NEW EXTENSION OF DARCY'S LAW TO UNSTEADY FLOWS

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ABSTRACT

The Darcy's experimental law is valid in the case of linear laminar steady flows through saturated porous media. Although this law is still widely used, it is not valid in the case of laminar unsteady or cyclic flows due, for instance, to the action of short waves within soils and earth structures, which can affect their behaviour and durability. The present paper presents a new extension of Darcy's law particularly suitable to these unsteady flows. The model is first derived from the Navier-Stokes equations combined with a capillary-type model. For validation of the model, an experimental device simulating flows due to short waves within soils was developed. 95 unsteady tests conducted on five different sands have confirmed the validity of the law identified. The results of the tests show the effects of acceleration, which generates an increase of the interstitial pressure within the soils.

Key words: cyclic flow, Darcy's law, earth structures, fluvial banks, levees, short waves, soil, unsteady flow (IGC: D4/E7/H7)

INTRODUCTION

Rapid and repeated variations of internal pressure due to waves can cause variations of stresses, local ruptures, and even instability in river banks, levees or other earth structures. It is well known that short waves induced by ship motions in some areas (for example in the Mississippi River, USA) cause local levee breaking. Although it is still widely used in internal flow modelling, the Darcy's law is not valid in the case of unsteady laminar flows, induced, for instance, by these short waves within soil and earth structures. Such structures play an important role, since they protect high areas against flooding. It is, therefore, essential to improve our basic knowledge of the modelling of their hydraulic behaviour.

The present paper reports a study conducted to develop a model, which, by considering the extension of the classical Darcy's law to unsteady laminar flows, makes it possible to model flows caused by short waves within granular soils. We will first present the theoretical principles of the model before addressing the problem of its experimental validation and of the characterisation of its parameters.

BACKGROUND

Darcy's law (1856) is undoubtedly the most widely used law governing flows within soils and earth structures. It is traditionally expressed as:

$$v = ki \quad (1)$$

with a possible alternative expression:

$$i = av \quad (2)$$

where

i = hydraulic gradient,

v = flow velocity, ms^{-1} ,

k = permeability of the medium, ms^{-1}

$a = 1/k$ = experimental coefficient of the Darcy's law (viscous term), sm^{-1} .

Generally, v (macroscopic velocity) is a vector in 3-D. In our particular case, v is related to experimental Forchheimer law. In this procedure, experimental tests are made in 1-D, and then only the 1-D velocity vector is taken into account, and v is considered as a scalar.

Darcy's law is a one-parameter (a) law describing hydraulic frictions between fluids and particles. This experimental law is valid within the well-known conditions on which it is based, i.e., linear laminar steady flow through saturated porous media.

Understanding of the extension of the law to non-linear laminar or turbulent steady flows studied by Forchheimer (1901) has greatly improved thanks to the experimental validations of many researchers (Ergun, 1952; Ahmed et al., 1969; Hannoura, 1978; Martin, 1990; Hansen, 1992; Wahyudi, 1998; Bingjun et al., 1998). This extension takes the form of Forchheimer law:

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Written discussions on this paper should be submitted before July 1, 2003 to the Japanese Geotechnical Society, Sugayama Bldg. 4F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101-0063, Japan. Upon request the closing date may be extended one month.

$$i = av + bv^2 \quad (3)$$

where b is an experimental coefficient of the inertial term, s^2m^{-2} .

Forchheimer law is a two-parameter (a and b) law, in which the additional term, b , describes the inertial effects. As the Reynolds number increases, microscopic inertial forces contribute to the change of stream lines (Comiti et al., 2000). In the geotechnical field, recent studies have made it possible to identify the range of validity of Darcy's law, beyond which Forchheimer's non-linear law should be used (Babendrier, 1991; Poux, 1997; Khalifa et al., 2000a). In the literature, it is generally agreed that the breakdown of Darcy's law initiates at an interstitial Reynolds number of particle ranging between 1 and 10. These studies have been conducted to examine the problems of scaling laws in centrifuged geotechnical models, where very large hydraulic gradients are often found.

On the other hand, an extension of Forchheimer's law to unsteady flows within porous media has been proposed by Palubarinova-Kochina (1962). The model equation, which includes an additional term related to the acceleration, is written:

$$i = av + bv^2 + c \frac{\partial v}{\partial t} \quad (4)$$

where

$\partial v / \partial t$ = acceleration of the fluid in the porous medium, ms^{-2} .

c = experimental coefficient of the acceleration term, m^{-1}s^2 .

Several studies have been conducted on coarse materials (gravel and rocks) in order to determine the three parameters (a , b and c). These studies were applied to the modelling of internal flow in coastal structures such as breakwaters (coarse soil and blocs), where viscosity term (av) can be considered negligible in comparison with the inertial (bv^2) and acceleration ($c(\partial v / \partial t)$) terms (Hannoura, 1978; McCorquodale et al., 1985; Burchart et al., 1991; Van Gent, 1993; Andersen, 1994; Wahyudi, 1998).

Although Darcy's law is still widely used, its validity remains uncertain in the case of laminar unsteady, or cyclic flows due, for instance, to the action of short waves within soils and earth structures. Special experimental devices (triaxial devices) are used to simulate the behaviour of poroelastic media subjected to cyclic stresses and measure the global pressure variation (Zen et al., 1998; Hyodo et al., 1999). The hydraulic and mechanical problems are therefore coupled. In this study, simplification is achieved by disconnecting both problems and carrying out the test on a adequately dense granular skeleton (with a given porosity) so that with regards to hydraulic stresses applied during the tests, the deformation of both the medium and the cell can be neglected. Therefore, only water pressure variation is studied as in studies by this same author conducted for Eq. (4). Using these studies to

understand the influence of short waves in earth structures (levees, river banks), where the inertial effects in fine soils are negligible ($bv^2 = 0$), we can obtain a reduction of Eq. (4) expressed as:

$$i = av + c \frac{\partial v}{\partial t} \quad (5)$$

Equation (5) presents an additional acceleration term to the classical term of Darcy's law. Therefore, the justification of this hypothesis gives a new extension of Darcy's law to unsteady flows.

We first present the theoretical justification of this law before addressing its experimental validation.

NEW RELATIONSHIP MODELLING

Flow in porous media can be described by the Navier-Stokes equation. In the three-dimensional case, it can be written (Padet, 1991; Andersen, 1994):

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i}{\partial x_j} u_j = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + F_i + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} \quad (6)$$

Where

ρ = density of the fluid, Kg m^{-3} ,

ν = kinematic viscosity of the fluid, m^2s^{-1} ,

u_i, u_j = interstitial velocity, ms^{-1} ,

p = pressure (in Pa or in Bar, 1 Bar = 10^5 Pa),

t = time, s,

x_i, x_j : coordinates, m.

In this study, we assume that fluid density ρ is constant. Equation (6) describes flow of any Newtonian fluid in any domain. The convective acceleration can be written as:

$$\frac{\partial}{\partial x_j} (u_i u_j) = \frac{\partial u_i}{\partial x_j} u_j + \frac{\partial u_j}{\partial x_j} u_i \quad (7)$$

As continuity equation gives $\partial u_j / \partial x_j = 0$, Eq. (6) becomes:

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_i u_j) = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + F_i + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} \quad (8)$$

The acceleration term has been investigated by several authors (Burchart, 1991; Andersen, 1994; Hannoura et al., 1978; Madsen, 1974; Sollit, 1972). Specific studies have been conducted by these authors in order to model this term. In addition to the fluid element, they added a virtual mass related to the volume of solids, which was then, according to the authors, distributed over the volume of water and considered as an extra force acting on the fluid element. They consider the resistance force due to the virtual mass to be equal to the product of the displaced fluid mass, the virtual mass coefficient, and the acceleration in the approach velocity.

Total volume forces F_i are due to gravitational acceleration acting on total volume and inertial forces due to fluid acceleration acting on solid volume (Burchart, 1991; Andersen, 1994). Then we can write:

$$-F_i = g + c_m \left(\frac{1-n}{n} \right) \frac{\partial u_i}{\partial t} \quad (9)$$

Where F_i are the volume forces, c_m is a virtual mass coefficient and n is the porosity of the medium.

Inserting Eq. (9) into Eq. (7) and writing $p_0 = p^+ \rho g h_0$, we obtain:

$$-\frac{1}{\rho} \frac{\partial p_0}{\partial x_i} = -\nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{\partial}{\partial x_j} (u_i u_j) + \left[1 + c_m \left(\frac{1-n}{n} \right) \right] \frac{\partial u_i}{\partial t}. \quad (10)$$

Where $\rho g h_0$ is the pressure induced by the particle level (the position is h_0 , considering an arbitrary datum). Introducing the expression of hydraulic gradient $i = -(1/\rho g)(\partial p_0/\partial x_i)$, we can write:

$$i = -\frac{\nu}{g} \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{1}{g} \frac{\partial}{\partial x_j} (u_i u_j) + \frac{1}{g} \left[1 + c_m \left(\frac{1-n}{n} \right) \right] \frac{\partial u_i}{\partial t}. \quad (11)$$

The capillary representation considers a porous medium as a bundle of straight identical cylindrical pores characterized by a length L and a diameter d_p . Total volume of pores is LS_p , where S_p is total pores section. We can write porosity as: $n = \text{void volume}/\text{total volume} = (LS_p)/(LS) = S_p/S$, where S is the total material section. If Q is the volumetric flow rate, we can write fluid velocity in pore sections as: $u = Q/S_p$. This velocity is commonly called microscopic velocity, interstitial velocity or Dupuit velocity. The macroscopic velocity ν , called Darcy velocity can be written as: $\nu = Q/S$.

The Dupuit assumption which gives the relationship between microscopic velocity and macroscopic velocity assumes the total flow rate is constant considering both formulations of velocity. Then $Q = \nu \cdot S = u \cdot S_p$, and then $\nu = u \cdot (S_p/S)$, giving $\nu = u \cdot n$.

In major porous media studies in Civil Engineering and Chemical Engineering (Dullien, 1975a; Dolejs, 1978; Agarwal et al., 1988) the Dupuit assumption ($u = \nu/n$) is very used as a relationship between macroscopic velocity, microscopic velocity and porosity.

By replacing microscopic velocity (u) by macroscopic velocity (ν), where $\nu = u \cdot n$, we obtain:

$$i = -\frac{\nu}{gn} \frac{\partial^2 v_i}{\partial x_j \partial x_j} + \frac{1}{gn^2} \frac{\partial}{\partial x_j} (\nu_i \nu_j) + \frac{1}{gn} \left[1 + c_m \left(\frac{1-n}{n} \right) \right] \frac{\partial \nu_i}{\partial t}. \quad (12)$$

Introducing V and d as characteristic velocity and length, respectively, and averaging over a sample, we can write Eq. (12) in the dimensionally form:

$$i = \alpha \frac{\nu}{gnd^2} V + \beta \frac{1}{gn^2 d} V^2 + \frac{1}{gn} \left[1 + c_m \left(\frac{1-n}{n} \right) \right] \frac{\partial V}{\partial t}. \quad (13)$$

Where d is characterising the material geometry (particle diameter). Empirical coefficients α and β are often denoted shape factors, because researchers generally explain α and β as dependent on the geometry of the granular medium.

Within the limit of validity of Darcy's law, the quadratic velocity (V^2) is negligible. Then the second term on the right-hand of Eq. (13) vanishes. Thus, Eq. (13) leads to:

$$i = \alpha \frac{\nu}{gnd^2} V + \frac{1}{gn} \left[1 + c_m \left(\frac{1-n}{n} \right) \right] \frac{\partial V}{\partial t}. \quad (14)$$

By replacing $d_p = (n/1-n)d$ (Dullien, 1975b), where d is the diameter of particle and d_p the pore diameter, we obtain:

$$i = \alpha \frac{\nu}{gd_p^2} \frac{(1-n)^2}{n^3} V + \frac{1}{ng} \left[1 + c_m \frac{(1-n)}{n} \right] \frac{\partial V}{\partial t}. \quad (15)$$

The model expressed here by Eq. (15) is identical to Eq. (5) and represents the extension of Darcy's law to an unsteady flow. Indeed final Eq. (15) can be written in the form:

$$i = a\nu + c \frac{\partial \nu}{\partial t} \quad (16)$$

where

$$a = \alpha \frac{\nu}{gd_p^2} \frac{(1-n)^2}{n^3} \quad (17a)$$

$$c = \frac{1}{ng} \left[1 + c_m \frac{(1-n)}{n} \right]. \quad (17b)$$

Equation (16) proposes an original relationship suitable for unsteady flows. Validation of this theoretical equation requires experimental results. In the following section of this paper, we will describe the experimental set up designed in order to test this equation.

EXPERIMENTAL STUDY

Description of the Experimental Design

In order to validate this model, we conceived an experimental device which can characterise flow parameters in both steady and unsteady flows from the same materials. A detailed description of the device is given by Khalifa et al. (2000b). The schematic diagram of the experimental device is shown in Fig. 1(a), and the photograph in Fig. 1(b).

In this device, the flow is forced through the specimen by a centrifugal variable speed pump, the characteristics of which have been specially selected to develop hydraulic gradients larger than 800 in fine sands. Water supply is provided by a tank and after use, the water is returned to the tank (Fig. 1(a)). The experimental device is equipped with a control valve, the purpose of which is to modulate an unidirectional flow with a sinusoidal variation set by the user. The sinusoidal signal order required by the user is sent by the computer to a controller, which then scans flowmeter measurements and sends back the automatic control order to the servovalve (partial opening or closing).

In the present study, we adopted a signal order for a sinusoidal flow rate simulating monochromatic waves limited by an upper and a lower bound in flowrate and a period. The ranges of the set flowrates vary according to the nature of the materials. The imposed wave periods range from 5 to 15 seconds.

The measurement bench, which consists of tempera-

ture sensors, differential pressure gauges and flowmeters, is connected to a microcomputer and a data logger used to provide full automation of the tests.

Studied Materials

The application of the new extension of Darcy’s law is carried out with five different sands, the grain-size distri-

bution of which is fairly representative of a wide range of natural materials. In order to make future comparisons with our results easier, we also tested an artificial material (uniform, 0.4 mm diameter polystyrene spheres). Table 1 presents the characteristics of the tested sands and spheres.

Steady Flow Tests and Darcy Limit

Firstly, a full series of steady flow tests is conducted to characterise the term of Darcy’s and Forchheimer’s laws (coefficients *a* and *b*), on the one hand, and to determine the limit of validity of Darcy’s law, on the other hand. All cyclic flow tests carried out later stand within this range of validity.

In order to determine the limit of validity of Darcy’s law, we have used a capillary-type modelling of the piezometric pressure drop through consolidated porous media consisting of anisotropic particles, developed by Comiti and Renaud (1989). This model is based on the representation of the porous medium by a set of identical cylindrical tortuous pores with a diameter of pore, *d_p*:

$$d_p = \frac{4n}{(1 - n)A_{vd}}$$

(18)

Fig. 1(a). Schematic diagram of the experimental device

Fig. 1(b). Photograph of the experimental device

Table 1. Characteristics of the materials tested

Sand	0.4 mm spheres	Fontainebleau	Labenne	Hostun	Le Rheu	Loire
Mean diameter (mm)	0.400	0.23	0.31	0.42	0.53	0.82
d_{85}/d_{10}	1.00	2.67	2.75	2.94	4.34	3.56
Porosity n	0.429	0.400 0.384 0.382	0.383	0.425 0.389 0.378	0.412	0.350

Fig. 2. Friction factor versus Reynolds number

where n is the porosity and A_{vd} is the dynamic specific surface area defined as the ratio of the total pore surface area, exposed to fluid flow, to the volume of solid.

The mean velocity in pores is:

$$u = \frac{\nu\tau}{n} \quad (19)$$

where τ is the tortuosity defined as the ratio of the mean length of the flow path to the crossed thickness of the porous medium.

According this model, the Forchheimer-type equation of the model becomes:

$$i = av^2 + bv \quad (20)$$

with

$$a = 0.0968\tau^3 A_{vd} \frac{(1-n)}{gn^3} \quad (21)$$

$$b = 2\mu\tau^2 A_{vd}^2 \frac{(1-n)^2}{\rho gn^3} \quad (22)$$

A dimensionless equation of this model is (Comiti et al., 2000):

$$f_p = \frac{16}{Re_p} + 0.194 \quad (23)$$

where:

$$f_p = \frac{\Delta P}{h} \frac{2n^3}{\tau^3 \rho A_{vd} (1-n) v^2} \quad (24)$$

Table 2. Tests summary

	For 0.4 mm spheres	For 5 sands
Periods (seconds)	5; 8; 10; 12; 15	5; 8; 10; 12; 15
Lower-higher flowrates (liter/hour)	300–400	300–400
For each period	250–450	250–450
	200–500	200–500
	150–550	—

$$Re_p = \frac{4\pi r v}{\mu(1-n)A_{vd}} \quad (25)$$

Where Δp is the piezometric pressure drop and μ is the dynamic fluid viscosity. This model is applied to all tests carried out in steady-state flow on five different sands. According to the dimensionless equation of the model (Eq. (23)) presented above, the data are plotted in Fig. 2 as f_p versus Re_p . The relevance of this kind of representation is to gather experimental data with a single equation whatever the porous medium. Therefore, a single value of the pore Reynolds number can determine the limit of Darcy's regime, considering a chosen error. For instance, if a tolerance of 5% for the deviation of Darcy's law is accepted, then the corresponding critical pore Reynolds number is: $Re_p^{crit} = 4.3$.

Unsteady Flow Tests

The simulation of short waves is achieved, under critical Reynolds number ($Re_p < 4.3$), through a series of tests with wave periods ranging from 5 to 15 seconds. Depending on the properties of the material, both the mean flowrate passing through the sample and its amplitude vary so as to cover a wide range in accelerations. 95 unsteady flow tests are thus carried out on the six different materials. Table 2 summarises the various combinations carried out.

Considering the number of tests and the variety of the curves achieved during the analysis phase, we restrict the detailed presentation of the results at two tests carried out on a natural material (Fontainebleau sand) and an artificial material (0.4 mm polystyrene spheres), respectively. However, major results are given for all the materials tested.

Figure 3 gives a raw example of these tests. The interpretation of the curves reveals that both the flowrate and

Fig. 3. Comparison between measured hydraulic gradients and calculated hydraulic gradients

Fig. 4. Fluctuating components of the velocity and hydraulic gradients

pressure drop time series are sinusoidal and confirms the ability of the experimental device to simulate short waves.

Figure 3 shows high deviation between unsteady (i_t , measured) and steady (i_p , calculated with Darcy's term) hydraulic gradients. Measured hydraulic gradients are different in the Fontainebleau sand and in spheres 0.4 mm. It is due to roughness and particle size differences. Hydraulic gradients are generally higher in the sand (0.21 mm mean diameter with angulous particles). This deviation represents the effect of acceleration. It reveals that the consideration of Darcy's classical law for unsteady flows tends to increase hydraulic gradients or pressure drops in the materials and, consequently, to reduce the actual pressures. This observation does not strive for safety since interstitial pressures have an important part in the stability of earth structures. As an

engineering matter the effect of acceleration on the deviation from Darcy's law is analysed below. Deviation is then given versus acceleration. Maximum deviation ratio depends on the the problem and accepted error.

Experimental tests conducted with various small granular materials (0.21 to 0.82 mm mean diameters) and different cells (5 to 10 cm diameters and 10 to 25 cm lengths) designed so that cell dimensions are very high regarding to particle sizes, show that transient hydraulic gradients are not affected by specimen sizes.

ANALYSIS AND EXPERIMENTAL VALIDATION

Validity of Darcy's Law

Experimental data presented in Fig. 3 are first smoothed by sinusoidal functions. Figure 3 also illustrates smoothed velocity and hydraulic gradient (i_p and i_t)

Fig. 5. Relationship between hydraulic gradient and velocity for tested materials

for both the Fontainebleau sand and the 0.4 mm spheres. Since the flow is periodic and independent from the mean velocity, we keep only the fluctuating part of the velocity and of the hydraulic gradients ($i_t - i_p$). Figure 4 presents

the fluctuating components of the velocity and hydraulic gradients ($i_t - i_p$) for the Fontainebleau sand and the 0.4 mm spheres. It shows that hydraulic gradient is in phase with velocity.

Fig. 6. Evolution of the hydraulic gradient ($i_t - i_p$) and of the acceleration (dv/dt) versus time (Fontainebleau sand and 0.4 mm diameter spheres)

The equations of the hydraulic gradients and velocities for the cyclic flow are then written in the form:

$$i_t = \frac{\hat{i}}{2} \sin(\omega t) \quad (26)$$

$$v = \frac{\hat{u}}{2} \sin(\omega t) \quad \text{and} \quad i_p = av \quad (27)$$

where

i_t = smoothed experimental hydraulic gradient,
 i_p = calculated hydraulic gradient from Darcy's law,
 $\hat{i}$ = amplitude of the hydraulic gradient,
 $\hat{u}$ = amplitude of the velocity, ms^{-1} ,
 $\omega = 2\pi t/T$,
 t = time, s,
 T = wave period, s.

Figures 5a), 5b), 5c), 5d), 5e) and 5f) illustrate the variation of the hydraulic gradient ($i_t - i_p$) according to the velocity for all six materials tested. Each straight line corresponds to a test. The bold lines represent the application of Darcy's classical law (steady flow).

These results show that the correlation remains linear for a given acceleration, whereas the slope of the straight line (coefficient a) varies, for the same material, according to acceleration (Fig. 6). Furthermore, whatever the material, the more the acceleration increases, the more the experimental line diverges from Darcy's line. This further demonstrates that Darcy's law is not valid for these unsteady flows.

New Extension of Darcy's Law

We have developed an original method for the treatment of the above data. This method allows keeping constant the steady flow parameter (a) and gives the unsteady flow parameter (c) in a simple way. As mentioned above, Figs. 3 and 4 show that the hydraulic gradients are in phase with velocity. However, as presented in Fig. 6, the velocity and hydraulic gradient ($i_t - i_p$) present a systematic phase shift of $\theta = \pi/2$ with the acceleration which is given by:

$$\frac{dv}{dt} = \frac{\hat{u}\omega}{2} \cos(\omega t). \quad (28)$$

The correlation between hydraulic gradient ($i_t - i_p$) and acceleration can be obtained into a more convenient form by using the function (dv_θ/dt), which represents a phase shift of $\pi/2$ with acceleration (dv/dt). This acceleration is written:

$$\frac{dv_\theta}{dt} = \frac{\hat{u}\omega}{2} \cos\left(\omega t + \frac{\pi}{2}\right). \quad (29)$$

The acceleration (dv_θ/dt) is then in phase with the hydraulic gradient ($i_t - i_p$). In Fig. 7, this hydraulic gradient is plotted versus acceleration for Fontainebleau sand and 0.4 mm spheres.

In this graph, the Darcy's law is represented by $i_t - i_p = 0$ and $dv_\theta/dt = 0$. The linear correlation deduced from Fig. 7 is given by:

$$i_t - i_p = c \left(\frac{dv_\theta}{dt} \right). \quad (30)$$

We can notice in Fig. 8 that for the same material, the values of " c " vary according to $(\hat{u}/T)$. We also note that the coefficient " c " increases when $(\hat{u}/T)$ decreases. The studying of the 95 flow tests carried out, as presented in Fig. 8, gives the relationship between coefficient " c " and $(\hat{u}/T)$:

$$c = p \left(\frac{\hat{u}}{T} \right)^q \quad (31)$$

where p and q are the experimental coefficients characterising the acceleration term. We note that the smaller the diameter of the material, the stronger the effect of the acceleration (increase of c).

Coefficients p and q obtained from the curves (Fig. 8) constitute the parameters of the term related to the acceleration in the law governing cyclic flows for the materials studied. Coefficients p and q are presented in Table 3 for studied materials. The correlation between the

Table 3. Parameters of new extension of Darcy's law (five sands and 0.4 mm spheres)

	0.4 mm spheres	Fontainebleau	Labenne	Hostun	Le Rheu	Loire
a (s/m)	2865	4681	2578	2029	1716	546
p	164	1427	175	265	480	24
q	-0.20	-0.11	-0.39	-0.23	-0.16	-0.50

Fig. 7. Correlation between the hydraulic gradient ($i_t - i_p$) and the acceleration (dv_θ/dt) (Fontainebleau sand and 0.4 mm spheres)

hydraulic gradient and the new expression of the acceleration becomes:

$$i_t - i_p = p \left(\frac{\hat{u}}{T} \right)^q \frac{dv_\theta}{dt} \quad (32)$$

From Eq. (32), we can deduce:

$$i_t = av + p \left(\frac{\hat{u}}{T} \right)^q \frac{dv_\theta}{dt} \quad (33)$$

Equation (33) is similar to Eq. (5) and represents the extension of Darcy's law to unsteady flow. The values of the

Fig. 8. Relationship between coefficient "c" and acceleration ($\hat{a}/T$) (5 types of sands and 0.4 mm diameter spheres)

different parameters of this equation are plotted in Table 3 below, for all materials tested.

Figure 9 shows a good agreement between the model (Eq. (33)) and the experimental values, for the two materials (Fontainebleau sand and 0.4 mm spheres). This comparison is made with a new set of tests.

DISCUSSION

The relevance of this study's contribution to geotechnics can be proved with a few examples. Indeed, we previously noticed that acceleration of the fluid generates a decrease of pressure drops and, therefore, an increase of the interstitial pressure. Since total stress within the material is constant, the increase of interstitial pressure generates reductions in the effective stress compared with steady flows. This effective stress reduction might induce mechanical instabilities in earth structures submitted to short waves.

Furthermore, interstitial pressure excess due to fluid acceleration can modify piping and sand boiling modeling due to ascendant hydraulic gradients. Indeed, in addition to the force generated by an ascending steady flow, there is an added force related to acceleration, which can induce sand boiling under hydraulic gradients lower than classical critical hydraulic gradients.

On the other hand, in the first part of this paper, we discussed the range of validity of Darcy's law for steady

Fig. 9. Comparison between measured and calculated hydraulic gradients (Fontainebleau sand and 0.4 mm spheres)

flows. For unsteady flows, we are now able to set a new limit of validity of Darcy's law according to acceleration, based on the deviation between hydraulic gradients (i_t and i_p). Figure 10 illustrates, for all sands studied, approximations of the hydraulic gradients according to acceleration, and shows that Darcy's law includes errors of 10% and 25% for average accelerations on the order of $2 \times 10^{-4} \text{ ms}^{-2}$ and $8 \times 10^{-4} \text{ ms}^{-2}$, respectively.

It is therefore advisable, when flow is unsteady, to check systematically that the acceleration does not exceed the admissible limit of validity of Darcy's law. If this limit is exceeded, then the new form of Darcy's law proposed above (Eq. (33)) should be applied.

CONCLUSION

In this paper we have shown that there are two limits for the validity of Darcy's experimental law.

The first breakdown of this law occurs in steady flow if the critical Reynolds number is reached. Precise determination of this limit is essential to studying the scaling law

Fig. 10. Deviation from Darcy's law due to acceleration

of flow parameters in centrifuged geotechnical models, within which very large hydraulic gradients are often found. For five different sands studied, we find that a single value of the pore Reynolds number characterises the end limit of Darcy's law. For a deviation equal to 5% from this law, the critical Reynolds number equals 4.3.

We demonstrate that the second breakdown of Darcy's law occurs if the critical acceleration of unsteady flows is reached. Deviations of 10% and 25% from Darcy's law are found in sands for average flow accelerations on the order of $2 \times 10^{-4} \text{ ms}^{-2}$ and $8 \times 10^{-4} \text{ ms}^{-2}$, respectively. Such unsteady flows can be generated by short waves due to wind or ship motions in river banks, levees or other earth structures.

The extension of Darcy's law presented in this paper particularly suits such unsteady flows. Theoretically, this law is first derived from the Navier-Stokes equations combined with a capillary-type model. This law consists of a viscosity term (identical to the classical Darcy's term) representing frictions between the fluid and the particles and of an additional term related to the acceleration of the fluid in the soil. The coefficient of the first term of the law is also identical to Darcy's law and depends on the properties of both the fluid and the soil. The second term depends on the characteristics of the flow (acceleration).

For the experimental validation of the new extension of Darcy's law, we have developed an experimental device used to simulate flows due to the periodical motion of short waves within natural and artificial materials in perfectly controlled conditions. An original method has been perfected for the test treatments. A first series of tests simulating steady flows made it possible to determine the range of validity Darcy's law. The second series of 95 flow tests, carried out on five different sands and on uniform spheres and conducted under critical Reynolds number ($Re_p < 4.3$), confirmed the validity of the law theoretically identified, and was used to characterise the coefficients.

It reveals the effects of the acceleration, which generate a decrease of the hydraulic gradient and an increase of the interstitial pressure. This result confirms the importance of the consideration of acceleration in earth structure modelling.

ACKNOWLEDGMENTS

This study has been partly conducted in collaboration with the Laboratoire Central des Ponts et Chaussées (LCPC) within the context of a joint research program. Funds were provided by grants from the French Department of Research and Technology and the European Regional Development Fund (FEDER). The assistance and cooperation of Dr. Garnier (LCPC), Pr. Comiti and Dr. Montillet (LGP) are sincerely appreciated. Collaboration of M. Bouchelaghem and M. Oger is appreciated. Special thanks to Mr. Coué for his involvement in the production of the experimental devices.

NOTATION

- a, b, c : Experimental parameters of the laws
 c_m : Virtual mass coefficient
 d : Grain diameter, m
 d_p : Pore diameter, m
 Fi : Volume forces, N
 dv/dt : Flow acceleration, ms^{-2}
 dv_θ/dt : Flow acceleration function shifted by $\theta = \pi/2$ from dv/dt , ms^{-2}
 g : Gravity acceleration, ms^{-2}
 i : Hydraulic gradient
 k : Permeability of the medium, ms^{-1}
 L : Cell length, m
 n : Porosity of the medium
 P : Pressure, Nm^{-2}
 p, q : Experimental coefficients of law
 Q : Flow rate, m^3s^{-1}
 S : Test cell section, m^2
 S_p : Total pore sections, m^2
 t : Time, s
 T : Flow period, s
 u : Interstitial velocity, ms^{-1}
 $\hat{u}$: Flow velocity amplitude, ms^{-1}
 v : Flow velocity, ms^{-1}
 α, β : Empirical coefficient
 ν : Kinematic viscosity, m^2s^{-1}

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