GENERALISED EFFECTIVE STRESS ANALYSIS OF STRENGTH AND SMALL STRAINS BEHAVIOUR OF A SILTY SAND, FROM DRY TO SATURATED STATE

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ABSTRACT

Very small to large strains properties of a silty sand were measured at several water contents ranging from a few percents to saturation, using three triaxial devices with different types of transducers. In the analysis of the data, attention was focused on the influence of water content on the maximum (Young's) modulus, the decay of modulus with strain, the effect of the confining stress, etc. Isotropic and triaxial tests, with measurement of negative pore water pressure, were performed on the same material. The effective stress concept, validated by a recent theoretical analysis by Coussy and Dangla (2002), was used to interpret the results. The predictions of a micromechanical model are compared with the experimental data in the elastic domain and on the failure criterion. The method highlights a simple way to relate the strength and stiffness of the soil to its negative pore water pressure.

Key words: effective stresses, small strains, suction, unsaturated soils (IGC: D6)

INTRODUCTION

Compacted soils find their most important applications in traditional civil engineering works: road and railways embankments, earth dams, etc. The small strain behaviour is of key importance to predict the performance of the earth structures themselves, but also to analyse the soil-structure interaction. Despite the wide use of compacted materials, their stress-strain pre-failure behaviour, particularly under unsaturated conditions, has not yet been deeply investigated. There is still a lack of experimental data concerning unsaturated soils in the small strains domain and a need for a more rational analysis based on the general framework of soil mechanical behaviour.

In this paper, results of an extensive laboratory study on a compacted silty sand are presented. The following aspects are considered:
—comparison of pre-failure behaviours, particularly in the very small strains domain, obtained by means of two different laboratory triaxial devices: a precision triaxial cell for large specimens at the Technical University of Lisbon—IST, and a precision triaxial cell for smaller specimens at the Ecole Centrale Paris,
—influence of negative pore water pressure, or suction, on the failure and pre-failure behaviours, noticeably in the range of very small strains,
—validation of an effective stress approach with a simple model to interpret the results.

Brull (1980), Wu et al. (1989), Quin et al. (1991), Kheirbek-Saoud (1994) have investigated the small strains behaviour of partially saturated soils. Picornell and Nazarian (1998) examined the effect of matrix suction on the small strains modulus of fine to coarse granular remoulded soils. Vinale et al. (1999) reported interesting data on Metramo silty sand from resonant column—torsional shear tests under suction-controlled conditions. Results obtained on specimens compacted to Modified Proctor optimum and wet of optimum water contents show that the initial shear modulus increases with suction up to a maximum value that depends on the net mean stress. The same trend was also observed by Gomes Correia et al. (1987) on Fontainebleau sand, and Wu et al. (1989). Vinale et al. (1999) remarked that wet compaction induces a weaker soil fabric than optimum and results in a strong reduction in the initial shear modulus of the Metramo silty sand, both in resonant and torsional shear tests.

Most of the data has been presented on the basis of the independent stresses approach, i.e. by considering separately the effect of the stress tensor and that of the negative pore water pressure or suction (Brull, 1980; Olof and Fredlund, 1998). However, Biarez et al. (1991), Wu et al. (1989), Kheirbek-Saoud (1994), Fleureau et al.
have shown that an effective stress approach could be used to take into account the effect of negative pore water pressure in the interpretation of data in the very small strains domain. Recently, Coussy and Dangla (2002), starting from thermodynamical considerations, confirmed the validity of the effective stress approach from a theoretical point of view as long as the behaviour of the unsaturated soil could be considered as elastic. On the other hand, Chateau and Dormieux (2002), using an homogenisation method, showed the possibility to take into account the role of the capillary forces between the grains in the failure criterion through an extension of the effective stress definition.

THEORY

Validity of the Effective Stress Approach in Elasticity

In recent years, Dangla and Coussy (1998), Coussy and Dangla (2002) have tried to extend Biot’s theory of poroelasticity used in saturated media to model the behaviour of unsaturated soils through an energy approach. This approach, which is briefly summarised below, yields a consistent general framework to formulate the constitutive relations.

For an element $d\Omega$ of saturated porous medium, external actions result in deformations and loss or gain of mass. Considering this open system, for reversible and infinitesimal transformations, the change in free energy ($\psi$) of the matter (both solid and liquid) contained in the element can be derived from the laws of thermodynamics (Coussy, 1995):

$$d\psi = \sigma_{ij}\cdot d\varepsilon_{ij} - s\cdot dT + \mu\cdot dm$$

(1)

where $\sigma_{ij}$ and $\varepsilon_{ij}$ are the stress and strain tensors, $\mu$ the chemical potential per unit mass of the fluid, $s$ the entropy, $T$ the absolute temperature and $m$ the mass.

Equation (1) implies that $\psi$ is a potential function of the state variables $[c_{ij}, m, T]$, which leads to the state equations of the porous medium:

$$\sigma_{ij} = \frac{\partial \psi}{\partial \varepsilon_{ij}}; \mu = \frac{\partial \psi}{\partial m}; s = -\frac{\partial \psi}{\partial T}.$$  

(2)

The previous developments are extended to unsaturated media by considering the two immiscible fluid phases filling the voids: liquid water ($w$), which is considered as a standard compressible fluid, and air ($a$), which is assumed to behave as an ideal gas. Then, the chemical potentials can be identified with the specific free enthalpies and Eqs. (1) and (2) can be written as (with the usual convention as to the sum of the terms with repeated indices):

$$d\psi_{ak} = \sigma_{ij}\cdot d\varepsilon_{ij} - s_{ak}\cdot dT + u_{aw}\cdot d\phi_{a} + u_{aw}\cdot d\phi_{w}$$

(3)

and:

$$\sigma_{ij} = \frac{\partial \psi_{ak}}{\partial \varepsilon_{ij}}; u_{aw} = \frac{\partial \psi_{ak}}{\partial \phi_{w}}; u_{aw} = \frac{\partial \psi_{ak}}{\partial \phi_{a}}; s_{ak} = -\frac{\partial \psi_{ak}}{\partial T},$$

(4)

with:

$$\psi_{ak} = \psi - \sum_{a \neq w} m_{a}\psi_{a} + s_{ak} = \sum_{a \neq w} m_{a}\psi_{a}$$

(5)

$a = \text{air}; w = \text{water}$.  

(5)

$\psi_{a}$ and $s_{a}$ represent the free energy and entropy of the fluids, $\psi_{ak}$ and $s_{ak}$, those of the skeleton, composed of the solid phase and of the interfaces (Dangla and Coussy, 1998). $u_{a}$ and $u_{aw}$ are, respectively, the pore air and pore water pressures, and $\phi_{a}$ and $\phi_{aw}$, the volumetric fractions of both fluids in the voids. In a soil, it can generally be assumed that deformations of the grains are negligible, compared to those of the skeleton. Then, the volumetric strain is equal to the variation of the porosity $\phi$:

$$\varepsilon = (\phi - \phi_{0}).$$

(6)

As $\phi = \phi_{0} + \phi_{aw}$, this relationship means that, among the 3 variables [$\varepsilon$, $\phi_{0}$, $\phi_{aw}$], only two are independent, for instance $\varepsilon$ and $\phi_{aw}$. When introduced in relation (3), it leads to:

$$d\psi_{ak} = [\sigma_{ij} + (u_{aw} + u_{aw})\delta_{ij}]d\varepsilon - \phi_{aw}dS_{a} - s_{ak}\cdot dT$$

(7)

where $u_{aw} = u_{aw} - u_{a}$ is the negative pore water pressure, or suction, of the soil, and $S_{a}$ and $S_{aw}$, the degrees of saturation, respectively in air and water.

When the material does not deform and is in isothermal conditions, the free energy of the skeleton is only due to the energy of the interfaces:

$$\phi_{0}\cdot \varepsilon_{w} = -\frac{\partial \psi_{ak}}{\partial S_{aw}} \Rightarrow \psi_{ak} = \phi_{0}\cdot U(S_{w}),$$

with $U(S_{aw}) = \int_{0}^{S_{aw}} u_{aw}(\cdot)\cdot dx$.

(8)

On the other hand, when the soil is deformable, $u_{aw}$ is also a function of the deformations and the relation (8) no longer stands. An additional hypothesis is necessary, e.g., to assume that the free energy of the skeleton is the sum of two terms: (i) the energy of the solid and of the interfaces in the reference saturated state ($U = 0$ when $S_{aw} = 1$), and (ii) the energy brought to the interfaces to decrease the saturation of the medium from 1 to its actual value. Then, it comes:

$$\psi_{ak}(\varepsilon_{ij}, S_{w}) = \phi_{0}\cdot U(\varepsilon_{iw}, S_{w}) + [\psi_{a}(\varepsilon_{ij})]_{S_{aw} = 1}.$$  

(9)

Now, $U$ is also a function of the deformations and $\psi_{a}$ is the free energy of the solids, independently from the interfaces. With this hypothesis, it is possible to express the constitutive equation of the solid components, as:

$$\sigma_{ij} + (u_{aw} + u_{aw})\cdot \delta_{ij} + \frac{\partial \psi_{ak}}{\partial \varepsilon_{ij}} = \frac{\partial (\phi_{0}\cdot U)}{\partial \varepsilon_{ij}} + \frac{\partial \psi_{ak}}{\partial \varepsilon_{ij}}.$$  

(10)

Introducing $p_{w} = u_{aw} + u_{aw} - (\partial (\phi_{0}\cdot U)/\partial \varepsilon_{ij})_{S_{aw} = 1}$, Eq. (10) can be written as:

$$\sigma_{ij} + p_{w}\cdot \delta_{ij} = \frac{\partial \psi_{ak}}{\partial \varepsilon_{ij}}.$$  

(11)

For a saturated medium under isothermal conditions,
as long as deformations are elastic, the relation between
the effective stress and strain tensors can be expressed as
a function of the elastic energy of the solid, \( w_e \):

\[
\sigma_{ij} = \frac{\partial w_e}{\partial \varepsilon_{ij}}.
\]

Identifying \( w_e \) and \( \psi_e \), leads to:

\[
\sigma_{ij} = p_e' \delta_{ij} + p_e' \delta_{ij}. \tag{13}
\]

\( p_e' \delta_{ij} \), which is termed the capillary stress, appears as an
isotropic tensor. Its expression is similar to that proposed
by Bishop (1959), with \( \chi \equiv S_m \), but with an additional term
corresponding to the work of the interfaces. Independently
from the expression of \( p_e' \), which can be obtained by
other methods, this approach validates the effective stress
concept when the behavior of the soil is elastic.

**Micromechanical Model**

Another approach to define effective stresses starts
from the expression of the intergranular forces between 2
particles in an idealized medium, at the microscopic level.
The method consists of calculating the force \( F_{\text{cap}} \) due to
water menisci between two grains of soil, modelled as
balls (Biarez et al., 1993). The balls are supposed to be
perfectly water-wettable, gravity is neglected; the menisci
are spherical tores, tangent to the particles. The pressure
in the air phase is atmospheric (\( u_a = 0 \)). Two cases are
considered:

(i) at low degrees of saturation, water is supposed to be
 discontinuous (\( k_u = 0 \)) and air continuous, water forms
menisci at the contact points between particles; the water
pressure inside the menisci is negative (\( u_{\text{in}} < u_a \)).
Experimentally, such conditions are observed for water
contents lower than the shrinkage limit. In that case, the
intergranular forces due to water are perpendicular to the
planes tangent to the particles at the contact points and
cannot, therefore, result in a rearrangement of the structure
or in a volume change. However, these forces contribute
to the strength of the medium. Considering two

spheres with the same diameter, the expression of the
attraction force resulting from a water meniscus ("capillary"
force) is:

\[
F_{\text{cap}} = S_{\text{men}} u_c \tag{14}
\]

where \( S_{\text{men}} \), the cross-section area of the meniscus in the
plane tangent to the spheres at the contact point (Fig. 1),
is directly related to the diameter of the balls and the curvature
radii of the menisci, i.e. to the negative pore
water pressure through Laplace’s law. The passage from
discontinuous to continuous medium is made by considering
regular arrangements of balls. Four types of
arrangements were considered, with densities ranging from
0.83 g/cm³ (tetrahedric) to 1.81 g/cm³ (dodecahedric)
(Fig. 1). In a representative elementary volume (REV),
the capillary stress in a direction is given by:

\[
\sum F_{\text{cap}} = \frac{S_{\text{REV}}}{S_{\text{REV}}} \tag{15}
\]

where \( \Sigma : F_{\text{cap}} \) is the vectorial sum of the capillary forces
acting in this direction and \( S_{\text{REV}} \), the cross-section area of the
REV in the plane normal to this direction. The variations of
the capillary stress \( p_e' \) with the negative pore
water pressure \( u_c \) can be derived from the model:

\[
p_e' = \frac{\pi \gamma}{2 g(e) R^2} \left[ 4 R + \frac{3 (3 \gamma - \sqrt{9 \gamma^2 + 8 \gamma R^2 u_c})}{u_c} \right]. \tag{16}
\]

The function of the void ratio, \( g(e) \), is derived from a
quadratic interpolation between the values of \( g(e) \) for the
4 considered arrangements:

\[
g(e) = 0.32e^2 + 4.06e + 0.11 \tag{17}
\]

where \( R \) is the radius of the balls, \( \gamma \), the surface tension of
the liquid. When the negative pore water pressure
becomes very large, \( p_e' \) tends towards a maximum value
given by:

---

**Fig. 1. Schematic view of the contact force due to water menisci between two isodiametral spheres and different types of studied regular arrangements**

<table>
<thead>
<tr>
<th>Tetraedric</th>
<th>Cubic</th>
<th>Octaedric</th>
<th>Dodecaedric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of contact points per ball:</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>( e = 1.95 )</td>
<td>( e = 0.91 )</td>
<td>( e = 0.47 )</td>
<td>( e = 0.35 )</td>
</tr>
<tr>
<td>( g(e) = \frac{16}{\sqrt{3}} \approx 9.24 )</td>
<td>( g(e) = 4 )</td>
<td>( g(e) = \frac{4}{\sqrt{3}} \approx 2.31 )</td>
<td>( g(e) = \sqrt{2} \approx 1.41 )</td>
</tr>
</tbody>
</table>
Table 1. Main properties of Perafita clayey sand

<table>
<thead>
<tr>
<th>w_i</th>
<th>w_p</th>
<th>I_p</th>
<th>&lt;2 μm</th>
<th>&lt;80 μm</th>
<th>w_pe</th>
<th>γ_i,\text{max}</th>
<th>G_s</th>
<th>M_p</th>
</tr>
</thead>
<tbody>
<tr>
<td>32.6</td>
<td>25.0</td>
<td>7.6</td>
<td>2.5</td>
<td>20</td>
<td>Standard: 17.6</td>
<td>Standard: 16.8</td>
<td>2.66</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Modified: 13.2</td>
<td>Modified: 18.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \rho_i^{\text{max}} = \frac{2 \pi \cdot \gamma}{g(\epsilon) \cdot R}. \]  

The concept of capillary stress can be extended to real soils, e.g. by deriving the "characteristic dimension" \( R \) from the experimental data (Biaræz et al., 1993, 1994).

(ii) at high degrees of saturation, the air forms isolated bubbles within the voids (\( k_s = 0 \)). The water phase is continuous and completely wets the grains. As there is no contact between the air bubbles and the solid grains, pore air pressure plays no part in the strength of the medium, but the presence of the bubbles makes it more compressible. In that case, Terzaghi's effective stress concept is valid and \( \rho_i \) is equal to \( u_c \). Many experimental validations of this model have been presented by the authors in past years (for instance, Biaræz et al., 1994; Fleureau et al., 1995, 1998, 2002c) that showed, in particular, its ability to model the failure criterion of unsaturated soils.

Recently, similar models have been developed (Chateau, 2001; Cho and Santamarina, 2001), that take into account both the role of the negative pore water pressure within the meniscus and the effect of a "tensile membrane" corresponding to the water-air interface. When the balls are in contact and the shape of the meniscus is approximated by a spherical tore, these approaches lead to expressions hardly different from that proposed by Biaræz et al. (1993).

**MATERIALS AND METHODS**

**Material and Specimens Preparation**

The tests were performed on a residual silty sand, hereafter called Perafita sand, made from weathered granite, which has been used as a building material for a road in the north of Portugal. Its main properties are shown in Table 1 and its grain size distribution in Fig. 2. The soil contains approximately 2% of particles smaller than 2 μm and nearly 20% smaller than 80 μm. The standard and modified Proctor curves are plotted on Fig. 3. Photos of the porous medium, made with a scanning electron microscope, are presented in Fig. 4. The grains appear as made of many assembled layers, the width of which is a few micrometers. This texture, which is characteristic of the residual nature of the soil, is similar enough to that of a clay, and gives it a liquid limit of nearly 33% and some original properties, especially with respect to saturation.

Drying and wetting tests were carried out on samples compacted to the Modified Proctor Optimum water content and nearly maximum density, using tensiometric plates and osmotic devices. Details of the experimental techniques are given, for instance, in Biaræz et al. (1988) or Verbrugge and Fleureau (2002). The drying and wetting paths, both starting from the compaction point, are shown in Fig. 5 in the void ratio and water content versus suction coordinate systems. When the soil is normally consolidated or slightly overconsolidated, these paths generally exhibit hysteresis (Fleureau et al., 1993). In this case, no cycle was performed but recent results from Fleureau et al. (2002b) indicate that, for a soil compacted under these conditions, the paths should be fairly reversible. The points corresponding to the specimens prepared for Ttu tests, i.e. compacted at different water contents and densities, have also been plotted on the graph. These points are quite apart from the drying and wetting paths, which cannot be used for the prediction of the negative pore water pressure of the compacted specimens.
The specimens preparation procedure is the same for all the tests: the soil is sieved to avoid the presence of coarse grains (maximum size: 4.75 mm), then it is mixed up with the right quantity of water; after that, it is placed in a sealed plastic bag for 24 hours to achieve uniform moisture conditions. Two mould sizes were used to prepare the specimens: 70 mm in diameter and 140 mm high for SPTT and TTu at ECP, and 160 mm in diameter and 320 mm high for LPTT at IST. The small specimens were compacted to the chosen dry density by a 24 N weight falling from a 305 mm height, with 3 layers and the required number of blows per layer. The large specimens were compacted in 7 layers by a vibrating hammer with a static weight of around 30 N and a 105 mm plate. The time of vibration was that necessary to obtain the chosen dry density. The compaction characteristics of the specimens used in the study are indicated in Tables 2 and 3, respectively for modulus and suction measurements, and the corresponding points are shown in Fig. 3.

**Experimental Devices and Test Methods**

Modulus measurements were carried out using two different triaxial cells with local strain measurements, one at Lisbon Technical University for tests on large speci-
mens (hereafter called LPTT) and the other one at Ecole Centrale Paris for tests on smaller specimens (SPTT). Suction measurements were performed at Ecole Centrale Paris in a conventional triaxial cell equipped with a semi-permeable porous stone (TTu).

Precision triaxial cell for large samples (LPTT)

The precision triaxial cell for tests on large samples, 320 mm high and 160 mm in diameter, is equipped with 3 LVDT for the measurement of axial strains and 3 LVDT for the measurement of radial strains. The frame and the armature rod of the vertical LVDT are fixed to target studs put in the specimen during compaction. The same type of studs is also used to measure radial strains, but the frame of the LVDT is supported by a ring mounted in the triaxial cell. The strain resolution is around $10^{-6}$ with a 16 bits $A/D$ converter. A standard pressure transducer and a sensitive load cell located inside the triaxial cell are used to measure the confining stress and axial force. The specimen is in contact with air at the atmospheric pressure through semi-permeable membranes at the top and at the bottom.

The test procedure uses the multistage technique. For each, confining pressure (26 and 52 kPa), the test starts with five unloading-reloading cycles of very small vertical stress amplitude. The amplitude of the cycles is controlled to be sure that the cycles are closed and linear, in order to evaluate the elastic Young’s modulus. Then, a deviatoric loading is applied up to an axial strain of about $5 \times 10^{-4}$ to obtain the decay curve of the secant Young’s modulus with vertical strain. The strain rate of the tests is approximately $1.4 \times 10^{-7} \text{ s}^{-1}$.

During the unloading process, very small unloading-reloading vertical stress cycles are performed at different steps. Figure 6 shows a typical result for a confining pressure of approximately 52 kPa. At the end of all these tests, an isotropic stress path is followed in order to evaluate the anisotropy of the soil.

Precision triaxial cell for small samples (SPTT)

The precision triaxial cell for tests on small samples, 140 mm high and 70 mm in diameter (SPTT), is fitted with two 80 mm-long Hall effect-based transducers (HET) for the measurement of axial strains and one HET for the measurement of radial strains. After a critical analysis of the possible sources of error, several improvements were made to the standard GDS equipment (Dufour-Laridan, 2002): very high precision voltage source (HP 3245A) and multimeter (HP 3458A) were used; the fixation of the transducers on the sample was improved; the ferrite magnets were replaced by Sm-Co magnets, which are more temperature-stable ($35.10^{-8} \text{ K}^{-1}$) and allowed an increase in the overall gain by a factor 10; finally, the tests were performed in temperature controlled conditions. The equipment was tested with a dummy brass specimen, with a resulting precision in axial strain as low as a few $10^{-7}$. On soil specimens, strains can be measured with a reasonable accuracy over a 5 $\times 10^{-6}$ range (Fleureau et al., 2001).

After compaction, the specimen is placed in the triaxial
cell and allowed to consolidate under the first isotropic stress during one day, then a deviator loading is applied up to $10^{-4}$ axial strain, in order not to damage the sample. Then, the confining pressure is increased to the second consolidation stress level, and so on for the other levels. For the last confining pressure, i.e. 79 kPa, the specimen stays in consolidation for 3 days (ageing) before the test. At the end of all the tests, a larger cycle is made, up to a few $10^{-3}$ axial strain. The drainage of the specimen remains open during the whole test. The strain rate is approximately $10^{-3}$ s$^{-1}$.

Triaxial cell with measurement of negative pore-water pressure (TTu)

In most cases, negative pore water pressure measurements were carried out separately from modulus measurements. A usual triaxial cell is equipped with a semi-permeable porous stone (with 1.5 MPa air entry pressure, from Soil Moisture), located in the pedestal, preventing the passage of air in the water circuit. The pressure in the water phase is measured by means of an absolute pressure transducer, while a relative pressure transducer, connected to the upper part of the specimen, is used to measure the pressure in the air phase. For water contents above the Proctor optimum water content, corresponding to water pore pressures higher than $-50$ kPa, the device is used as a tensiometer (i.e. with $u_w = 0$ and $u_a < 0$). For water contents below the optimum, the axis translation technique is used: a positive air pressure is applied to the sample, resulting in positive pore water pressure; in that way, cavitation problems in the measuring devices are avoided as long as the passage of air through the porous stone is prevented, which was the case in all these tests. Volume changes are derived from the water volume coming in or out of the cell, measured by the confining pressure controller. Considering the errors resulting from the expansion of the perspex envelope, the compression of the latex membrane or the compression of air bubbles during isotropic loading, this method was only used during the application of the stress deviator, when the confining pressure is constant.

The consolidated undrained tests with measurement of pore water pressure and control of air pressure were performed in two steps: (i) isotropic consolidation under stresses of 7, 14, 28, 52 and 79 kPa, (ii) compression under 79 kPa confining stress at constant strain rate ($\approx 10^{-4}$ s$^{-1}$), up to failure. During the consolidation phase, the changes in negative pore water pressure were recorded until equilibrium was reached.

In some compacted samples, the initial negative pore water pressure (under $\sigma_3 = 0$) was controlled by means of calibrated filter papers (Whatman #42). In this technique, after compaction, the filter paper is inserted between two protective paper layers within the specimen, which is then wrapped up in an airtight plastic foil; after 10 days, once the suction equilibrium between the paper and the sample is reached, the water content of the filter paper is measured with a high precision balance. The negative pore water pressure of the specimen is derived from a calibration curve.

![Fig. 7. Influence of isotropic stress and water content on negative pore water pressure](image)

1: $w = 3.74\%$, 2: $w = 6.25\%$, 3: $w = 7.5\%$, 4: $w = 8\%$, 5: $w = 9.5\%$, 6: $w = 10\%$, 7: $w = 10.4\%$, 8: $w = 14.1\%$, 9: $w = 16.1\%$, 10: $w = 17.9\%$, 11: $w = 18.3\%$.

**Normalisation of Data**

The results of SPTT and LPTT were all corrected to eliminate the effect of the different initial void ratios ($e_0$) of the tested specimens and a 'normalised' value corresponding to $e_{ref} = 0.5$ was derived from the expression of Iwasaki et al. (1978):

$$E_{(e=0.5)} = E_{(e_0)} \times \frac{f(0.5)}{f(e_0)},$$

with $f(e_0) = \frac{(2.17 - e_0)^2}{1 + e_0}$ and $f(0.5) = 1.859$. (19)

**EXPERIMENTAL RESULTS**

**Isotropic Compression Paths**

When the specimen is submitted to an isotropic compression stress, its negative pore water pressure $u_t$ tends to decrease, but the change is very dependent on the water content of the specimen (Fig. 7): when the soil is on the dry side of the optimum water content, the change is negligible; on the other hand, wetter specimens exhibit a decrease in suction with stress, all the more important as their water content gets higher. These results are consistent with volume change measurements showing that, even for a specimen compacted to the Proctor optimum water content and maximum density, the relative change in void ratio remains smaller than 1% when the confining stress increases from 0 to 78 kPa (Fleureau et al., 2002a).

When plotted against the initial volumetric water content of the specimens, $\Theta$, there is a good correlation between this parameter and the negative pore water pressure, independently from the initial water content and void ratio (Fig. 8). This can be due to the fact that the volumetric water content takes into account both the density of the soil and its degree of saturation. The curve also appears to be relatively independent from $p$, but it must be noted that the range of isotropic stresses investigated
remains limited and that a larger range would probably lead to an increased scatter of data. In that case, a major improvement of the correlation would be to include the effect of $p$ on the volume changes of the specimens, i.e. to plot suction versus actual (and not initial) volumetric water content.

\[
\begin{align*}
\text{Suction: } u_c &= 21089.6^{1.6} - 100 \\
\text{Coefficient of determination: } r^2 &= 0.96
\end{align*}
\]

Fig. 8. Influence of volumetric water content on negative pore water pressure for all the isotropic stresses

**Deviatoric Compression Paths**

At the end of the isotropic compressions, consolidated undrained triaxial tests, with measurement of the negative pore water pressure, were performed on the specimens under the same confining stress of 78 kPa. The test results are shown on Fig. 9. When the water content increases, there is a progressive change from brittle to plastic behaviour. For water contents lower than 8%, the stress-strain curves present a peak and the specimens seem strongly dilatant; at the other end, for water contents larger than 16%, perfectly plastic and slightly contractant behaviours are observed. However, in all the tests, the volumetric deformations remain very limited (1 to 1.5%). In the first case, the negative pore water pressure in the menisci creates strong bonds between the grains, that increase the strength of the material and prevent its deformation. In fact, localisations of deformations are frequently observed in the samples under those conditions, so that the measured increase in volume is probably not wholly representative of the real behaviour of the soil, but partly due to an apparent increase in the global volume after the formation of the discontinuity. A similar behaviour was observed by Verbrugge in a quasi-dry loam (Verbrugge and Fleureau, 2002). When the water content increases, the grains begin to be sur-

![Graphs showing deviatoric compression paths](image_url)

Fig. 9. Changes in stress deviator, volumetric strain and pore water pressure versus axial strain for consolidated undrained triaxial tests on unsaturated specimens of Peralta sand

1: $w = 3.74\%$, 2: $w = 6.25\%$, 3: $w = 7.5\%$, 4: $w = 8\%$, 5: $w = 9.5\%$, 6: $w = 10\%$, 7: $w = 10.4\%$, 8: $w = 14.1\%$, 9: $w = 16.1\%$, 10: $w = 17.9\%$, 11: $w = 18.3\%$. 
Fig. 10. Normalised Young's modulus versus total vertical stress for specimens of Perañita sand at different water contents and densities

Fig. 11. Influence of water content and vertical stress on the Poisson's ratio of Perañita silty sand

rounded by water, and the behaviour of the soil tends towards that of the saturated soil at the same void ratio.

The changes in pore water pressure $\Delta u = (\Delta u_s)$ during triaxial tests are presented only for water contents larger than 9.5%. The curves for the driest samples (between 3.7 and 9.5%) are not shown in this figure as the pressure remains constant during the tests, at the same value as at the end of the isotropic compression tests (refer to Fig. 7). The trend is the same for all the curves: (i) first, an increase in pressure, corresponding to the contractant behaviour already mentioned, (ii) then a reduction of pressure, characteristic of dilatant behaviour; this second phase is observed even in the nearly saturated specimens which exhibit an overconsolidated behaviour, corresponding to what is usually observed in saturated compacted specimens. In all the cases, the changes in pressure are small ($\pm 20$ kPa or less), as a consequence of the low compressibility of the sand.

Very Small Strains Behaviour

The very small strains properties of Perañita sand were measured in separate tests from those used to measure the large strains properties, but approximately under the same water content and void ratio conditions. Two tests (at $w = 6.5$ and 13.2%) were performed under several confining stresses and stress deviators, while the others were performed under isotropic stress states only. The influence of several factors has been investigated.

Influence of water content on maximum modulus

Figure 10 shows that, for the unsaturated specimens, the variations of normalised modulus versus total vertical stress approximately follow a power law, with an exponent $n = 0.35 - 0.40$. On the other hand, the lines for the dry and quasi-saturated specimens are nearly superimposed, with a stiffer slope ($n = 0.68$). For the same void ratio and under the same vertical stress, there is a general increase in the modulus when the water content decreases, as long as the water content is strictly larger than 0.

Poisson's ratio versus water content

In most of the tests, the radial deformations were measured to derive the value of Poisson's ratio. As an example, the results of two tests, at water contents of 6.5 and 13.2%, are presented in Fig. 11. The scatter of data is rather large, mainly because radial strains measurements are often less accurate than axial strains measurements, due to the presence of the latex membrane. However, nearly all the values are comprised between 0.20 and 0.26, with a slight trend to increase with the confining stress, which confirms the results of previous investigators. There is a small shift between the curves for $w = 6.5\%$ and $w = 13.2\%$, but the difference in the values of Poisson's ratio is well within the scatter of data, in spite of the wide changes in the water content and degree of saturation of the soil (from $S = 40$ to $S = 84\%$).

Influence of loading rate on maximum modulus

Tests have been made at different loading rates to examine the possible viscosity of the material (Fleureau et al., 2001; Dufour-Laridan, 2002). Variations of 1 to 1000, here expressed as loading frequencies ($10^3$, $2\times10^3$, $10^3$ and 1 Hz), have been carried out. The results, plotted on Fig. 12, show that there is only a slight increase in the modulus with the loading rate. Several investigators have shown that the loading rate has little effect on the maximum modulus of stiff materials, but that this influence increases in the case of silty sands and clays (Bray et al., 1999; Tatsuoka et al., 1999; Santucci de Magistris et al., 1999). In the case of Perañita sand, the limited effect of the loading rate is probably due to the partial saturation of the soil, which increases its stiffness.

Influence of confining stress and water content on the decay curve

For the range of pressures investigated (26 to 78 kPa), the confining stress does not seem to have any influence on the decay curve of the secant modulus versus strain, either at the water content of 6.5% or at $w = 13.2\%$ (Fig. 13). A similar observation has been made in the case of resonant column tests performed at IST on the same...
soil (Fleureau et al., 2002a). The elastic limit of Perafita sand derived from these tests is very low, corresponding to a strain of approximately $10^{-6}$, while a larger value ($5 \times 10^{-5}$) has been observed in resonant column tests, probably because, in the case of triaxial tests, the elastic limit is measured during the first loading-unloading cycle while, in resonant column tests, its value is obtained after a large number of cycles during which the soil may experience hardening.

The initial water content of the specimens does not appear to influence the decay curve of the secant modulus either (Fig. 14). Here, the range of water contents is very large, from 6.5 to 18%, covering all the states from nearly dry to saturated.

**INTERPRETATION OF THE RESULTS USING EFFECTIVE STRESSES**

The interpretation of the triaxial tests is based on the assumption that the effective stress concept is valid for the determination both of the failure criterion and of the elastic moduli $E_0$, and that there is a unique relationship between $p'$ and $q$ in the first case, log ($\sigma'$) and log ($E_0$) in the second, whether it is for saturated or unsaturated specimens. When the soil is quasi-saturated, $p'$ is given by Terzaghi's expression; in the case of unsaturated soils, the values of $p'$ are derived from the micromechanical model proposed by Biarez et al. (1993), using Eq. (16), provided the "characteristic dimension" $R$ has been determined.

—Considering first the tests on quasi-saturated specimens (i.e. with $S > 75\%$), it is possible to derive the maximum strength criterion from the large strains triaxial tests, with the equation:

$$q = \eta_{\text{max}} (p' + p'_0),$$

where $p'_0 = -u_a$. (20)

—From that, the value of the "characteristic dimension" $R$ of the micromechanical model is chosen so that the final points of the tests on the unsaturated samples (with $S < 50\%$) are located near the previously determined maximum strength criterion. In the case of the triaxial tests on Perafita sand, this leads to the function $p'_0(u_a)$ shown in Fig. 15. The "experimental" values of $p'_0$ are derived from:

$$p'_{\text{exp}} = p' - (p - u_a) = (q/\eta_{\text{max}}) - (p - u_a).$$

(21)

—The same approach is used to interpret the results of the small strains measurements, with the same value of the parameter $R$ in the case of the unsaturated specimens.

**Interpretation of Large Strains Triaxial Tests**

The stress paths for the different tests are represented in Fig. 16 versus the effective mean stress $p'$. For the quasi-saturated specimens, the slope of the maximum strength criterion is $\eta_{\text{max}} = 1.5$. With a characteristic dimension $R$ equal to 1.7 $\mu$m, the paths of the unsaturated specimens also finish near the same maximum strength criterion as the quasi-saturated tests. The shape of the stress paths is rather that of drained specimens, especially for the driest samples corresponding to the largest values of $p'_0$, and therefore of $p'$. The reason is the fact that the negative pore water pressure remains constant under these condi-
tions as the deformations of the samples are small. At the other end, for the wettest specimens (at low suction values), the behaviour is that of overconsolidated specimens. For some of the driest specimens, the localisation of deformations already mentioned results in premature brittle failure of the specimens, which do not reach perfect plasticity.

**Interpretation of Small Strains Modulus Measurements**

The normalised values of the modulus for the different tests, shown in Fig. 10, have been plotted in Fig. 17 versus the effective vertical stress: \( \sigma'_v = \sigma_v + \rho'_c \) (\( u_c = 0 \)). To interpret the results, the values of the negative pore water pressure were derived from correlations based on the measurements made at different water contents, void ratios and confining stresses. For quasi-saturated specimens, \( \rho'_c = -u_c \) and for dry specimens, \( \rho'_c = 0 \). For unsaturated specimens, the values of \( \rho'_c \) were derived from the micromechanical model, using the same “characteristic dimension” \( R = 1.7 \mu m \) as for the other triaxial tests and the function presented in Fig. 15. Expressed against the effective stress, all the results (from \( w = 0\% \) to \( w = 18.3\% \)) are correctly located near the regression line of the dry and quasi-saturated specimens, with the equation:

\[
E_{vo} = 200(\sigma'_v/p_v)^{0.66}.
\]  

Considering that the results were obtained under very different conditions, using different experimental devices, on samples of different sizes, with correlation-based values of pore water pressure and for deformations smaller than 10^{-3}, the scatter of data appears rather small.

**CONCLUSIONS**

A significant number of tests was performed on Perafita silty sand, under many different conditions of water content and density, isotropic and deviatoric stress, using non standard devices for the measurement of very small strains or negative pore water pressures. The data from all these tests provide a fairly wide picture of the properties of the sand, from nearly dry to nearly saturated, from the elastic to the perfectly plastic domain.

The use of the effective stress concept allows a unified interpretation of the data both in elasticity and on the failure criterion. The micromechanical model, which features a single parameter \( R \), leads to a very good agreement with the experimental results for the determination of both the \( q(p') \) and \( E(p') \) relationships and is a very simple way of predicting the strength and stiffness of a soil when its negative pore water pressure is known.

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NOTATIONS

- $D$: Diameter of specimen
- $e$: Void ratio
- $f$: frequency
- $E$, $E_0$: Secant modulus, maximum Young's modulus
- $F_C$: Capillary force due to a meniscus
- $g(e)$: Empiric function of void ratio
- $G_s$: Specific weight of solid grains
- $H$: Height of specimen
- $I_p$: Plasticity index
- $k_s$, $k_w$: Air permeability, water permeability
- $m$: Mass
- $M_{p}$: Slope of perfect plasticity criterion in $[p', q]$ plane
- $p$: Mean stress
- $p_a$: Atmospheric pressure
- $p_c$: Capillary stress
- $q$: Stress deviator
- $R$: Radius of balls
- $s$: Entropy
- $S_a$, $S_b$, or $S_c$: Degree of saturation in air, degree of saturation in water
- $S_{max}$, $S_{rev}$: Cross section area of a meniscus, of a Representative Elementary Volume
- $T$: Absolute temperature
- $u_s$, $u_w$: Pore air pressure, pore water pressure
- $u_c = u_s - u_w$: Negative pore water pressure, or suction
- $U$: Energy of deformation of interfaces
- $w$: Water content
- $w_L$, $w_p$: Liquid limit, plastic limit
- $w_{opt}$: Proctor optimum water content
- $w_S$: Elastic energy of the solid
- $\epsilon$: Bishop's effective stress parameter
- $L_0$: Kornecker symbol
- $\epsilon_0$: Strain tensor
- $\phi$: Porosity
- $\phi_a$, $\phi_c$: Volumetric fraction of air, water, in the pores
- $\gamma$: Surface tension of water
- $\gamma_{max}$: Proctor maximum unit weight
- $n_{max}$: Maximum value of $q/p$
- $\mu$: Chemical potential per unit mass
- $\nu$: Poisson's ratio
- $\sigma_{ij}$: Total or effective stress tensor
- $\sigma_{ij}'$: Total or effective vertical stress
- $\psi$: Free energy

REFERENCES


