A CONSTITUTIVE MODEL FOR CRUSHED GRANULAR AGGREGATES WHICH INCLUDES SUCTION EFFECTS

C. CHÁVEZⁱ⁾ and E. E. ALONSOⁱⁱ⁾

ABSTRACT

The mechanical response of rockfill is largely dominated by particle breakage, which is controlled by stress level and the prevailing relative humidity (RH) in the large rockfill voids. The paper presents a model for rockfill behaviour under triaxial stress states, which is based on: (a) existing knowledge on rockfill compressibility, (b) experimental and theoretical developments on sand behaviour and (c) a series of suction-controlled triaxial tests performed in a new large-diameter triaxial cell, in which samples of gravel were tested under two relative humidities: 36% (“dry”) and 100% (saturated). Changes in grain size distribution were measured in all the tests performed. The model is formulated within the framework of work hardening plasticity and critical state concepts. Specimen stiffness and limiting conditions depend on the accumulated plastic work and on the prevailing RH. The model reproduces in a satisfactory way the set of triaxial tests performed.

Key words: elastoplastic model, relative humidity, rockfill, suction, triaxial test (IGC: D6)

INTRODUCTION

The behaviour of rockfill under laboratory-controlled conditions has been reported by several authors (Lee and Farhoomand, 1967; Marachi et al., 1969; Nobari and Duncan, 1972; Marsal, 1973; Oldecop and Alonso, 2001). Particle breakage has been identified as a fundamental factor to explain the observed behaviour. More specifically, particle breakage was the reason behind the qualitative differences observed between the behaviour of sand (at low and moderate stress levels) and rockfill. Particle breakage in rockfill depends on the strength of individual particles, the grain size distribution, the stress level and the relative humidity prevailing at the rockfill voids. Oldecop and Alonso (2001) have shown that the last factor (RH at the rockfill voids) explains the collapse behaviour observed in rockfill structures when they undergo some total or partial wetting. It was also shown that relative humidity controlled the observed macroscopic stiffness of rockfill when loaded under confining stresses.

The issue of the effect of RH on rockfill performance is by no means an academic one. In fact, due to the very large rockfill permeability, water never fills the large rockfill voids unless structures are submerged. Environmental changes and rainfall in particular modify relative humidity. The extreme case of rockfill flooding is also of concern in rockfill structures which become inundated, such as the

upstream shells of rockfill of zoned earth and rockfill dams.

Relative Humidity is a measure of the concentration of water vapour in the atmosphere. In our case, the “atmosphere” refers to the void space. RH is the ratio of the pressure of water vapour actually present in the air to the vapour pressure when the air is saturated with water vapour (u_v/u_{v0}). There is an alternative measure of water vapour concentration: the total suction of water in equilibrium with a given Relative Humidity. The psychrometric law:

$$s_t = -(RT\rho_w/M_w) \ln(u_v/u_{v0}) \quad (1)$$

provides an easy transfer from Relative Humidity to total suction, s_t . In this equation, R is the gas constant, T the absolute temperature of the reference system, M_w the molecular mass of water and ρ_w the density of water at the reference temperature. Total suction is preferred as a measure of the energy of the water since water transfer laws are, in general, expressed in terms of gradients of suction.

Sand particle breakage may be induced at high confining stresses. Therefore, insights into the behaviour of rockfill may be also derived from testing programs in which sand was subjected to stresses capable of inducing particle breakage. Tests on sand at high stresses have been reported by Been et al. (1991), Yamamuro and Lade (1996) and Biarez and Hicher (1997) among others. The

ⁱ⁾ Professor, Department of Geotechnical Engineering and Geosciences, Universidad Politécnica de Cataluña, Gran Capità s/n. Módulo D-2, Barcelona 08034 Spain (eduardo.alonso@upc.es).

ⁱⁱ⁾ ditto.

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1. Triaxial chamber.	10. Vacuum gauge.
2. LVDT for axial measure.	11. Hydrometer.
3. Load cell.	12. Pressure transducer.
4. DPT.	13. Power supply.
5. Diametral Measure Transducer (DMT).	14. Pressure regulator.
6. Window.	15. Air bomb.
7. Car to move the cell.	16. Pot with salt solution.
8. Load frame.	17. Hygrometer sensor.
9. Data acquisition system.	

Fig. 1. Scheme of the triaxial equipment developed

transition from a “normal” regime (no particle breakage) to the breaking particle regime was observed in the tests mentioned. Been et al. (1991) investigated the critical state of sands for a wide range of confining stresses and proposed a bilinear critical state (e vs. $\log p'$) line. The angular point marked the beginning of particle breakage. Similar results were reported by Biarez and Hicher (1997) for Hostun sand at different densities.

This paper presents an elastoplastic constitutive law for rockfill, which takes into account the phenomena outlined. The aim was to develop a relatively simple model, capable however of capturing the essential physical phenomena behind observed rockfill behaviour. It is primarily based on some triaxial tests on a gravel-like material performed in a relative humidity control triaxial device recently developed. The model benefits also from some modelling work on sand reported by Wood et al. (1994) and Wan and Guo (1998). The experimental program will be first presented with the purpose of discussing

some key features of rockfill behaviour under deviatoric stress. Rockfill behaviour under compression (oedometer paths) was discussed in detail in Oldecop and Alonso (2001). On the basis of the observed behaviour, an elastoplastic work hardening model for rockfill, which incorporates the effect of water suction, will be proposed. Finally a comparison of model calculations and observed stress-strain behaviour will be given.

EXPERIMENTAL PROGRAM

Triaxial Apparatus

A new triaxial apparatus, which can test samples up to 25 cm in diameter and 50 cm in height under suction controlled conditions was developed and installed at the Soil Mechanics Laboratory of UPC. The apparatus follows the earlier design and construction of a large diameter suction-controlled oedometer cell. Figures 1 and 2 show a schematic layout of the system and a photograph

Fig. 2. View of the triaxial apparatus

of the equipment during one of the tests reported here. Only a brief account of the apparatus will be given.

The cell has a double-walled chamber to allow for a direct measurement of sample volume changes. For redundancy, horizontal strains can also be measured by means of specially developed sensors for diameter changes. Vertical load on the sample and axial displacements are also recorded by means of a load cell and LVDTs located inside the cell. The LVDTs measure the displacement of the upper polished stainless steel loading plate in direct contact with the sample. The nature of the direct contact between the platten and the gravel is illustrated in Fig. 3(a): it implies a loss of the granular structure of the specimen. The number of contacts of the first layer of the sample is reduced and therefore these particles receive larger concentrated contact forces than the rest of the sample. As a result, they break more easily. It was found (Oldecop, 2000) that this effect introduced significant changes in sample compressibility in oedometer tests which had a similar platten-sample contact. The procedure adopted to reduce these effects is shown in Fig. 3(b). A thin protection layer of small gravel particles fills the voids between the larger gravel particles of the aggregate and the platten without changing the overall vertical dimension of the specimen. At the bottom, a thin layer of fine gravel was uniformly extended before placing on the first layer of the coarse aggregate. The additional weight of material was always the same in all tests performed and was later discounted when the grain size distribution analysis was performed at the end of tests (the correction was negligible, however). In the case of oedometer tests this procedure increased the sample stiff-

Fig. 3. Particle breakage due to a discontinuity of the granular structure at the edges of the specimen: a) Tests without protection layer and b) Tests with protection layer (Oldecop, 2000)

ness and drastically reduced the dispersion of the curves of compressibility. No bedding errors associated with the penetration of grains into membranes or the compressibility of lubricating grease, reported in other cases (Dendani et al., 1988), are expected here because of the direct contact of the steel platens and the sample.

Total suction is controlled by means of a closed loop circuit, which induces a vapour flow through the sample. Air relative humidity is imposed by circulating the flowing air through acid or salt solutions at a predetermined concentration (Oldecop, 2000). Suction changes are induced by changing the solute concentration. In the tests reported here on “dry” samples, the low relative humidity was controlled with a solution of Sodium hydroxide, $\text{Na}(\text{OH})_2$. RH was also measured by means of a hygrometer sensor located in the air circuit. Therefore, two independent indications of the imposed RH are available. Errors in the determination of RH, due to the precision of both techniques are similar: 1% to 2% of the measured value (Oldecop, 2000). In terms of suction, in view of Eq. (1), the error is high (20%–30%) for the low suction range (3 to 10 MPa and smaller (2%–3%) for a high suction range (100–300 MPa). However, mechanical effects induced by suction on granular aggregates tend to depend on the logarithm of suction and therefore a limited mechanical effect of the error in the determination of RH should be expected.

Only tests at constant suction will be reported here. The system has a number of additional features, which have been indicated in Fig. 1. Confining stresses as high as 2.5 MPa can be applied to samples. The vertical stress may reach a maximum value of 18.5 MPa.

Material Tested

Crushed Cambrian slate, which was selected as the rock-fill material for the Jiloca dam design (Teruel, Spain), was used in all tests. This material was also used by

Oldecop and Alonso (2001). Basic properties of the rock material are: uniaxial compression strength 20.5 MPa (14.2–31.9 MPa), solid specific gravity 2.754, porosity 8% (6.3–11.8%). The material obtained in a quarry was further crushed in the laboratory and sieved. The maximum particle size was fixed at 40 mm. Particles may be described as irregular prisms with sharp edges with a dominant dimension along the direction of schistosity. The raw material used to compact samples had a low coefficient of uniformity ($U_c = 2.9$).

Samples, 25 cm in diameter and 50 cm in height, were directly compacted in the triaxial cell base, with a 3 mm thick membrane of neoprene placed into a split mould. A tamping method was applied to 6 layers, and 117 blows were applied per layer with the help of a Marshall Hammer. Compaction energy varied among samples between 600 to 700 joule/litre, which is close to the nominal compaction energy of the Normal Proctor Test. The mean initial void ratio at the beginning of tests was 0.628 ± 0.03 . The mould was removed after compaction and the membrane provided a small confinement to maintain the sample geometry. Then the measuring and control devices were mounted and the cell and auxiliary devices were assembled for testing.

Drained Triaxial Tests

The first testing campaign, reported here, involved two series of tests. The first series was performed under fully saturated conditions. Once compacted, samples were confined under 0.1, 0.3, 0.5 and 0.8 MPa and then sheared under constant lateral stress, σ_3 . In a second series, samples compacted in a similar manner were maintained at a constant low relative humidity ($RH = 36\%$, equivalent to a total suction $s = 142$ MPa). Similar stress paths were then applied to the dry samples. Tests on saturated samples are named S1, S3, S5 and S8. The number refers to the intensity of the confining stress. In a similar manner, the three tests performed on “dry” samples are marked as D1, D3, D5 and D8. Grain size distributions were measured for the following states: original material, after compaction, and after the performance of the triaxial tests. They are indicated in Figs. 4(a) and (b) for saturated and “dry” conditions respectively.

Measured deviatoric ($q = \sigma_1 - \sigma_3$) vs. axial strain (ϵ_1) response and volumetric deformations vs. axial strain are plotted in Fig. 5 for the saturated samples. Unloading-reloading cycles were imposed at some strain levels to estimate the elastic stiffness of the material. Samples were taken to large deformations in an attempt to approximate critical state conditions.

The saturated sample tested under $\sigma_3 = 0.1$ MPa (Fig. 5) shows a marked dilatant behaviour from the beginning of the test. The application of higher confining loads delays the development of dilatancy to higher vertical strains (7% for $\sigma_3 = 0.3$ MPa; 12% for $\sigma_3 = 0.5$ MPa). The sample tested under $\sigma_3 = 0.8$ MPa shows a compressive behaviour along the test. The application of suction (Fig. 6) increases the measured strength and enhances the dilatant behaviour of the gravel.

Fig. 4. Grain size distributions of specimens tested: a) Saturated samples and b) Samples at $RH = 36\%$

The effect of applied suction is now clearly observed comparing the Figs. 5 and 6, plotted on the same scale. It is also interesting to note that the deviatoric behaviour for saturated and “dry” conditions is similar at low strains.

Significant dilatancy rates were observed at the end of most of the tests performed, as shown in Figs. 5 and 6. In all cases, samples deformed in a barrel-like shape. The question of the appropriateness of critical state for these gravel samples is therefore difficult to elucidate on the basis of the triaxial tests performed. However, the concept provides a useful framework to interpret the data and to formulate the constitutive model.

Analysis

Vectors of incremental plastic strain ($\dot{\epsilon}_q^p = 2/3(\dot{\epsilon}_1^p - \dot{\epsilon}_3^p)$; $\dot{\epsilon}_p^p = \dot{\epsilon}_1^p + 2\dot{\epsilon}_3^p$) have been plotted along the stress paths (Figs. 7 and 8). Plastic strains were derived from total strains by subtracting the calculated elastic strains for the considered stress increment. Elastic parameters (E , ν) were derived from the response of the sample in the unloading-reloading paths imposed in tests, as shown in Figs. 5 and 6. Plastic strains develop at the start of the deviatoric stress path. Lines of constant ratio $\eta = q/p$ ($p = 1/3(\sigma_1 + 2\sigma_3)$; $q = \sigma_1 - \sigma_3$) have also been plotted in Figs. 7 and 8. It appears that plastic strains may be linked

Fig. 5. Results from triaxial tests: Saturated specimens

Fig. 6. Results from triaxial tests: Specimens maintained at RH = 36%

to changes in stress ratio η , as it is observed in sands. Yield loci described by $q/p = \eta$ seem therefore to be a natural choice to describe deviatoric-induced yield. The plots in Figs. 5 and 6 also indicate the curved nature of the strength envelope. In general, all the samples tested behaved in a ductile manner.

Yield along K_0 lines was discussed in a previous paper (Oldecop and Alonso, 2001), where samples of an identical material, compacted at the same initial density were tested in a suction controlled oedometer cell. Yield was observed at the very beginning of deformation in the oedometer tests performed. Compaction does not result in a well defined initial yield locus and so an elastic state could be defined in the stress space. It was checked that, once compacted, any subsequent loading immediately induces plastic strains. The implication of this previous research is also that the yield locus cannot be represented only by the deviatoric lines ($\eta = \text{constant}$). A "cap" that accounts for yield in compression is also necessary.

The evolution of the specific volume ($v = 1 + e$) along the stress path is plotted in Fig. 9 as a function of the mean confining stress p (natural scale) for the saturated and dry specimens respectively. This plot is useful to

identify a critical state line. It was stressed before, however, that the final points of the paths represented do not correspond precisely to critical state conditions since dilatancy was still being measured. The plot, however, provides an interesting indication of the state of the samples tested and the development of dilatational responses. The initial state (in terms of initial void ratio and confining stress) of samples lies, in all tests performed, within the dilatant zone. An estimation of the expected critical state conditions which will be used later in the model calculations, is plotted in both figures. If dilation rates are small at the end of the test the critical state line is plotted close to the state of the specimen. This is the case for the tests performed at the highest confining stress (0.8 MPa). These tests provide a reliable point for the position of the CSL. The slope of the lines was further adjusted on the basis of the comparison of model predictions and measured response of samples. A comparison of the two sets of specimens clearly indicates that suction contributes significantly to maintain higher critical state void ratios for a given confining stress. The conclusion is that a critical state (v vs. p) relationship strongly depends on the prevailing suction (or RH).

Fig. 7. Incremental plastic strain vectors along stress paths: Tests on saturated specimens

Fig. 8. Incremental plastic strain vectors along stress paths: Tests on "dry" specimens (RH = 36%)

Values of the "critical state" ratio $\eta_c = M_c$ at the end of the tests performed can be derived for the saturated and dry samples. They are plotted in Fig. 10 as a function of the confining stress σ_3 . A drop of M_c with confining stress is well marked for the saturated and dry samples. The

Fig. 9. Evolution of v during the tests and estimated position of critical state lines

Fig. 10. Variation of critical state parameter M_c with σ_3 for the two series of tests performed

data shows the significant effect of applied suction in increasing the M_c values.

Wood et al. (1994) proposed a hyperbolic hardening relationship to relate the position of the deviatoric yield locus to plastic deviatoric strains. This hardening relationship is written:

$$\frac{\eta_s}{M_p} = \frac{\epsilon_q^p}{b + \epsilon_q^p} \quad (2)$$

Where $\epsilon_q^p = 2/3(\epsilon_1^p - \epsilon_3^p)$, M_p is the available peak stress ratio and b is a parameter linked to the initial stiffness of the soil. M_p was then related to the current void ratio of the soil:

$$M_p = M_c(1 - k\psi) \quad (3)$$

where $\psi = v - v_c$ is the distance between the critical state specific volume v_c and the current value v (Been and Jefferies, 1985). M_c is the critical state stress ratio and k is a constant. In this way, strain-softening effects could be reproduced in a simple manner. This idea is retained here

Fig. 11. Variation of parameter b with σ_3 for the two series of tests performed

and will be used for the modelling of rockfill, although strain softening effects are limited in the samples tested.

In the case of a rockfill, M_c is no longer a constant since it was found to depend on confining stress and suction. If the empirical relationships given by Eqs. (2) and (3) are accepted, parameters M_c and b may also be derived for the tests performed by means of fitting procedures. This is facilitated by the structure of the simple hyperbolic law (2). Derived b values are plotted in Fig. 11 for dry and saturated conditions for the confining stresses used in the tests. Parameter b is shown to depend on confining stress and on the relative humidity.

Parameter b is proportional to the inverse of the normalized initial tangent stiffness of the curve fitting the model (Eq. (2)) to actual tests results. It provides an estimation of initial stiffness even if no measurements at very low strains are performed. The actual deviatoric stress strain curves indicate that the initial stiffness increases with confining stress. However, in the case of a gravel, the stiffness is controlled by two opposing mechanisms: the "pure" effect of increasing confining stress and the progressive damage, associated with particle breakage, due to sample deformation under increasing confining stress. The normalization by p , introduced in Eq. (2), is a procedure to remove " p effects", provided that stiffness and confining stress are linearly related. Figure 11 shows, therefore, that breakage associated with confining stress reduces the initial stiffness. This result was also found by Yamamuro and Lade (1996) testing sands at very high stresses. It is also shown that suction increases the normalized initial stiffness for all the applied confining stresses.

Changes in the critical stress ratio (M_c) and in the compliance parameter (b) with the confining stress σ_3 , as shown in the Figs. 10 and 11, are mainly attributed to particle breakage. However, the confining stress is obviously not a good indicator of conditions leading to particle breakage. Instead, particle breakage may be

Fig. 12. Breakage factors obtained after the tests

associated with some effective internal work induced by the applied stress. Plastic work is the first choice. It is believed, however, that plastic work accounts also for irreversible phenomena not necessarily linked to particle breakage. Particle rearrangement, in particular, leads to a component of plastic work which should not be involved in particle breakage.

Two components may be distinguished in the plastic work: volumetric and deviatoric. In the series of suction controlled oedometer test reported by Oldecop and Alonso (2001), a threshold vertical stress (σ_y) was identified which marked the transition between particle rearrangement and particle breakage. The plastic work performed during the initial part of the loading (for $\sigma_y < \sigma_y$) may be identified as an essentially spherical component of the plastic work which does not induce particle breakage. It is recognized that oedometer and triaxial tests are different with respect to particle breakage but Yamamuro and Lade (1996) have also shown that breakage in sands subjected to triaxial loading only started beyond a given confining stress. An examination of grain size distributions in Fig. 4 shows that the triaxial test performed on the "dry" sample under the confining stress of 0.1 MPa did not modify the "as compacted" grain size distribution. These findings lead to the proposal of a threshold work, W_R^P , below which there is no particle breakage.

Breakage factors as proposed by Marsal (1973) and Hardin (1985) have been derived from the grain size distribution curves given in Fig. 4 and are plotted in terms of confining stress in Fig. 12. The initial point, for $\sigma_3 = 0$, corresponds to the state after compaction. The plot shows that the dry sample D1 has not experienced a significant particle breakage. However, all the saturated samples modify their grain size distribution during testing. Breakage intensity will depend on confining stress, Relative Humidity and the type of rock particle. The idea of introducing a threshold work will be retained in general even if, for some rocks and RH conditions, this threshold is a low value.

Based on these considerations, it is proposed that M_c

Fig. 13. Variation of M_c with effective plastic work, W_E^P

will evolve as an “effective” plastic work, W_E^P increases. The effective plastic work is defined as the excess of total plastic work W_T^P over the plastic work associated with particle rearrangement, W_R^P :

$$W_E^P = W_T^P - W_R^P \quad (4)$$

The variation of M_c with the effective plastic work for the “dry” samples, taking as a threshold rearrangement-type of work, the work computed for D1, has been plotted in Fig. 13. The plastic work is expressed in units of work per unit volume of specimen (stress).

For the saturated specimens, the grain size distributions in Fig. 4 indicate that particle breakage was already present for the specimen tested at the lower confinement stress ($\sigma_3 = 0.1$ MPa). The threshold plastic work cannot be reliably estimated in this case and it has been taken as zero. Therefore M_c is plotted against the total plastic work in Fig. 13 for the saturated samples.

Parameter b has also been related to the effective plastic work dissipated during the test. Note that b incorporates information of the stress-strain curve because it is found by adjusting the proposed hyperbolic model (Eq. (2)) to the experimental data. The computed values are given in Fig. 14. The influence of RH on parameter b seems to be small and it has been disregarded in the model presented later.

Wan and Guo (1998) examined the effect of initial density and stress ratio on the volumetric behaviour of sands. Figure 15 reproduces their proposal, which is a modification of the original Rowe dilatancy theory. In this plot, the dilatancy factor $D = 1 - d\varepsilon_v^p/d\varepsilon_1^p$ is plotted as the stress ratio $R = \sigma_1/\sigma_3$ increases. Paths for both, dense or loose sands, generally start at a value $D < 1$, which reflects the initial contracting tendency of granular soils when subjected to shearing. Dense sands evolve towards a dilatant behaviour but eventually they reach a critical

Fig. 14. Variation of parameter b with W_E^P for the two series of tests performed

Fig. 15. Modified Rowe's equation, Wan and Guo 1998

state ($D = 1$). In loose materials, contraction accumulates until steady state critical conditions are met. This background is used to interpret the dilatancy plots shown in Figs. 16 and 17. Trends are similar in both graphs. However, dry samples exhibit a stronger dilatant behaviour.

Compare, for instance, the curves for $\sigma_3 = 0.3$ MPa for both states (see Figs. 16 and 17). The dry sample starts at an expansion rate and maintains higher dilation rates for all stress ratios. Similar results are obtained when comparing the D parameter at a common confining stress. The final state of all samples is, however, far from the theoretical value $D = 1$, which should be found at high stress ratios. This difficulty to find critical state conditions in triaxial testing of very coarse materials has already been pointed out.

Constitutive Model

For model development it will be assumed that the rockfill material tends towards a critical state when sub-

Fig. 16. Dilatancy observed in saturated samples tested at different confining stresses

Fig. 17. Dilatancy observed in dry samples tested at different confining stresses

jected to shear stresses. This hypothetical state has been questioned for granular materials but it remains a useful concept for modelling purposes. Based on the previous discussion, a work hardening dependence will be introduced in some of the model parameters which describe the effect of particle breakage.

Yield Surfaces

Following the proposal of Wan and Guo (1998), two yield surfaces are postulated for the saturated materials: a shear and a cap yield surface (Fig. 18). Yield is activated at the initial unloaded (as compacted) state. Therefore, point A in Fig. 18 is also viewed as the current stress state of a sample loaded along the indicated stress path. A similar behaviour is expected when a suction, s , is applied to the rockfill. The experimental results analysed before suggest also that, at a given deformation, a dry sample is capable of sustaining larger deviatoric stresses (Figs. 5 and 6). Therefore, it is expected that the peak and critical state stress ratio, M_c , increases with suction.

Yield in shear is described by the yield surface:

$$f_q = \frac{q}{p} - \eta_s = 0 \quad (5)$$

The hardening rule is given by Eqs. (2) and (3), which are written here as:

$$\frac{\eta_s}{rM_c(W_E^p, s)} = \frac{\epsilon_q^p}{b(W_E^p) + \epsilon_q^p} \quad (6)$$

where

$$r = 1 - k\psi = 1 - k(v - v_{cr}) \quad (7)$$

and W_E^p is the effective plastic work which is capable of breaking the rockfill particles. W_E^p is defined in Eq. (4). The total plastic work is a known quantity along a given stress-strain path and the plastic work W_R^p may be considered as a model parameter. A discussion of the physical meaning of W_R^p was given before.

In Eq. (7), ψ is the state parameter defined by Been and Jefferies (1985); v is the current specific volume at a given

Fig. 18. Yield surfaces of proposed model

mean stress p ; v_{cr} the corresponding value at the critical state and k is a constant. The value of v_{cr} depends also on suction and confining stress (see Fig. 9). This figure suggests a linear relationship between v_{cr} and p :

$$v_{cr} = y(s) - m_{cr}p \quad (8)$$

where the function $y(s)$ is assumed to depend on the logarithm of suction,

$$y(s) = y_0 + \alpha_y \ln \left(\frac{s + p_{at}}{p_{at}} \right) \quad (9)$$

where y_0 , m_{cr} and α_y are constants. The atmospheric pressure, p_{at} , is introduced in Eq. (9) to recover saturated conditions when $s = 0$. Equation (9) describes the variation of specific volume with suction. The specific form of Eq. (9) is derived from results presented in Oldecop and Alonso (2001).

M_c defines the stress ratio at critical state and is a function of effective plastic work and applied suction. It has a limiting value given by M_{cr} for saturated conditions: M_{cr} . Based on previous results, the following expression is proposed for M_c :

$$M_c(W_E^p, s) = M_{cr}(W_E^p) \left(1 + k_s \ln \left(\frac{s + p_{at}}{p_{at}} \right) \right) \quad (10)$$

k_s is a constant, which describes the gain of strength induced by suction. A simple linear law relating M_c with suction was introduced in Eq. (10) for simplicity but this relationship may be adapted to new available experimental information. In addition, the plot given in Fig. 13 suggests that $M_{cr}(W_E^p)$ may be expressed as:

$$M_{cr}(W_E^p) = M_{c_{res}} + (M_{c_0} - M_{c_{res}}) e^{-aW_E^p} \quad (11)$$

In Eq. (11), $M_{c_{res}}$ is a residual stress ratio for saturated conditions. It corresponds theoretically to the limiting stress ratio as the effective plastic work accumulates. a is a model parameter. Analytical expressions 10 and 11 have been fitted to data in Fig. 13.

Parameter b , which describes the initial stiffness (it is proportional to $1/b$) of the rockfill, depends also on effective plastic work and suction. Following the results shown in Fig. 14, b has been defined as:

$$b = B \left(\frac{W_E^p}{c + W_E^p} \right) \quad (12)$$

where B is the maximum change in b as the effective plastic work increases. Equation (12) defines a hyperbolic reduction of the initial plastic tangent modulus, from an infinite value at no degradation of the rockfill structure. This is a simple choice which reduces the number of parameters. Some high, but finite, initial tangent plastic modulus may be suspected for the rockfill. It can be easily introduced by suitably modifying Eq. (12). Data in Fig. 14 suggests, however, that Eq. (12) may reproduce adequately the experiments performed.

The yield Eq. (5) and the hardening rule (6) define, in an implicit way, a nonlinear failure envelope for peak and critical state conditions.

The volumetric (cap) component of yield is based on the elastoplastic compressibility model developed for rockfill in Oldecop and Alonso (2001). Under isotropic compression, a threshold yield stress, p_y , marks the beginning of particle breakage. Below this yield stress, no suction effects were measured in compression tests. Therefore, the yield locus describing the cap position is:

$$p(s) = p(0) = p^* \quad (p < p_y) \quad (13)$$

or

$$f_p(p) = p(s) - p^* = 0 \quad (p < p_y) \quad (14)$$

For mean stresses above p_y , the following yield line was described by Oldecop and Alonso (2001):

$$f_p(p, s) = p[\lambda^i + \lambda^d(s) - \kappa] - p_y \lambda^d(s) - p^*(\lambda^i - \kappa) \quad (15)$$

where

$$\lambda^d(s) = \lambda_0^d + \alpha_s \ln \left(\frac{s + p_{at}}{p_{at}} \right) \quad (16)$$

defines the isotropic delayed compressibility of the rockfill, which depends on the applied total suction, s . λ_0^d is the isotropic compressibility coefficient for saturated condi-

tions and α_s is a model parameter.

κ defines the elastic compressibility index and λ^i is the instantaneous compressibility index. p^* was taken as the hardening parameter in Eqs. (14) and (15). It was identified as the yield stress for a very dry rockfill. The following strain hardening law controls p^* :

$$dp^* = \frac{d\epsilon_q^p}{\lambda^i - \kappa} \quad (17)$$

Flow Rules

For the deviatoric part, a plastic potential, Q_q , which follows a Mohr-Coulomb format with an appropriate dilatancy angle ψ_m is proposed:

$$Q_q = \frac{1}{2} q - \left(\frac{1}{6} q \right) \sin(\psi_m) \quad (18)$$

The dilatancy angle follows the proposal of Wan and Guo (1998) discussed previously. The dilatancy angle ψ_m proposed by these authors on the basis of the original Rowe's dilatancy angle was:

$$\sin \psi_m = \frac{\sin \varphi_m - \left(\frac{e}{e_{cr}} \right)^\alpha \sin \varphi_{cr}}{1 - \left(\frac{e}{e_{cr}} \right)^\alpha \sin \varphi_m \sin \varphi_{cr}} \quad (19)$$

where φ_m is the mobilized friction angle at a given yield state and φ_{cr} is the critical state value. φ_{cr} may be obtained from the expressions for M_c given previously (Eqs. (10) and (11)). e_{cr} is the void ratio at critical state and e the current void ratio. e_{cr} is given by the expressions for v_{cr} (Eqs. (8) and (9)). α is a constant model parameter.

Finally, for the volumetric part, an associated law:

$$Q_p = f_p \quad (20)$$

is adopted.

An incremental set of elastoplastic stress-strain relations was derived on the basis of the previous relationships. The input for calculations was the imposed stress path on the specimens.

Model Predictions

A single set of model parameters was derived from the set of tests performed. The analysis of test results, presented before, provides a description of the procedures followed to derive them. Parameters are indicated in Table 1. A classical elastic model using the bulk modulus, $K = \nu p / \kappa$, and constant Poisson's ratio ($\nu = 0.29$) was added to compute elastic strains. The value of ν is the average of the values estimated for different tests (which were in the range 0.20–0.35). The unloading-reloading elastic compressibility, κ , was determined in an isotropic test performed on a saturated specimen. The shear modulus is given by $G = 3(1 - 2\nu)K/2(1 + \nu)$.

The plastic work associated with particle rearrangement was simply made equal to the plastic work computed in D1, for the "dry" specimens, as explained before. The actual modelling laws adopted for M_c and b are plotted in Figs. 13 and 14. Critical state lines for the two

Table 1. Model parameters

A) Shear behaviour	
Yield surface and hardening rule	
Peak stress ratio	$k=0.3$
Critical void ratio	$y_0=2.0$; $\alpha_y=0.156$; $m_{cr}=0.37 \text{ (MPa)}^{-1}$
Critical stress ratio. Influence of suction	$k_s=9.65 \times 10^{-3}$
Critical stress ratio. Influence of effective plastic work	$M_{c0}=1.93$; $M_{cres}=1.45$; $a=0.7 \text{ (MPa)}^{-1}$
Initial compliance modulus	$B=3.7$; $c=6 \text{ MPa}$
Flow rule	
Dilatancy angle	$\alpha=0.3$
B) Isotropic compression (CAP)	
Yield surface and hardening rule	
Threshold confining stress	$p_y=0$
Compressibility indexes	$\lambda^i=0$; $\lambda_0^d=7.48 \times 10^{-3} \text{ (MPa)}^{-1}$; $\alpha_s=0.786 \times 10^{-2} \text{ (MPa)}^{-1}$
C) Elastic behaviour	$\kappa=1.41 \times 10^{-3}$; $\nu=0.29$

series of samples tested are given in Fig. 9. They are an estimation, discussed before, of the expected steady state void ratio of the specimens.

A comparison of model predictions and experimental results is given in Fig. 19 for saturated conditions and in Fig. 20 for “dry” conditions ($RH=36\%$).

The agreement is good in general, especially for the deviatoric curves (q vs. ε_1). More difficulties are found in reproducing the volumetric response of the samples. Computed dilation rates, especially for low confining stresses, tend to underestimate measured values. Measured dilation rates at the end of the experiments are significantly larger than model predictions, a question related to the assumption of critical state conditions in the model. In fact, the attainment of critical state conditions for a gravel material is probably outside the possibilities of a triaxial test. Very large strains and a fully broken gravel structure are probably required. In addition, localization will complicate the interpretation. In a gravel, the shape of particles is probably a relevant issue. The material tested has elongated particles with sharp edges. Interlocking and dilation, even after some significant breakage and deformation seems very likely and the proper observation of limiting conditions would require, again, large displacements which are not achieved in a triaxial cell.

The experimental strength envelopes for peak conditions determined in the tests performed are indicated in Fig. 21. When no peak could be found, the maximum deviatoric stress at the end of the test was selected. A marked nonlinear relationship between the peak deviatoric stress and the mean confining stress is found. Also plotted in Fig. 21 is the strength envelope given by the model. Strength values were determined with the same criterium. A good correspondence between model and experimental results is found.

Fig. 19. Comparison between model predictions and measured response: Triaxial tests on saturated rockfill specimens

CONCLUSIONS

A series of triaxial tests performed on a large diameter suction controlled cell has provided information on the behaviour of rockfill under triaxial stress paths. The material tested is a fairly uniform gravel of quartzitic shale, compacted under energies equivalent to the Normal Proctor test. Previous developments on the behaviour of rockfill materials under oedometric compression paths at different relative humidities and current understanding on the behaviour of sands, specially under high stresses provided a convenient framework to develop a new constitutive model for triaxial stress states.

Rockfill deformations are a combination of particle rearrangement and particle breakage. Both, stress level and Relative Humidity, control particle breakage. This underlying phenomena explains the influence of suction, which virtually affects all the macroscopic constitutive parameters derived in the paper.

Yielding is described by two yield surfaces: deviatoric and isotropic or “cap” surface. The proposed deviatoric

Fig. 20. Comparison between model predictions and measured response: Triaxial tests on "dry" specimens (RH=36%)

Fig. 21. Peak strength envelopes: Comparison of experimental results and model predictions

yield equations followed the work of Wood et al. (1994), which describes a simple procedure to model strain softening in sands. The effect of suction and confining stress was incorporated into the model on the basis of the experiments performed. Rockfill degradation (through particle breakage) was made dependent of an effective plastic work which was defined as the difference between total plastic work and the plastic work employed in particle rearrangement. The latter may be considered as a model parameter but may be also estimated on the basis of experimental information. The deviatoric flow rule followed the extension of Rowe's dilatancy theory proposed by Wan and Guo (1998). Again, suction and confining stress effects were introduced. The isotropic cap behaviour was directly derived from previous work by the authors.

The complete model offers a comprehensive description of rockfill behaviour and incorporates the effect of suction. It incorporates a variety of observed effects such as the curved strength envelopes, strain hardening/softening, effect of suction on stiffness and strength, dilatancy controlled by initial state, stress level and suction and collapse effects upon suction reduction. The model has a built-in mechanism for rockfill degradation as the effective plastic work accumulates. Not all the effects mentioned have been investigated experimentally in detail. However, the set of triaxial tests performed could be described in a satisfactory way by the model developed. An area open for further research is the relevance of a critical state for rockfill behaviour. The model developed uses this concept as a convenient procedure to derive the constitutive model. However, difficulties in matching the observed high rates of dilation at large strains were encountered.

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