SEISMIC SOIL-STRUCTURE INTERACTION OF CROSS SECTIONS OF FLEXIBLE UNDERGROUND STRUCTURES SUBJECTED TO SOIL LIQUEFACTION

YUKIO TAMARI and IKOU TOWHATA

ABSTRACT

The soil-structure interaction of underground structure was studied considering soil liquefaction of the backfill soil. A series of 1G shaking table tests was conducted on an aluminum fixed base structure model embedded in saturated cohesionless soil. A special attempt was made to experimentally build a stress-strain relationship as well as an effective stress path of soil around the embedded structure by installing a variety of transducers for acceleration, displacement, pore pressure and earth pressure. Moreover, earth pressure and bending strain of the flexible wall of the embedded structure model were measured. The time histories of shear modulus of liquefied soil, amplification of shaking, and the phase difference between displacement and earth pressures on the structure’s side face were recorded. It was observed that the natural period of the ground along with structure and the dilative nature of the soil significantly affect the type of soil-structure interaction around the studied embedded structure. The problem of the determination of dynamic pressures on a flexible wall during liquefaction is solved analytically considering the dynamic feature of embedded structure observed in a series of shaking table tests. Finally, a comparison is made between measured and calculated dynamic pressure for several test results in order to show that the proposed solution of dynamic pressure on flexible wall gives good results.

Key words: dynamic interaction, earth pressure, liquefaction, model test, stress-strain curve, underground structure (IGC: F7/H8)

INTRODUCTION

The importance of seismic design is being increasingly recognized in such large public underground structures as ducts for utility pipelines or conveyances of sewage, subway stations and embedded tunnels. Furthermore, the nonlinear behavior of soils as well as structures now must be considered in seismic design procedures (JSCE, 2000). The seismic soil-structure interaction associated with the large underground structures is one of the important research topics in the field of structural dynamics and geotechnical earthquake engineering. In the past few decades, considerable research has been conducted to study seismic soil-structure interaction effects for box-shaped cross sections of large underground structures (Watanabe et al., 1991a; Kawashima, 1994).

To facilitate the practical seismic design of buried concrete box sections of underground structures, the dynamic earth pressures acting on a box section during earthquakes have been examined by experimental and theoretical study (Watanabe et al., 1991b, 1991c). Practical simplified methods in a quasi-static manner capable of seismic design calculations considering soil-structure interaction were proposed based on the seismic deformation method using a frame-spring model (Tateishi, 1992), and the finite element method (Penzioni et al., 1992; Tateishi, 1995). The numerical method using soil-structure model with non-linearity of both reinforced concrete (RC) structural members and soil by the equivalent linear method were verified by simulating a damaged RC underground subway structure at 1995 Hyogoken-Nanbu earthquake (Matsuda et al., 1997). The effects of slipping at the interface between the outer surface of a cut-and-cover tunnel and the backfill were examined by shaking table tests to evaluate the interactive load during strong earthquakes (Kawama et al., 2001). Non-linear seismic interaction as well as seismically induced plastic deformation performance of RC underground structures were studied by large scale shaking table tests using half scale RC structures (Ohtomo et al., 2002). It is noted in the above-mentioned studies that the discussions are limited to non-liquefied soil during earthquakes. Neither a build-up of the excess pore water pressure due to cyclic loading nor a reduction of shear modulus accompanied by pore

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pressure build-up in surrounding soil were taken into account.

The seismic interaction problems associated with the liquefied soil have been studied chiefly focusing on such structures as semi-buried roads (Koga et al., 1996), and quay walls of gravity type (Matsuo et al., 1960; Tsuchida, 1968; Inagaki et al., 1996; Kohama et al., 1998; Ghali and Tsuchida, 1998), and others (Tanaka et al., 1994). Although the seismic behavior of these types of structures could be different from that of the box-shaped cross sections which usually are subjected to shear-type deformations, the fundamental characteristics of interaction are drawn by those studies.

Fundamental investigations by shaking table tests have made on dynamic pressures due to completely liquefied soil acting on a rigid wall which translates in the horizontal direction. Tsuchida (1968) divided the total pressures acting on the wall into two components, i) the monotonic component which is exerted by the unit weight of liquefied soil, and ii) the fluctuating component which oscillates around the pressures of the monotonic component. The amplitude of pressures of fluctuating component was found to be approximately equal to the calculated values by Westergaard’s formula (Westergaard, 1933), which was originally proposed for reservoir water in dams.

Seismic earth pressures acting on walls of semi-buried roads were studied by Koga et al. (1996). A series of shaking table model tests as well as dynamic centrifugal model tests were conducted to investigate seismic earth pressures as well as bending moments of the wall in the process of pore pressure build-up. Simplified equations to evaluate the monotonic component as well as the fluctuating component of earth pressure are proposed introducing the ratio of excess pore pressure as a considerable parameter. The Westergaard’s formula (Westergaard, 1933) was extended to consider fluctuating earth pressures in the process of liquefaction. It was shown that the calculated bending moments in a wall during liquefaction were conservatively consistent with the bending moments observed in dynamic centrifugal model tests.

Shaking table model tests on quay walls of gravity type have been conducted by using a scaled model to clarify the mechanism of the interactive behavior as well as the damage due to past earthquakes. Ghali and Tsuchida et al. (1998) detected that the dynamic earth pressure acting on the wall and the wall displacement with inertia force became in-phase. This supports the idea of seismic coefficient when liquefaction in the foundation sand beneath the wall has occurred, inducing the lateral displacement of the wall. Kohama et al. (1998) indicated that the interactive behavior of quay walls was quite variable, depending on the occurrence of liquefaction in the backfill. Namely, the fluctuating earth pressure on the quay wall suppresses the movement of the quay wall before liquefaction, whereas the sliding of the quay wall is enhanced after liquefaction of the backfill. The amplitude of the observed fluctuating component of earth pressures on the quay wall due to liquefied soil appeared to agree reasonably with the calculation by Westergaard’s formula.

The seismic stability of a rectangular rigid block constructed by the stabilization of the Deep Mixing Method in the liquefied soil was investigated by Tanaka et al. (1994). The solution of dynamic pressures acting on a rigid block was derived by introducing rocking motion to Westergaard’s theory (1933). The experimental fact that the rigid block in liquefied soil was stable against overturning and sliding was accounted for by the calculation using the solution.

One of the limitations of past studies on soil-structure interaction of buried box-shaped cross sections was that the effect of liquefied soil was not studied experimentally, with only the non-liquefied condition attracting much attention. This was because the severe damage of large underground structures due to earthquakes had not occurred until recent years (Iida et al., 1996), and the effect of liquefaction was not clear for damaged underground structures (Matsuda et al., 2000). However, it was inevitable that it would become necessary to take the effect of soil liquefaction into consideration in seismic design calculations of civil structures owing to the consideration of strong earthquakes in recent seismic design in Japan (JSCE, 2000). In this regard, considerations of the effect of soil liquefaction on underground structures during earthquakes is now required in seismic design codes (e.g., JSCE, 1998; RTRI, 1999).

From the viewpoint of response analysis of cross sections, a seismic deformation analysis using a frame-spring model has a greater capability by taking into account the interactive behavior under the control of deformation, excluding the case where the backfill liquefies (Tateishi, 1992; Kawashima, 1994). However, many studies associated with liquefied soil and rigid walls have shown that the dynamic earth pressure due to liquefied soil can be evaluated by Westergaard’s formula (1933) which calculates the dynamic pressures by acceleration of wall (Tsuchida, 1968; Koga et al., 1996; Ghali and Tsuchida et al., 1998; Kohama et al., 1998). This suggests that the response of the cross section can be under the control of inertia force when the soil liquefies. In contrast, past researches have not determined the dominant factor in the response of a flexible wall of a box-shaped cross section for underground structures in the process of pore pressure build-up, as well as at complete liquefaction.

Accordingly, there is a need to study further the detailed mechanism of the seismic interaction between underground structures and liquefied soils. Particularly, one of the distinctive features of a box-shaped cross section is that the cross sections are subjected to shear-type deformations as a result of vertically propagating waves during earthquakes (Clough and Penzien, 1993). In most previous experimental studies on interaction between liquefied soil and structure, discussions are limited to rigid structures whose stiffness is relatively very high in contrast with that of the soil. Seismic soil-structure interaction is not clear in the cross sections with shear-type deformation of flexible walls.

The authors have studied experimentally the seismic
soil-structure interaction of a flexible underground structure subjected to liquefaction of the backfill soil. A box-shaped cross section of an underground structure which is subjected to the shear-type deformation is used in tests. A special attempt is made to reproduce the stress-strain relationship as well as the effective stress path of backfill soil during shaking. Theoretical study is conducted for the evaluation of the dynamic effect due to liquefied soil on the flexible wall of a cross section.

**SHAKING TABLE TESTS**

A series of 1G shaking table tests for a scaled soil-structure model was carried out by using a laminar box which measured approximately 1.2 meters in length, 1.0 meter in height, 0.8 meters in width at the Power Engineering Research and Development Center of the Tokyo Electric Power Company.

**Model Specification**

One of the key issues in this experiment is the interactive behavior of the shear-type deformation of a box section subjected to soil liquefaction. The cross section of the structure model was designed so that enough shear-type deformation could occur during shaking. The inner space of the structure model was kept vacant, filled with air.

First, the dimension of the rectangular cross section of the structure was determined as 0.85 m high, 0.28 m wide by the limitation of the size of the container used. A configuration of a structure model is depicted in Fig. 1. The side walls and the top plate of the structure model were divided into three in the transverse direction as illustrated in Fig. 1(a). A physical gap of 5 mm was maintained between side walls, while 10 mm was maintained between the structure model and the container. Complete waterproof works were made at the physical gaps to prevent leakage of both water and sand into the culvert box. Measurement was conducted at the middle part of the structure, which was not in contact with the side walls of the container.

To simplify the analysis of the tests, side walls of the model were bolted to the thick bottom plate, which was fixed to the container base. Neither translation nor rotation was allowed at the bottom of the wall, and no rotation at its top. Owing to difficulties in preparing reinforced concrete models, the wall as well as top and bottom plates of the model were made of A5083 alloyed aluminum plates (Young’s modulus = 75 GPa and Poisson’s ratio = 0.32, Yield stress = 146 MPa, Model-A). A1070P aluminum plates (Young’s modulus = 69 GPa and Poisson’s ratio = 0.32, Yield stress = 86 MPa) were also used in a limited number of experiments (Model-B).

The thickness of the side wall was determined by 2 factors: i) to be as thin as possible so that the magnitude of the shear deformation of the cross section during shaking might become larger, ii) thick enough so that the yielding of aluminum might not occur before shaking. Calculations were made on three kinds of thickness of wall of 3 mm, 5 mm, and 10 mm, applying the static water pressure and the horizontal effective earth pressure at rest. The yield stress of aluminum was conservatively estimated as 86 MPa (A1070P) for a criterion. By the results, the wall of 5 mm thickness was selected to satisfy the requirement. The thickness of the top plate was determined as 15 mm so that the bending stiffness would become relatively high as compared with that of the walls. The top plate was 3 times thicker than the side wall, and 27 times stiffer than the side wall in terms of the flexural stiffness. Model sizes and properties of members are shown in Table 1. The thickness of the wall shown in the table is the measured value of the walls (nominal thickness of 5 mm) by using a slide caliper with accuracy of 1/20 mm. The natural frequency of the structure itself in air was approximately 7 Hz according to a free vibration test.
The backfill soil used was Toyoura sand (uniform, sub-rounded, fine sand, $D_{10} = 0.2$ mm, specific gravity of 2.65, and maximum and minimum void ratio of 0.966 and 0.642 respectively). The backfill was made by pouring sand into de-aired water in the box to achieve the initial relative density of approximately $D_r = 35\%$.

A special attempt was made to build a stress-strain relationship as well as an effective stress path of the backfill soil during shaking around an embedded structure by installing a variety of transducers such as those for acceleration, displacement, pore pressure and earth pressure as shown in Fig. 1. Earth pressure and strain transducers were attached to the flexible wall of an embedded structure model. Special care was taken of the earth pressure transducers that were buried in liquefiable saturated soil. The apparent densities of earth pressure transducers were approximately adjusted to that of saturated sand by using a lightweight material of vinyl chloride plastic. The transducers were fixed on a thin plastic plate in order to minimize the rotation and settlement during liquefaction.

The performance of earth pressure transducers in saturated soil was examined by a different series of shaking table tests in which only a uniform saturated soil deposit in a soil container was shaken in the vertical direction to exert the dynamic earth pressure in the saturated soil (Tamari et al., 2001). In this test, the dynamic earth pressures were evaluated by two different procedures; i) measured directly by earth pressure transducers which were buried in the soil and, ii) calculated by integrating the inertia forces of soil mass with respect to the depth in which the inertia forces were evaluated by the measured accelerations. It was therein supposed that the dynamic earth pressures obtained from the accelerations were correct since the acceleration measurements were more reliable. Figure 2 shows the ratio of amplitude between those earth pressures. It is seen that the values of the ratio are almost 1.0 under a variety of conditions such as the intensity of input motion, the frequency of input motion, the relative density of soil, and the ratio of excess pore water pressures. This suggests that the earth pressure transducers give reasonable dynamic earth pressures when the transducer is buried in saturated soil.

The soil-structure system was shaken in several stages in the horizontal direction. The input motions consisted of thirty or forty sinusoidal cycles at 3 Hz or 5 Hz of frequency. The intensity of acceleration was increased in stages in which peak accelerations were approximately $0.5 \text{ m/sec}^2$ (50 gal), $2.0 \text{ m/sec}^2$ (200 gal) and $5.0 \text{ m/sec}^2$ (500 gal) in the test. Each stage of shaking was followed by a stationary period to allow for dissipation of developed excess pore water pressure as well as the measurement of settlement at the ground surface.

### Measurements of Initial Earth Pressure and Strain

Before shaking, accelerations on the wall of the structure are zero. Therefore, only lateral earth pressures contribute to strains in the wall. The measured initial earth pressures and initial curvatures based on the measured bending strain of the wall for 35 and 67% relative density of sand are plotted in Fig. 3. The curvatures calculated from measured earth pressures are also shown in the figures. In this case, the wall was assumed to be an elastic fixed-end beam. As is seen, the agreement is good. Such a

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**Table 1. Model sizes and properties**

<table>
<thead>
<tr>
<th></th>
<th>Thickness</th>
<th>Moment of inertia</th>
<th>Flexural stiffness</th>
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<tbody>
<tr>
<td>Wall (Model-A)*</td>
<td>4.97 mm</td>
<td>$1.02 \times 10^{-4}$ m$^3$/m</td>
<td>765 Nm/m</td>
</tr>
<tr>
<td>Wall (Model-B)*</td>
<td>4.87 mm</td>
<td>$9.63 \times 10^{-4}$ m$^3$/m</td>
<td>664 Nm/m</td>
</tr>
<tr>
<td>Top plate</td>
<td>15 mm</td>
<td>$2.81 \times 10^{-3}$ m$^3$/m</td>
<td>20960 Nm/m</td>
</tr>
</tbody>
</table>

*Approximate yield strain: 1875 μ
good agreement between measured and calculated initial curvatures was also observed in other tests. This suggests that the initial quantities measured on the wall such as lateral earth pressures and bending strains were internally consistent.

**Measurements of the Test**

Selected dynamic measurements from the test shaken by the peak acceleration of 50 gal at 35% relative density are presented in Fig. 4. The locations of transducers are shown in Fig. 1. The initial horizontal portion of the pressure and curvature time histories indicate the static quantities before shaking.

In the response horizontal accelerations AC2, the intensity at around 7.5 to 8.0 seconds of motion is larger than the rest. It is seen that the response vertical accelerations AC9 is much smaller than the horizontal response accelerations AC2. The displacements at the same cycles of motion are also significantly amplified. These amplifications of motion are related to the oscillations in time histories for the curvatures of the wall and the lateral earth pressures. It can be seen that the curvatures and the lateral earth pressures continue to increase during shaking accompanied by the build-up of pore water pressure becoming equal to the initial overburden pressure at about 8.0 seconds.

**STRESS-STRAIN RELATIONSHIP OF SOIL DURING SHAKING**

One of the key investigations in this study is to reproduce the stress-strain relationship and the effective stress path of backfill soil near the structure. They will be referred to as fundamental information to understand the mechanism of dynamic soil-structure interaction in the following section.

This kind of attempt has been made previously focusing on the foundation ground of such structures as embankments (Koga and Matsuo, 1990) and a quay wall of gravity type (Ghahaladzarzadeh et al., 1998). In the previous studies, shear stress generated in a horizontal plane at a depth was estimated using several measurements. From the test results in this study, it is observed

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**Fig. 3.** Measured initial earth pressure and curvature of the wall

**Fig. 4.** Measured time histories from the test (peak acc. = 50 gal, relative density $D_r = 35\%$)
that the horizontal responses are much more predominant than the vertical responses (see Fig. 4(a)). Since the vertical response is relatively insignificant, the force equilibrium only in the horizontal direction is taken into account herein. The shear stress generated in a horizontal plane is reproduced considering the effect of the structure’s wall.

A free-body diagram of the simplified problem of a saturated soil mass is shown in Fig. 5. Considering the force equilibrium in the horizontal direction, it can be shown that

$$\rho \alpha_x = \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau}{\partial z}$$

where \( \rho \) is the mass density of saturated soil. Integrating Eq. (1) once with respect to \( z \) and considering the boundary condition of \( \tau(x, 0, t) = 0 \), the shear stress at any depth and a particular time instance \( t \) at a particular horizontal coordinate \( x \) is calculated as follows (Tamari et al., 2000).

$$\tau(x, \xi, t) = \int_0^\eta \{ -\rho \alpha_x(x, \xi, t) \} + \frac{\partial}{\partial x} \sigma_x(x, \xi, t) \} \\delta \xi$$

where \( \delta \) is a dummy integration valuable. Acceleration \( \alpha_x(x, \xi, t) \) can be evaluated by interpolating the measured ground accelerations and wall accelerations. Normal stress difference can be specified as

$$\frac{\partial}{\partial x} \Delta \sigma_x(x, \xi, t) = \Delta \sigma_x(x, \xi, t) / \Delta x$$

where \( \Delta \sigma_x \) is a difference of the measured lateral earth pressure (EP) both on the wall and in the soil, and \( \Delta x \) is the width of soil mass (\( \Delta x = 0.225 \) m).

The shear strains of the soil mass adjacent to the wall (see Fig. 1) were calculated considering soil displacements at AC5 and AC6, and wall displacements at SG2 and SG3. The soil displacements were calculated by integrating twice the measured accelerations with respect to time. The wall displacements were calculated by integrating twice the measured wall curvatures with respect to depth. Although this method is correct, it was difficult to detect the small range of shear strain due to many noises in time histories. In this study, shear strain less than about 0.5% was evaluated by the average angle of deformation of the wall, which can be calculated by dividing the displacement at the top (LV1) by the height of the structure (\( H = 0.7 \) m).

The effective stress in the vertical direction, \( \sigma'_v \), is calculated from measured excess pore pressures as

$$\sigma'_v = \sigma_{i0} - \Delta u = \gamma' z - \Delta u$$

where \( \sigma_{i0} \) is the initial effective vertical stress and \( \Delta u \) is the excess pore water pressure, \( \gamma' \) is the effective unit weight of soil, and \( z \) is the depth. The effective unit weight was assumed constant in the vertical direction.

The stress-strain relationship and the effective stress path of backfill soil near the structure at various intensity of input motion are depicted in Fig. 6. The analyzed soil mass is located at GL-0.30m to GL-0.45m. Figures 6(a)(i) is the stress-strain relationship from the test shaken by the peak acceleration of 50 gal. It is initially close to a linear elasticity as shown in the central part of the figure, and gradually changes into a behavior of liquefaction; i.e., the shear strain grows rapidly in the amplitude. The pore water build-up and the consequent onset of liquefaction can be observed in the stress path shown in Fig. 6(a)(ii). The stress-strain relationship with the peak acceleration of 200 gal for 41% relative density is shown in Fig. 6(b)(i). The amplitude of shear strain reaches a level of about 1.5% in this case. The effective vertical stress, being initially about 3 kPa in the stress path in Fig. 6(b)(ii), reduces to zero after several cycles of the motion.

Figure 6(c) shows the results for 32% relative density of sand shaken by 500 gal of intensity. When the shear strain reaches a level of about 4%, the shear stress is only 2 or 3 kPa. The effective vertical stress, which is initially about 3 kPa, reduces to zero in one or two cycles of motion. Figure 6(d) shows the results of Model-B structure for 36% relative density of sand shaken by 500 gal of intensity. In this case, the soil yield just before the liquefaction (see Fig. 25(c) in the following section). In spite of the yielding of the wall, the same feature can be seen in the stress-strain relationship and corresponding stress path. In the case of the same structure (Model-B) and intensity of motion (500 gal), the feature of stress-strain relationship and stress path for 78% relative density shown in Fig. 6(e) is different from those for 36% relative density. The shear stress at about 4% shear strain reaches about 20 kPa at A in Fig. 6(e)(i). The effective vertical stress shown in Fig. 6(e)(ii) reaches about 10 kPa at B in spite of the initial effective vertical stress of about 3 kPa. The same feature is seen at 67% relative density shown in Fig. 6(f) in which the wall of the structure yielded (see Fig. 25(e) in the following section). The acceleration responses at the top of structure (AC2) are shown in Fig. 7. In the test for 32% relative density, the response acceleration was about 7 m/sec² as shown in Fig. 7(a). The higher response acceleration for dense sand deposit was produced as shown in Figs. 7(b)(c) by the dilatant behavior of sand which allowed the greater magnitude of
This suggests that the soil around the embedded structure behaves in a dilative manner during seismic shaking when the backfill soil is dense.

Shear modulus of soil in each cycle of motion can be obtained by drawing the secant line of a hysteresis loop. An example of a reproduced hysteresis loop between 5.67 and 6.00 seconds from the test for 35% relative density shaken by 50 gal of intensity is illustrated in Fig. 8. A line for determination of approximate shear modulus is also drawn in the figure. In this cycle, the shear modulus was reduced to about 0.11 of the initial value.

Figure 9 shows the shear modulus ratios at each cycle, and the measured excess pore water pressure time histories under various intensity of input motion (50 gal, 200 gal, 500 gal). It is seen that the reduction of shear modulus becomes rapid as intensity of input motion becomes larger. In the case of 500 gal shaking, the excess pore water pressures change cyclically during liquefaction in the range of about 1 kPa to 5 kPa. This is considered to be the effect of longitudinal waves resulting from the dynamic interaction between the structure and the liquefied soil.

SEISMIC RESPONSE OF EMBEDED STRUCTURE

The nature of the seismic response of the embedded structure is investigated in terms of the natural frequency of the ground. The natural frequency, \( v \), of a soil column adjacent to the embedded structure can be calculated as

\[
v = \sqrt{\frac{G}{4H}} \rho
\]

where \( G \) is the shear modulus of soil, \( \rho \) is the mass density, and \( H \) is the depth of the ground adjacent to the embedded structure. The shear modulus of soil \( G \) was specified by drawing the secant line of a hysteresis loop at
Fig. 7. Acceleration responses at the top of structure

Fig. 8. Examples of stress-strain relationship in a cycle and a secant line for determination of shear modulus

Figure 11(a) shows the relationship between the amplifications and the natural period of ground. In the case of 3 Hz shaking, the natural period of ground when the peak of amplifications occur is 0.3 to 0.5 seconds. This range of the period almost coincides with the input period of 0.33 seconds. In the case of 5 Hz shaking, the natural period of ground when the peak of amplification occurs became about 0.25 seconds. This period also coincides with the input period of 0.20 seconds.

Figure 11(b) illustrates the relationship between phase difference and natural period of ground. It is seen that the phase difference between top (AC2) and bottom (AC1) of the structure, initially in phase (0°), changes up to about 150° as the natural period of ground becomes long due to shaking. It is remarkable that the phase difference is almost 90° when the peak amplification appears. This behavior of the embedded structure is resonance by the sinusoidal input motion. It suggests that the reduction of shear modulus of backfill soil during the pore pressure build-up causes resonance of the soil-structure system. The approximate values of amplification and phase difference between the top and bottom of structure obtained in the series of shaking table tests are summarized in Table 2.

The natural frequency of a soil column is about 2 to 3 Hz when the resonance occurs by the 3 Hz input motion. Considering that the initial natural frequency is approximately 23 Hz, the shear modulus of soil at the instance of resonance reduced from about 1/60 to 1/130 of the initial values.

**DYNAMIC EFFECTS OF LATERAL EARTH PRESSURES ON STRUCTURE**

Typical measurement of lateral earth pressure acting...
on the structure in tests of 35% relative density is presented in Fig. 12 (dotted line). The frequency of input motion is 3 Hz, and the peak acceleration is 50 gal. As seen in the shape of the measured time history, the earth pressures consists of, i) static lateral earth pressure, ii) increase in lateral earth pressure due to accumulation of pore water pressure, and iii) cyclic change of lateral earth pressure in phase with sinusoidal input motion. In this study, the earth pressure iii) is regarded as the dynamic earth pressure. An example of time history of dynamic earth pressure is shown at the bottom part of Fig. 12 (solid line).

Phase Difference between Earth Pressures and Displacements

It is possible that the dynamic earth pressures, which act on the upper part of structure, strongly affect the displacements at the top of the structure. Therefore, the relationship between the dynamic earth pressures at 0.3 m from the surface (EP6, EP7) and the displacement of the structure was examined. The resultant force $F_d$, as defined in Fig. 13, was introduced to represent the effect of dynamic pressures EP6 and EP7. The earth pressure at the ground surface was assumed as zero, and interpolated.
Table 2. Approximate values of amplification and phase difference

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<thead>
<tr>
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<th>Amplification</th>
<th>Phase difference</th>
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<tr>
<td>When resonance occur</td>
<td>2.5 to 3</td>
<td>90 deg.</td>
</tr>
<tr>
<td>When soil liquefy</td>
<td>1 to 1.5</td>
<td>150 deg.</td>
</tr>
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</table>

Fig. 12. Component of measured lateral earth pressures

Fig. 13. Definition of resultant force $F_d$ due to dynamic earth pressures

by a piecewise linear function in the backfill soil.

The time histories of the resultant force $F_d$ and displacements at the top of the structure are presented in Fig. 14. The positive displacement is illustrated in Fig. 13. It is observed that the time histories of the resultant force $F_d$ and the displacements are out-of-phase at the time period of 3.0 to 5.0 seconds. This means that the dynamic earth pressure constrains the motion of the structure in the soil because soils are more rigid than the wall. In contrast, both time histories are in phase in the time period of 7.0 to 9.0 seconds when the high excess pore water pressure (P6) has made the soil very soft (see Fig. 4(e)). In this time, the dynamic earth pressures act as a load which increases the dynamic deformation of the structure.

To understand this phenomenon more clearly, the natural periods of ground at each time period were investigated. The relationships between the phase angle differences and the natural period of ground are shown in Fig. 15. In this figure, the 180 degree difference of phase angle means the out-of-phase response (see Fig. 14 top), and zero means that of in phase (see Fig. 14 bottom). The natural period of the structure itself (0.14 sec) is indicated in the figure. It is seen that the phase difference is $-180$ to $-120$ degree when the natural period of ground is shorter than that of the structure. In contrast, the phase angle difference increases to around zero as the natural period of ground become longer than that of the structure. This suggests that the dynamic earth pressure does not act as a load on the structure until the natural period of ground near the structure becomes longer than that of the structure itself.
Effect of Inertia Force on Dynamic Earth Pressures

As shown in Eq. (1) in the previous section, the inertia force per unit volume of saturated soil is equal to the sum of normal stress difference and the shear stress difference of unit depth in the horizontal direction. Figure 16 shows a typical example of the time histories of the inertia force per unit width, \(-\rho \alpha \cdot \Delta Z\) and the normal stress difference per unit width, \(\Delta \sigma_n / \Delta x \cdot \Delta Z\) (= \(\Delta \sigma_s / \Delta x \cdot \Delta Z\)). The height of the soil mass, \(\Delta z\), is 0.30 m and the width, \(\Delta x\), is 0.225 m. The amplitude of normal stress difference, \(\Delta \sigma_n / \Delta x \cdot \Delta Z\) near the structure is much smaller than that of the inertia force in the time period from 5.0 to 6.0 seconds. The amplitude of the normal stress difference starts to increase at around 6.5 seconds, and the amplitude becomes similar to that of the inertia force at 8.0 seconds, when the soil liquefied completely (see Fig. 4(c)). Considering the equilibrium of force in the soil mass, there is no shear stress difference after 8.0 seconds. This change is considered to be due to reduction of shear modulus of the soil by the liquefaction.

Figure 17 presents the effect of natural period of ground on the normal stress difference of dynamic earth pressures during pore pressure build-up from various tests. The axis of ordinates represents the ratio of the amplitude between the normal stress difference \(\Delta \sigma_n / \Delta x\) and the inertia force \(-\rho \alpha\). The ratio increases toward unity as the natural period of ground becomes longer. This means that the inertia force of the soil mass adjacent to the structure becomes dominant in the dynamic earth pressures when the soil liquefies.

Effect of Dilatancy of Soil

In a previous study based on the stress-strain relationship of soil (see Fig. 6), it was observed that dense backfill soil near the embedded structure behaves in a dilative manner during shaking. The effect of dilatancy on dynamic soil-structure interaction is examined here.

Figure 18 presents the relationship between the resultant force \(F_g\) (see Fig. 13) and the displacement of the structure (LV1) at a relative density of 32%, 78%, 67% in which the peak acceleration of the input motion is 500 gal. Different from Fig. 14(a) with 50 gal, the 500 gal shaking shows that liquefied sand generates load basically on the wall.

The shape of the hysteresis loop at 32% relative density is almost positive proportionality. When the wall translated away from the backfill soil (positive displacement is illustrated in Fig. 13) the earth pressure still increased. Thus, it is interesting that the liquefied soft sand tends to move laterally more significantly than the wall, making this positive proportionality. On the other hand, the shape of the hysteresis loop at 78% and 67% relative density is very complicated. The spiky earth pressures at the negative displacement (C in Figs. 18(b)(c)) appears in the hysteresis loop. According to the stress-strain relationship shown in Figs. 6(c)(f) and acceleration response presented in Figs. 7(b)(c), the dilative behavior of soil was observed at 78% and 67% relative density. The dilative behavior of soil influenced the relationship.

Figure 19 depicts one cycle of the hysteresis of Fig. 18(c) in the time period of 4.0 to 4.3 seconds. The path of the hysteresis loop is indicated in the figure by the arrows. It is seen that the earth pressure increases as the displacement increases to the positive direction between B and A. The liquefied soft backfill tends to move laterally more significantly than the wall. The earth pressures act as a load to the structure in this moment. The quick decrease of earth pressure with positive displacement is observed at A to D. In this time, the structure moved away from backfill soil that became more rigid than the
structure instantaneously by increasing the effective stress. In contrast to A to D, the earth pressure increased rapidly from B to C with negative displacement. The backfill soil hardened again more than the structure by increase of effective stress, and it constrained the motion of the structure in negative direction instantaneously. As a result of this, the earth pressure acts as a reaction.

Figure 20 demonstrates the hysteresis loops in the time periods of 4.0 to 4.3 seconds and 5.0 to 5.3 seconds for (i) relationship of the resultant force $F_d$ and displacement, (ii) stress-strain relationship and (iii) effective stress path. The relative densities of backfill in the tests are 32%, 78% and 67%.

As shown in Fig. 20(a)(ii) at 32% relative density, the hysteresis loop is of positive proportionality. In this case, the effective stress path stays at the origin as indicated in Fig. 20(a)(iii), indicating complete liquefaction of the backfill soil.

The spiky earth pressures are seen in Fig. 20(b)(i) when the displacement is minimum at 4.14 and 5.14 seconds respectively. In this time, the earth pressure, which once decreased to the level of about $-1.5$ kN/m, increased to the level of 2 kN/m. This time moment corresponds to the time when the minimum shear stress occurred as shown in the stress-strain relationship of Fig. 20(b)(ii). And the effective vertical stress reached to the level of 10 kPa as shown in Fig. 20(b)(iii). It shows that the spiky earth pressures occurred due to the effect of dilatancy. The same feature can be seen in Fig. 20(c) at 67% relative density. Consequently, the observed earth pressures which act as a reaction of the structure is due to the effect of dilatancy of dense backfill.

Dynamic Earth Pressure and Horizontal Normal Strain, Acceleration on the Wall

Often in practical calculations, the horizontal relative displacement between the ground and the structure and the acceleration on the wall are taken into consideration to evaluate the dynamic earth pressure.

The horizontal relative displacement $\delta$ is defined as

$$\delta = \delta_b - \delta_i$$

where $\delta_i$ is the horizontal displacement at the free field, and $\delta_b$ is the horizontal displacement of structure as shown in Fig. 21. When the width of ground $B$ in the figure is wide enough, $\delta_i$ can be regarded as displacement of free field. In this section, the horizontal normal strain $\varepsilon_{hn}$ is defined in a simplified manner instead of the horizontal relative displacement as

$$\varepsilon_{hn} = (\delta_b - \delta_i)/B$$

where $B$ is the width of the ground, 0.45 m. The measured displacement at the edge of container (LV3, LV4) and the displacement on the wall by the measured bending strain (SG2, SG4) are regarded as the displacement $\delta_b$ and $\delta_i$ respectively. The acceleration on the wall was derived by differentiating $\delta_i$ twice with respect to the time.
(a) Relative density Dr=32%

(b) Relative density Dr=78%

(c) Relative density Dr=67% (the wall started to yield at about 4 sec.)

Fig. 20. Hysteresis loop of earth pressure and displacement of the structure (i), stress-strain relationship (ii), effective stress path (iii) shaken by the same intensity of input motion (3 Hz, 500 gal)

Figure 22 illustrates the hysteresis loops between the dynamic earth pressure and the horizontal normal strain \( \varepsilon_h \) at each depth. The time period 1.5 to 2.5 seconds when the maximum amplification occur, and 6.0 to 7.0 seconds when the soil completely liquefy (see Fig. 9, bottom) are selected in each figure. Figure 23 shows the hysteresis loops between the dynamic earth pressure and the acceleration on the wall at the corresponding depth. The correlation is not clear between the earth pressure and normal strain. In contrast, the correlation of negative proportionality is recognized clearly between the earth pressure and the wall acceleration.

The coefficients of correlation between the time histories are calculated. The results are shown in Table 3. The coefficients for the acceleration at both depths are in the range of \(-0.6\) to \(-1.0\) indicating that the inertia force of structure at each depth has a close relation with the dynamic earth pressures. The coefficients for the relation...
Fig. 21. Definition of horizontal normal strain of ground

\[ \varepsilon_h = (\delta_s - \delta_b) / B \] (compressive strain as positive)

Fig. 22. Hysteresis loops between the dynamic earth pressure and the normal strain in horizontal direction

Fig. 23. Hysteresis loops between dynamic earth pressure and the horizontal acceleration on the wall

of the normal strain are in the range of \(-0.5\) to \(0.5\).

The measured acceleration during the time periods at the top of structure (AC2) and the surface of ground (AC3) are shown in Fig. 24. The phase difference between the accelerations is not significant during 1.5 to 2.5 seconds when the maximum amplification occurs, whereas the phase difference about 1/5 of the period is seen between them at the period of 6.0 to 7.0 seconds. It is considered that the change of the phase difference between the structure and soil causes the low correlation between the earth pressure and the normal strain of ground.

**Yielding of Structure**

Figure 25 shows the time histories of bending strains at the bottom of the wall (SG7, SG14) and the relation between the bending strain and effective vertical stress at the point of P2 (GL-0.375m, see Fig. 1) at the 500 gal shaking of input motion. The approximate yield bending strain of the wall (\(\pm 3750\mu\)) is depicted in each figure.

Figure 25(a) illustrates the bending strain of Model-A at the 32% relative density. The positive bending strain is depicted in the inset of Fig. 25. The bending strains of wall are smaller than the yield strain through the duration of shaking. The effective vertical stress is almost zero as shown in Fig. 25(b) so that the soil liquefies completely at time period of 4.0 to 8.0 seconds.

Figure 25(c) shows the bending strain of Model-B at the 36% relative density. The tensile strength of the wall in Model-B is about 3/5 of the strength of Model-A. The bending strain at the bottom of wall at left side (SG7) gradually increased with cyclic change up to about 8 seconds, and exceeded the yield bending strain at 3.4 seconds. The bending strain at the bottom of wall at right side (SG14) is gradually decreasing with cyclic change, and exceeded the yielding strain at 3.2 seconds. The effective vertical stress keeps around zero as shown in Fig. 25(d), indicating that the loose soil behaves in the manner of negative dilatancy.
Figure 25(e) illustrates the bending strain of Model-A at 67% relative density. The bending strain of the wall exceeded the yield strain cyclically during the time of 4.0 to 8.0 seconds. The effective vertical stress of the backfill soil increased significantly due to dilatancy (see Fig. 6(f) and Fig. 7(c)) when the bending strain exceeded the yield bending strain as shown in Fig. 25(f). It is considered that the yielding of the aluminum caused large deformation of the structure, consequently, the dense soil strain becomes large and the soil behaves in a more dilative manner.

**SOLUTION OF DYNAMIC LATERAL PRESSURES ON FLEXIBLE WALL SUBJECTED TO SOIL LIQUEFACTION**

In this section, a solution for the dynamic lateral pressure on the flexible wall of underground structure caused by liquefied soil will be developed. Acceleration of the structure is selected as the major input parameter for the solution since a good correlation between the dynamic earth pressure and the acceleration was detected in the previous study.
Consider a wall-soil model consisting of a semi-infinite, uniform layer of liquefiable soil of height $H$, that is free at its surface, underlain by a base unliquefied layer, and retained along one of its vertical boundaries by a flexible wall of structure. Neither translation nor rotation was allowed at the bottom end of the flexible wall, while no rotation at its top end. The base of liquefied layer is assumed to be excited by a seismic motion, the acceleration of which at any time $t$ is $\ddot{\zeta}_s(t)$. A cross section of this system along the $(x, y)$ plane is shown in Fig. 26. The figure also illustrates the positive directions of the horizontal and vertical displacements which are denoted by $\xi$ and $\eta$, respectively.

Assumptions are made to enable the calculation of the dynamic lateral pressures:

(ASM.1) The horizontal dynamic displacement of flexible wall $(x=0)$ is approximated by a deflection of beam with a settled end in a vertical cross section as follows:

$$\xi = \xi_0 \left(1 - \frac{y}{H}\right)^2 \left(3 - 2 \left(1 - \frac{y}{H}\right)\right)$$

(8)

where $\xi_0$ is the horizontal relative displacement at the top of the flexible wall. Figure 27 illustrates the measured deflections of wall shaken by 500 gal of excitation. The deflections approximated by Eq. (8) are also shown in the figures. It can be seen that the approximation by Eq. (8) is satisfactory as they show such a good agreement.

(ASM.2) The motion of liquefiable soil is governed by the following equations which apply to sound in liquid (Westergaard, 1933) assuming no shear stresses in the soil,

$$\frac{\partial^2 \xi}{\partial t^2} = \rho \frac{\partial^2 \xi}{\partial x^2}$$

(9)

$$\frac{\partial^2 \eta}{\partial t^2} = \rho \frac{\partial^2 \eta}{\partial y^2}$$

(10)

$$p = K \left(\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y}\right)$$

(11)

where,

$$\rho$$: mass density of liquefiable soil

$p$: dynamic pressure (compression as negative)

$K$: bulk modulus of liquefiable soil

The Eqs. (9)(10)(11) are solved subject to the boundary conditions:

$$p = 0 \quad \text{at} \quad y = 0 \quad \text{(surface)}$$

$$\eta = 0 \quad \text{at} \quad y = H \quad \text{(bottom)}$$

$$p = 0 \quad \text{at} \quad x = \infty$$

$$\ddot{\xi}_s(t) = -\frac{1}{\omega^2} \alpha_{\xi\xi} e^{i\omega t} = \dot{\xi}_0 e^{i\omega t}$$

at $x = 0$ and $y = H$

(14)

$$\ddot{\xi}_s(t) = -\frac{1}{\omega^2} \alpha_{\eta\eta} e^{i(\omega t + \delta)} = \dot{\xi}_0 e^{i\omega t}$$

at $x = 0$ and $y = 0$

(16)

$$\ddot{\xi}(y, t) = (\ddot{\xi}_s(t) - \ddot{\xi}_s(t)) \left(1 - \frac{y}{H}\right)^2 \left(3 - 2 \left(1 - \frac{y}{H}\right)\right)$$

$$\ddot{\xi}_s(t) \quad \text{at} \quad x = 0$$

(17)

where, $\ddot{\xi}_0$ is amplitude of displacement at the bottom of structure, $\dddot{\xi}_0$ is amplitude of displacement at the top of structure, $\delta$ is the phase angle between the top and the bottom of structure.

The solution of the dynamic pressure on the flexible wall $(x=0)$ is derived as follows:

$$p(y, t) = -\frac{2\rho H}{\pi} e^{i\omega t} \sum_{n=1}^{\infty} \frac{A_n}{(2n-1)C_n} \sin \left(\frac{2n-1}{2} \pi \frac{y}{H}\right)$$

(18)

in which

$$A_n = 2B_n \{1 + 6B_n^2(1 - 2(-1)^{n+1}B_n)\} \{\alpha_{\xi\xi} e^{-i\omega t} - \alpha_{\eta\eta}\} + \alpha_{\xi\xi}$$

(19)

$$B_n = \frac{2}{\pi(2n-1)}$$

(20)

$$C_n = \sqrt{1 - \frac{16\rho H^2}{(2n-1)^2 KT^2}}$$

(21)
where $T$ is a period of the input motion ($= 2\pi/\omega$).

Note that the solution is applicable to liqueifiable backfill soil which behaves in the manner of negative dilatancy during earthquake.

To verify the validity of the solution of Eq. (18), the dynamic pressures were calculated using the measured acceleration at the top ($\alpha \omega e^{i(\omega - 3)}$) and the bottom ($\alpha \omega e^{i\omega t}$) of the structure, and compared to the measured dynamic pressures on the wall. Figure 28 shows the calculated dynamic pressures by using the proposed solution in various cases of experiment at the cycle when (i) the resonance occurred and the amplification became the maximum, (ii) the soil liquefied completely. In this calculation, the bulk modulus of liquefiable soil was specified as $2.25 \times 10^{6}$ kPa assuming the velocity of sound in water $1500$ m/sec. The measured pressures on the flexible wall and the calculated dynamic pressures on the rigid wall (Westgaard, 1933), which are often used in practical calculation, are also shown in each figure. The simplified equation of dynamic pressures on the rigid wall is expressed using same system shown in Fig. 26 as,

$$p(y, \omega) = -\frac{7}{8} \cdot \alpha \omega e^{i\omega t} \cdot \rho \sqrt{H' y} \quad (22)$$

where $\rho$ is mass density of liquid.

The dynamic pressures calculated by the proposed solution when the resonance occurred in Model-A (figures of (i) in Figs. 28(a)(b)(c)) show good agreements with the measured dynamic pressures under the condition of various intensity of input motion. The good agreements exist in spite of the assumption of no shear stresses as expressed in Eqs. (9)(10). Note that the shear modulus of backfill soil is about 1/60 to 1/130 of initial value according to the previous study. The dynamic pressures by proposed solution when the soil completely liquefies (figures of (ii) in Figs. 28(a)(b)(c)) agree with the measured dynamic pressures through the depth very well. While the dynamic pressures calculated assuming the rigid wall became larger than the measured pressures at the bottom of wall in all results.

Figure 28(d)(i) presents the dynamic pressures in the third cycle of shaking in the test using Model-B. This result also shows good agreement. As presented in Fig. 25(c), the bending strain of wall in Model-B exceeded the yield bending strain at its bottom (SG7) after 3.4 seconds. Figure 28(d)(ii) presents the dynamic pressures in the 15th cycle (about 6 seconds) when the bending strain of wall exceeded the yield strain. It is seen that the agreement is fairly good in spite of the yielding of the aluminum. Those results suggest that the dynamic pressures on flexible wall during liquefaction can be evaluated by the proposed solution. The solution seems to be applicable not only for the complete liquefied soil but also for the soil which has the shear modulus reduced to the order of 1/100 of initial value.

The vertical distribution of dynamic earth pressure by the proposed solution depends on the amplitude and the phase difference between the top and the bottom of the
wall. From the viewpoint of application to a practical problem, it is important to estimate the maximum amplification when the resonance of the structure occurs. In this case, the range of the approximate values of the maximum amplification in Table 2 could be referenced. It should be noted that these are results of a type of small scale model tests. It is a future topic of study to confirm the maximum amplifications of full scale underground structures when the backfill liquefies.

CONCLUSIONS

A series of 1G shaking table tests were performed to study the soil-structure interaction of underground structure considering liquefiable backfill. The base motions consisted of sinusoidal cycles at 3 Hz, or 5 Hz frequency. In order to reproduce the stress-strain relationship and the effective stress path of backfill soil, a variety of transducers were used for the tests. The conclusion drawn from this study is that the natural period of the ground with structure and the dilative nature of the backfill soil significantly affect the type of soil-structure interaction around the studied embedded structure.

Major findings as obtained from the present tests are shown as follows:

1) The reduction of shear modulus of soil during the pore pressure build-up causes resonance of the soil-structure system.
2) The dynamic earth pressure does not act as a load on the structure until the natural period of ground near the structure becomes longer than that of the structure itself.
3) After liquefaction, the dynamic earth pressures act not only as a load but also as a reaction of the structure when the effective stress is increased by the dilative nature of soil.
4) The yielding of the walls causes large deformation of the structure and soil. Consequently, the soil strain becomes large and the soil behaves in a more dilative manner.
5) The dynamic pressures on flexible wall during liquefaction have closer relation to accelerations on the flexible wall than the normal strain of soil adjacent to the wall.
6) The proposed solution of dynamic pressure on flexible wall gives good results.

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