional Undrained Triaxial Tests

When a soil specimen is sheared in undrained condition, the condition of $\delta e_{vd} = \delta e_s + 2\delta e_b = 0$ gives the undrained stress path to be:

$$\lambda = \frac{\delta \sigma_i}{\delta \sigma_s} = \frac{(5-3a)(56a-8a^2-175\chi + 14\alpha\chi + 29a^2\chi)}{2(5+a)(28a-4a^2+175\chi - 98\alpha\chi - 17a^2\chi)}.$$

(15)

The undrained stress path directions calculated for various sets of $a$ and $\chi$ are shown in Fig. 6. When a soil is stiffer in the vertical direction ($a > 0$), the undrained stress path aligns to the right of the vertical in the $p' \cdot q$ diagram ($\theta < 90^\circ$). When a soil is stiffer in the horizontal direction ($a < 0$), the undrained stress path aligns to the left of the vertical in the $p' \cdot q$ diagram ($\theta > 90^\circ$).

Substituting Eq. (15) into Eq. (12), the undrained triaxial shear modulus ($G_{\text{ud}}$) where subscript ud denotes undrained condition) can be compared to $G_{\text{micro mech}}$ and $G_{\text{micro mech}}$ for a given set of $a$ and $\chi$, as shown in Figs. 7 and 8. The small-strain shear modulus derived from undrained triaxial compression and extension tests is within $\pm 10\%$ of $G_{\text{micro mech}}$, when $a < 0.4$, and within $\pm 10\%$ of $G_{\text{micro mech}}$, when $-0.2 < a < 0.15$.

Discussion

Based on the results shown in Figs. 4, 5, 7 and 8, Fig. 9 summarizes the typical conditions where the small-strain moduli from various tests match. The figure shows that the possibility for the small-strain shear moduli from conventional drained or undrained triaxial tests to coincide with $G_{\text{ud}}$ or $G_{\text{ud}}$ from shear tests is only when the degree of anisotropy is in a narrow region around the isotropic fabric state.

The agreement between ($G_{\text{ud}}$) and $G_{\text{ud}}$ is applicable for a wide range of $a$, which is the conditions for all clays and some sands (see Figs. 7 and 9). This may be the possible reason for the coincidence between the small-strain shear moduli from undrained triaxial tests and from wave propagation tests reported in some experimental works (e.g. Tatsuoka and Shibuya, 1991; Coop et al., 1997; Tatsuoka et al., 1997; Mukabi and Tatsuoka, 1999).

As indicated in Figs. 8 and 9, ($G_{\text{ud}}$) is smaller than $G_{\text{ud}}$ for the case of typical fabric conditions of clays. This implies that, for clay, the undrained small-strain shear modulus derived from pressuremeter test (which is $G_{\text{ud}}$) should be larger than the small-strain shear modulus obtained from undrained triaxial test when compared at the equivalent strain level. This hypothesis is substantiated by the experimental results on London Clay by Yimsiri
match the small-strain shear modulus in the shear modes (except for the case of $(G_{is})_{ud} = G_{vb})$. Using the micromechanics model, it is possible to investigate how the shear modulus from a triaxial test sheared in a specific stress path direction compares with the shear modulus estimated from different shear modes.

As shown in Eq. (12), the shear modulus $G_{is}$ measured from a stress-path-controlled triaxial test depends on the stress path direction applied ($\lambda$ or $\theta$). Herein, $G_{is}$ is compared with $G_{rh, micromech}$ and $G_{bh, micromech}$ (Eqs. (2c) and (2d)). Figures 11 and 12 show the ratios of $G_{is}/G_{rh, micromech}$ and $G_{is}/G_{bh, micromech}$ with different $\chi ( = K_b/K_s)$ and $a$ for various stress path directions. When a soil has isotropic soil fabric ($a=0$), it is evident that $G_{is}$ is independent of the stress path directions and $G_{is}$, $G_{vb}$ and $G_{bh}$ are the same. However, when the soil becomes anisotropic, the ratios depend on the stress path direction. There are some cases which result in negative shear modulus due to the definition applied in this analysis. The shear modulus is defined in terms of vertical and horizontal stresses and strains, not major and minor principal stresses and strains; therefore, the cases where the horizontal strain becomes the major principal strain will give negative values of shear modulus.

According to the results shown in Fig. 11, for a given set of $a$ and $\chi$, there is a stress path direction that gives $G_{is}/G_{rh, micromech}$ equal to one (for example $\theta=100^\circ$ for $a=-0.8$ and $\chi=1.0$). If a specimen is sheared along this stress path, $G_{is}$ is equal to $G_{vb, micromech}$. Similar observation can be made for $G_{is}$ and $G_{bh, micromech}$ in Fig. 12 (e.g. for $a=-0.8$ and $\chi=1.0$, $G_{is}/G_{bh, micromech}=1$ when $\theta=125^\circ$). In addition, for a given set of $a$ and $\chi$, there is one specific stress path direction that results in $\delta \varepsilon_s = \delta \varepsilon_h$, making $G_{is}$ indeterminable (e.g. $\theta=153^\circ$ for $a=-0.8$ and $\chi=1.0$).

The analytical results shown in Figs. 11 and 12 suggest that it is possible to estimate $G_{vb}$ and $G_{bh}$ from triaxial tests by shearing the soil at a specific stress path angle for a given set of $a$ and $\chi$. Figures 13 and 14 summarise the stress path angle $\theta$ that gives $G_{is}/G_{vb}$ or $G_{is}/G_{bh}$ equal to
unity for a given set of $a$ and $\chi$. For the representative values of $K_R/K_N=0.5$ and $1.0$ and typical values of the degree of fabric anisotropy $a$, $G_{is}$ and $G_{ih}$ are approximately the same when the stress path angle $\theta$ is between $95^\circ$–$105^\circ$, whereas $G_{is}$ and $G_{ih}$ are approximately the same when the stress path angle $\theta$ is between $120^\circ$–$130^\circ$.

The analytical results from the micromechanics model are compared with the published experimental data as shown in Fig. 15. The test conditions are given in Table 3. In Fig. 15, the ratios of $G_{is}/G_{ih}$ and $G_{is}/G_{ih}$ from the experimental data are plotted by taking $G_{is}$ and $G_{ih}$ from the triaxial results at $\theta=100^\circ$ and $125^\circ$, respectively (according to the discussion made earlier). The analytical results for the case of $a=-0.8$ and $K_R/K_N=1.0$ are also plotted in the figures. Although the procedure to match the data is quite subjective, the comparisons show some degree of agreement. The experimental results also suggest that the degrees of fabric anisotropy $a$ for the three clays are less than zero; the clays are stiffer in the horizontal direction, which is consistent with the summarised data discussed earlier.

Some discrepancy between the analytical and experimental data, and among the experimental data themselves, are inevitable due to plastic behaviour beyond elastic strain range as suggested by Wheeler and Houlby (1994) and Smith et al. (1994), as well as being due to the difference in the degree of fabric anisotropy of the individual clays tested and the effect of recent stress history. Further experimental data on small strain stiffness at different stress paths are needed to validate the model.

CONCLUSIONS

According to the published experimental data (see also
Table 3. Test conditions of constant stress path shearing by various researchers

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Test conditions</th>
<th>Strain ranges</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undisturbed Winnipeg clay</td>
<td>$K_T$ consolidation to in situ stress + constant stress path shearing</td>
<td>0.02-0.7% shear strain 0.1-1.8% volumetric strain</td>
<td>Graham and Houlbys (1983)</td>
</tr>
<tr>
<td>Undisturbed Bothkennar clay</td>
<td>Reconsolidation to in situ stress + drained probing test</td>
<td>0.01% shear strain 0.01% volumetric strain</td>
<td>Smith et al. (1992)</td>
</tr>
<tr>
<td>Undisturbed Pisa clay</td>
<td>Reconsolidation to in situ stress + constant stress path shearing</td>
<td>0.01-0.3% shear strain depending on stress path 0.01-0.3% volumetric strain depending on stress path</td>
<td>Callisto and Calabresi (1998)</td>
</tr>
</tbody>
</table>

Fig. 14. Variation of stress path angle for $G_{th}/G_{sh} = 1.0$ with $a$

Fig. 9), clays are generally more anisotropic than sands in terms of small-strain stiffness and fabric conditions. The micromechanics model (for the representative values of $K_R/K_N$ considered in this study) indicates that, if $E_h/E_v$ is less than 1.0, $G_{sh}/G_{th}$ also has to be less than 1.0, and this means that the soil is stiffer in the horizontal direction, and vice versa.

The analysis using the micromechanics model shows that, when a soil is stiffer in the horizontal direction, the shear modulus evaluated from the conventional triaxial drained tests underestimates $G_{sh}$ and $G_{th}$. The opposite is true when a soil is stiffer in the vertical direction. When a soil is sheared in undrained condition, the measured shear modulus is closer to $G_{sh}$ than $G_{th}$, especially when the soil is stiffer in the horizontal direction ($a < 0$).

The analytical results indicate that there is a possible theoretical reason for the agreement reported in the literatures between the undrained small-strain shear modulus from triaxial tests and $G_{sh}$ measured in shear tests or wave velocity measurements. The results also show that, for clays, the undrained small-strain shear modulus derived from pressuremeter test should be larger than the small-strain shear modulus from undrained triaxial test at the equivalent strain level.

When a soil is anisotropic, the shear modulus $G_{sh}$ measured in triaxial tests depends on the stress path direction applied. The computed results from the micromechanics model are consistent with the published experimental data and suggest that it is possible to determine $G_{sh}$ and $G_{th}$ from triaxial tests by shearing the soil at a specific stress path angle. For typical values of $a$ and $\chi$, $G_{sh}$ and $G_{th}$ are approximately the same when the applied stress path is between 95°-105°, whereas $G_{sh}$ and $G_{th}$ are approximately the same when the applied stress path is between 120°-130°.

ACKNOWLEDGEMENTS

The authors would like to thank Prof. Malcolm Bolton for valuable discussions and comments.
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