NUMERICAL SIMULATION OF PROGRESSIVE FAILURE IN CUT SLOPE OF SOFT ROCK USING A SOIL-WATER COUPLED FINITE ELEMENT ANALYSIS

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ABSTRACT

In the present paper, based on an elastoplastic model with strain hardening and strain softening, a soil-water coupled finite element analysis is conducted to investigate the progressive failure of a cut slope in a model ground. In order to verify the validity of the analyses related to the strain-softening behavior, numerical analyses are firstly conducted for plane-strain compression in different meshes and loading steps under complete drained condition. It is confirmed by the analyses that the analysis conducted in this paper has a small dependency on the mesh size. Then, the mechanical behaviors of a cut slope, such as the change of excessive pore-water pressure, the redistribution of stress in ground due to strain softening, the propagation of shear band and the progressive failure are discussed in detail by the soil-water coupled finite element analysis. It is found that a soil-water coupled analysis based on an elastoplastic model can describe the time dependent behavior of soft rock in boundary-value problems. It is also found that a soil-water coupled analysis based on a strain-softening model can simulate the progressive failure of a cut slope.

Key words: effective stress, FEM, progressive failure, slope, strain softening, time dependency (IGC: G6)

INTRODUCTION

It is commonly known that soft sedimentary rock can be linked to many geotechnical engineering problems, such as the instability of cut slopes and foundations. Generally speaking, the mechanical behavior of soft sedimentary rock is elastoplastic, dilatant, strain hardening-strain softening and time dependent. Physically, soft sedimentary rock has an unconfined compressive strength of less than 20 MPa and its mechanical behavior is between the behavior of soil and rock. Cementation plays an important role in its shearing strength. Compared to other geological materials formed in same epoch, the void ratio is relatively large and is a special structure formed during sedimentation. Its mechanical behavior during shearing is largely dependent on the confined stress and the pore-water pressure. The cementation deteriorates due to the breakdown of the structure. The breakdown is caused by various processes, such as large shearing deformation, cyclic drying-wetting or stress relaxation. The softening behavior of soft sedimentary rock becomes a very important factor in the long-term stability of geotechnical engineering problems.

There are two different viewpoints on the softening behavior of geologic materials. The first viewpoint is that the softening behavior is an inherent characteristic in the stress-strain relation, that is, strain softening or material softening. The researchers who share this viewpoint try to establish a constitutive model that can describe strain softening (Hoeg, 1972; Nayak and Zienkiewicz, 1972). On the other hand, however, researchers who share a second viewpoint point out that the softening behavior is merely a consequence of non-uniform deformation occurring in a material which has undergone loading, in other words, the result of localization in deformation (Read and Hegemier, 1984). The researchers who share this viewpoint insist that softening is related to localization where a decrease of apparent stress occurs not by strain softening but by a reduction of effective area due to the coalescence of void and micro cracks, and that strain-hardening is an inherent characteristic of geologic materials. Because the localization of deformation is the main cause of softening behavior, the softening behavior can be illustrated by solving the boundary value problems based on strain hardening models.

It is, however, difficult to say that softening behavior can be fully described by the analyses based on strain hardening models. The decline of strength caused by the deterioration of cementation in geologic materials can never be neglected. It seems reasonable to say that the softening behavior is caused both by localization in deformation and structural deterioration in geologic...

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materials. For an engineering approach, the choice of a procedure for providing a drop in nominal stress with strain can be based on convenience because a rigorous development of the methods based on two different viewpoints should provide the same results on a macroscopic basis.

As some researchers have pointed out, when a strain-softening model is applied to finite element analysis, the stiffness matrix will become negative in the softening region and the boundary value problem cannot be defined and solved uniquely. Such kind of problem has been considered by many researchers, for example, Aifantis (1984), Bazant and Pijaudier-Cabot (1988).

Adachi et al. (1991) conducted a finite element analysis based on an elastoplastic model that is able to describe the strain hardening and the strain softening behaviors of soft rock. The finite element analysis was conducted under complete drained condition and can lead to a unique solution for initial value and boundary value problems in Valanis’s sense (1985). The results were found to be less dependent on the mesh size compared with the results from non-local calculation.

It is well known that progressive failure in a cut slope is usually caused by the following two factors, namely, (a) deterioration of the cementation of geologic materials due to weathering during and/or after the cut of slope, and (b) reduction in the apparent shear strength due to the dissipation of excessive negative pore-water pressure developed by the rapid excavation of the cut slope.

Many researches related to progressive failure have been done and some of them can be found in the works by Bjerrum (1967), Palmer and Rice (1973) and Burland et al. (1973). In their researches, the mechanism of the progressive failure of overconsolidated clay, clay shale in slope and deep excavation have been discussed in detail, especially the influence of the softening behavior of overconsolidated geomaterials. Restricted by computational technology, boundary value problems with finite element analysis based on strain softening model could not be conducted at the moment. Asakura et al. (1999) conducted a numerical simulation of triaxial compression test on heavily overconsolidated clay with boundary value problem considering progressive failure. The model for the clay is a Cam-Clay type model using the concept of subloading.

In general, there are two types of time-dependent behavior of geologic materials, one is the so called apparent viscosity due to the coupling of the soil skeleton with pore water, and the other is an inherent viscosity of soil skeleton, e.g., Oya Tuff (Okubo and Chu, 1994). Because the soil skeleton is not time-dependent in a soil with apparent viscosity, the time dependent behavior of the soil can be simulated with a soil-water coupled analysis based on an elastoplastic constitutive model. For those materials with inherent viscosity, however, the time-dependent behavior should be simulated with a time-dependent constitutive model.

Adachi et al. (1994) proposed an elasto-viscoplastic model that can describe not only time dependent behaviors, such as strain rate dependency, creep and stress relaxation, but also strain softening of geologic materials. Based on the model, a finite element analysis was carried out under total stress condition, to investigate the instability of cut slope of model ground, taking into consideration both the time dependency and the strain softening of soils.

In this paper, based on an elastoplastic model with strain hardening and strain softening (Oka and Adachi, 1985), a soil-water coupled finite element analysis was conducted to investigate the progressive failure of cut slope in a model ground.

**ELASTOPLASTIC MODEL WITH STRAIN SOFTENING**

Oka and Adachi (1985) proposed an elastoplastic model with strain softening, using a strain measure whose increment is expressed as

\[ \Delta \epsilon = (\Delta \epsilon_0, \Delta \epsilon_1) \]

where \( \Delta \epsilon_0 \) is an incremental strain measure. A stress history tensor is expressed by introducing a single exponential type of kernel function, namely,

\[
\sigma_i = \sigma_i(0) + \int_0^\tau \frac{1}{\tau} \exp (- (z - z')/\tau) (\sigma_i(z')) dz'
\]

where \( \tau \) is a material parameter which expresses the retardation of stress with respect to the strain measure and \( \sigma_i \) is the stress tensor and \( \sigma_i(0) \) is the value of the stress tensor at \( z = 0 \). From the definition of the stress history, present stress possesses the largest influence. The older the stress is, the less the influence will be. Therefore, those stresses at \( z < 0 \) are not necessarily taken into consideration in the stress history. In this paper, the superscript * denotes the stress history. It is known that the strength of soft sedimentary rock can usually be divided into two parts, one from the cementation and another from the friction of the material. The stress history, thereby, represents the frictional part of the strength. A detailed explanation of the physical meaning of the stress history can be referred to from the reference Adachi and Oka (1995). The strain measure is \( z = \int \epsilon_0 \). In proportional loading and one-dimensional deformation, it equals to \( \sqrt{\epsilon_0 \epsilon_1} \).

The total strain increment tensor, \( \Delta \epsilon_{ij} \), is composed of the elastic and plastic components:

\[ \Delta \epsilon_{ij} = \Delta \epsilon_{ij}^e + \Delta \epsilon_{ij}^p \]

where \( \Delta \epsilon_{ij}^e \) is an incremental elastic strain tensor and \( \Delta \epsilon_{ij}^p \) is an incremental plastic strain tensor. The plastic strain increment is given by a non-associated flow rule as,

\[ \Delta \epsilon_{ij}^p = \Lambda (f_p / \sigma_i) d\Gamma \]

where \( f_p \) is the plastic potential function, \( f^* \) is the yield function and \( \Lambda \) is a positive function describing the strain hardening-softening characteristics. The subsequent yield function is defined by
\[ f_\gamma = \eta^* - \kappa = 0, \quad \eta^* = \sqrt{\frac{s_0^* s_0^*}{s_{0m}^*}} \]

where \( s_0^* \) is the deviatoric stress history tensor, and \( s_{0m}^* \) is the mean stress history. \( \kappa \) is the plastic strain hardening factor and is given by the following evolution equation:

\[ d\kappa = \frac{G'(M^p - \kappa)^2}{M^{p2}} dy^p \quad \text{and} \quad dy^p = (de^p_\gamma/de^p_\kappa)^{1/2} \]

In the case of proportional loading, it can be integrated as

\[ \kappa = \frac{M^p G' y^p}{(M^p + G' y^p)} \]

\[ y^p = \int dy^p \]

\[ M^p = \sqrt{s_0^* s_0^*/s_{0m}^*} \text{ at residual} \]

where \( de^p_\gamma \) is an increment of deviatoric plastic strain tensor. \( M^p \) is the value of the stress invariant ratio at residual state and \( G' \) is the strain hardening parameter and can be determined with the value of the stress ratio \( \sqrt{s_0^* s_0^*/s_{0m}^*} \) at the unloading-reloading process under the residual state (Adachi and Oka, 1995).

For the yielding function defined in Eq. (5), a consistency rule (Prager condition) should be satisfied,

\[ df_\gamma = d\eta^* - d\kappa = 0 \]

The loading criteria is given by the following relations:

\[ de^p_\gamma \neq 0 \text{ if } f_\gamma = 0, \quad d\eta^* > 0 \text{ loading} \]

\[ de^p_\gamma = 0 \text{ if } f_\gamma = 0, \quad d\eta^* > 0 \text{ neutral loading} \]

\[ de^p_\gamma = 0 \text{ if } f_\gamma = 0, \quad d\eta^* > 0 \text{ unloading} \]

It is clear from the above equation that the loading criterion in this model is defined in stress history space instead of stress space. It is well known that strain hardening, perfectly plastic and strain softening is ordinarily defined according to the movement of the yielding surface, that is, expansion, stationary or shrinking when plastic strain develops. It is not necessary, however, to define strain hardening, perfectly plastic and strain softening in this model, because the loading criterion is defined in the stress history space. From the ordinary viewpoint, the model only describes the strain hardening behavior because no criterion for the strain softening is used; it can describe the softening behavior in that the stress invariant ratio decreases along with the increase of the shearing strain.

It is assumed that the plastic potential function is expressed by the relation as

\[ f_p = \eta + M \ln [(\sigma_m + b)/(\sigma_m + b)] = 0 \]

\[ \bar{\eta} = (s_0^* s_0^*/(\sigma_m + b)^2)^{1/2} \]

where \( s_0 \) is the deviatoric stress tensor, \( \sigma_m \) is the mean stress and \( M \) is a variable that controls the development of plastic volumetric strain. \( b \) is the plastic potential parameter that represents the tensile strength of soils. The plastic potential parameter \( \sigma_{0m} \) is determined by isotropic consolidation tests and takes the value of the

- **Fig. 1.** Plastic potential and boundary surface

- **Fig. 2.** Schematic illustration of Darcy law and boundary condition in soil-water coupling analysis
pre-consolidated stress. The following relation expresses
a boundary surface, which defines the normally consoli-
dated and overconsolidated region as shown in Fig. 1:

$$f_b = \bar{\eta} + \bar{M}_m \ln \left[ \frac{(\sigma_m + b)}{(\sigma_{mb} + b)} \right] = 0$$ (12)

Based on this relation, the value of $\bar{M}$ in Eq. (10) can be
calculated easily as:

$$\bar{M} = -\bar{\eta} \ln \left[ \frac{\sigma_m + b}{\sigma_{mb} + b} \right]$$ if $f_b \leq 0$

$$\bar{M} = \bar{M}_m$$ if $f_b > 0$ (13)

Combining Eqs. (4), (5), (6), (8), (10) and (13), the plastic
strain increment tensor can be evaluated by the following
equation as:
Table 1. Material parameters of model ground

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus E</td>
<td>100.0</td>
</tr>
<tr>
<td>Poisson’s ratio ν</td>
<td>0.30</td>
</tr>
<tr>
<td>γi (kN/m²)</td>
<td>9.8</td>
</tr>
<tr>
<td>k (cm/sec)</td>
<td>10⁻⁶</td>
</tr>
<tr>
<td>G'</td>
<td>452</td>
</tr>
<tr>
<td>ε₀</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Note: G = E/(1 + ν)/2, K = E/(1 - 2ν)/3.

\[
d e_{ij}^e = \frac{M_{ij}^e}{G'(M_{ij}^e - \eta^e)^2} \left[ \frac{\delta_{ij}}{\eta} + (\Delta M - \Delta \eta) \frac{\delta_{ij}}{3} \right] \left[ \eta_{ij}^e - \eta^e \right] \frac{\delta_{ij}}{3} \frac{d \sigma_{ij}^e}{\sigma_m^e}
\]

(14)

where, \( \eta = \sigma_m / \sigma_m^e \)

\[
d e_{ij}^b = \frac{d \sigma_{ij}}{2G} + \frac{d \sigma_m}{3K} \delta_{ij}
\]

(15)

On the other hand, the elastic strain increment is given as:

(Finite Element Analysis of Progressive Failure in Cut Slope of Model Ground)

Formulation and Analytical Procedures

In this section, a soil-water coupled finite element analysis based on the model introduced in the previous section is conducted to analyze the progressive failure of a cut slope in a model ground with soft rock. For a boundary-value problem related to a soil-water coupled analysis, based on Biot’s theory, the finite element method (FEM) was used for the spatial discretization of the equilibrium equation, while the finite difference method (FDM) was used for the spatial discretization of the excessive pore-water pressure in the continuity equation. As for the continuum equation, a backward finite difference scheme (Akai and Tamura, 1978) is adopted for water flow, in which for an arbitrary element, the quantity of the water drained from the element to the neighboring elements in a time interval \( dt \), as shown in Fig. 2, is

\[
q = \sum_{i=1}^{4} \frac{k u_r - u_n}{b_i} s_i dt
\]

(17)

where, \( k \) is the coefficient of permeability of the ground and \( \gamma_w \) is the unit weight of water. By some algebraic calculation, the incremental form of the field equations for a single element in the finite element-finite different scheme under plane-strain condition can be given as follow (see APPENDIX),

\[
\begin{bmatrix} K & L \r^T \end{bmatrix} \begin{bmatrix} d \delta \r^T \end{bmatrix} + \begin{bmatrix} 0 \\ \sum_{i=1}^{4} \alpha_i u_{i \alpha} \end{bmatrix} = \begin{bmatrix} (dF) + (dF_r) + \Delta [L(u_{i \alpha})] \r^T \end{bmatrix}
\]

(18)

Figure 3 shows the failure patterns of geologic materials considered in the constitutive model. Two types of failures, that is, residual failure and tensional rupture are
considered. The residual failure here means the state where the shear stress reaches a post-peak value and the cementation of soil particles diminishes to zero. Therefore, in residual state, the shear resistance of the soil is only from the friction between soil particles. The residual failure state in the model is described by the equation as \( \eta^* - M^* = 0 \), where \( M^* \) is the value of the stress invariant ratio at residual state. Theoretically, the maximum stress history ratio \( \eta^* \) cannot exceed the value of \( M^* \). In calculation, however, the situation of \( \eta^* \geq M^* \) may occur due to the Euler-type explicit integration scheme adopted in the numerical analysis. If \( \eta^* \geq M^* \), then the material is regarded to be failed, as shown in Fig. 3(a). Meanwhile, if the mean stress is less than \(-b\), that is, the maximum tensional mean stress is larger than the tensional strength, a tensinal rupture will occur, as shown in Fig. 3(b).

It is known that in an explicit integration scheme, the present stress state is calculated based on the stress and the strain calculated in the previous step. Therefore, in judging a residual failure state, it is better to consider the stress state in the next step. In the present calculation, as shown in the figure, an estimation of \( \sigma_{ij}^* \) in the next step is conducted so that if the stress at "future" reaches residual failure, the stress is judged as failure. Figure 4 shows the integration scheme adopted in the numerical analysis.

From Eqs. (14) and (15), it is clear that a numerical instability appears at the residual failure state \( \eta^* - M^* = 0 \). In order to avoid the unstable calculation, a numerical treatment is introduced to remedy the instability. That is, if an element is judged to be failed, the element will be defined as a non-resistance element in which the Young’s modulus will be as small as 0.00001 MPa and Poisson’s ratio is 0.499. This means that the element at failure state has little resistance to shearing and the volumetric change is restricted, which can be linked to the concept of critical state of soil in the Cam-Clay model. Meanwhile, the incremental strain \( \delta e_{ij}^* \) is assigned be equal to \( \delta e_{ii} \). The treatment discussed above implies that the calculation cannot be applied at the unloading-reloading process after the rock reached the residual failure. It, however, does not hinder the calculation conducted here because the unloading-reloading process is not involved in progressive failure.

Mesh Size Dependency and Influence of Calculating Steps

In order to verify the validity of the analysis for the simulation of a progressive failure in cut slope, a plane-strain compression is simulated in different mesh sizes and loading steps under complete drained condition. The specimen under compression is 10 cm in height and 5 cm at width. The initial stress condition of the specimens is assumed to be isotropic and has the value of 0.1 MPa. The material of the specimen is the same as the model ground that will be discussed later related to the progres-
PROGRESSIVE FAILURE OF CUT SLOPE

![Graphs showing progressive failure of cut slope](image)

**Fig. 7. Influence of calculating step in different meshes**

Progressive failure in cut slope. Table 1 shows the material parameters of the ground. Two cases of boundary condition were considered. In case 1, the friction at the top, where a prescribed vertical displacement was applied, is assumed to be zero; while in case 2, the displacement in the horizontal direction at the top is supposed to be fixed. In both cases, two side faces were free and the bottom was free in the horizontal direction and fixed in the vertical direction, as shown in Fig. 5(a). Figure 5(b) shows the comparison of stress-strain-dilatancy relations obtained from the theory and the finite element analysis in case 1. The result from theory here means the result obtained directly from the integration of the constitutive model expressed with Eqs. (14), (15) and (16). In case 1, because the friction at the top is zero, the calculated deformations and the stresses at all elements are uniform, which makes the comparison between the results from FEM and theory meaningful. It is found that the calculated relations agree well with the theoretical results. Figure 5(c) shows the influence of the restrained condition at the top of the specimen. It is found that the peak strength was not affected at all by the restrained condition while the residual strength changed much, with an increase of 20% in the strength under the condition of restrained top.

It should be pointed out in advance that the results given in the following Figs. 6–8 were calculated under the condition that the horizontal displacement at the top of the specimen was fixed.

Figure 6 shows the mesh size dependency in shear strain ($\sqrt{2I_2}$, $2I_2 = e_{ij}e_{ij}$) and the deformation pattern of the specimen under the condition that the average vertical strain was equal to 5.0%. Though the overall distributions of the shear strain were similar in all meshes, it was much sharper in finer mesh than in coarser mesh. However, if the number of elements were more than 800, then the difference would be very small. On the other hand, there was not much difference in the deformation patterns in all meshes.

Figure 7 shows the influence of the calculating step in different meshes. If a coarse mesh was used (the number of elements is less than 400), then the relation of the average vertical stress and strain would be the same in all calculating steps. In a fine mesh, however, the calculating steps might substantially affect the results. From the figure, it is known that in order to obtain a stable result, more than 2000 calculating steps were needed to get an apparent strain of 5%.

Figure 8 shows the mesh size dependency in different calculating steps. It is found that the peak strength was not affected at all in different meshes while the residual strength changed in such a tendency that the finer the mesh is, the less the residual strength will be. The maxi-
mum difference of the residual strength due to the mesh size dependency is about 10%. If the number of elements is more than 800, then the difference will almost be zero.

Based on the above discussion, it is found that if a relatively fine mesh and a large number of calculating steps are used, the calculating results are reliable.

**Progressive Failure in Cut Slope of Model Ground**

Figure 9 shows the finite element mesh used in the simulation of progressive failure in a cut slope of imaginary ground. The material parameters of the ground are the same as the ones of the specimen discussed above, and are listed in Table 1. The size of the ground is 1,000 m in
length and 360 m in height. The height and the gradient of the cut slope are 150 m and 5:1 respectively. The reason why such a big slope was chosen is that the imaginary ground will not fail for a small cut slope. Moreover, because an excessive pore-water pressure (E.P.W.P.) is taken as the unknown variable in the soil-water coupled analysis, the excavation considered here is an excavation beneath water. The numbers of the nodes and 4-node isoparametric element are 1,120 and 1,053 respectively. The boundary condition is given as: (a) for displacement, it is fixed at the bottom in both x, y directions and is fixed at the vertical boundaries in the x direction; (b) for E.P.W.P., the surface of the ground is a drainage boundary and the others are impermeable. It is known that it is difficult to determine an initial stress field for a ground made from soft rock, the initial stress field of the imaginary ground in the present calculation is assumed to be a gravitational field with a value of $K_0 = v/(1-v) = 0.429$. In the calculation, the first stage is to calculate the initial stress field of a horizontal ground; the second stage is then to excavate the slope, in which all excavated elements are dug away at the same time and the initial stresses along the excavated faces are released in 500 steps with a rate of 0.2%/step and in a total time of one month. After the completion of the slope excavation, a 30,000-step calculation with a time interval of 6,000 sec/step is conducted to simulate the dissipation of the E.P.W.P caused by the excavation of the slope.

In order to fully study the process of the progressive failure, the following two points are discussed,

1. Overall changes of the field quantities such as plastic strain, E.P.W.P and stress invariant ratio
2. Time history of stress, strain, strain rate and dilatancy in each element.

Figure 10 shows the change of the distribution of stress-history ratio. In the residual state, the cementation of the geologic material tends to be zero and only the frictional strength that depends on a confining stress remains. In this case, the stress-history ratio will be the same as the stress ratio and takes the value of $M^*$. In the figure, $T=0$ means the time immediately after the completion of the excavation. At the beginning, the value of $\eta^*$ remained in a constant value of about 0.84. 4.57 years after the completion of the excavation, it increased abruptly at the toe of the slope and then the phenomenon propagated to other regions. 6 months later, a failure band formed from the toe to surface, in which the $\eta^*$ reached a residual value. Finally, an unstable block appeared in the slope, taking the failure band as its boundary that connects the stable area of ground. It should be pointed out that the stress at the toe of the slope has a singularity in elastic calculation. In plastic calculation, the stress at the toe reached the residual state immediately after the excavation, which, however, cannot be regarded
as the main reason of the slope failure. Because even after the stress of the toe reached residual state, the slope kept stable for a long time.

Figure 11 shows the change of the distribution of plastic shear strain. Similar to the stress-history ratio, shear strain developed very quickly in a zone at the time of 4.57 years. The propagation of the shear zone in which a large shear strain occurs took the same form as the failure zone shown in Fig. 10.

Figure 12 shows a change of distribution of E.P.W.P. with time. At the time immediately after the completion of excavation, a large E.P.W.P. developed in the ground, resulting in a large apparent shear strength that kept the slope stable. After 4.57 years, it dissipated gradually and the failure zone shown in Fig. 10 began to develop due to the loss of the apparent shear strength. At the moment, the E.P.W.P. has reached its minimum value. When the shear zone formed, strain softening occurred and a dilatancy developed in some areas, resulting in an increase of a negative E.P.W.P., as shown in Fig. 12.

From Figs. 10-12, it is clear that because of the dissipation of an E.P.W.P., the ground of cut slope lost its apparent strength and a strain softening occurred in some areas. Then a redistribution of stress lead to the propagation of the softening zone, resulting in the formation of the failure band and the shear zone. The failure band developed gradually and finally caused a global failure of the cut slope.

In order to clarify the mechanism of the progressive failure, time histories of some field quantities such as stress ratio, E.P.W.P., strain rate etc. in each element are studied in detail. Two groups of elements located in the shear zone, one grouped along a horizontal line and the other grouped along the slope surface, are considered.

Figure 13 shows the change of stress-history ratio with time in the elements. Obviously, the stress-history ratio
kept constant for a long time at first and then increased abruptly to the failure line.

Figure 14 shows the change of stress ratio with time and the stress-strain relations. It is also known that the stress ratio increased very slowly but did not change for a long time, meanwhile the plastic strain was very small. When the ratio reached its peak value, strain softening occurred and the plastic strain developed very quickly, resulting in a sharp reduction of the stress ratio. The time at which strain softening occurred was different for different elements, showing a clear propagation of the softening zone. In group A, the softening propagated from inner to outer, while for group B, it propagated from the lower to the upper part. In both groups, the softening started from the shear band.

Figure 15 shows the relation of strain rate ($\sqrt{2I_2}$) with time. It is found that although the strain softening occurred at different time for different elements, the creep failure that is usually marked by an acceleration of strain rate occurred at the same time in all elements, implied that a global failure does not depend on a single element, but depends on the deformation of surrounding ground.

Figure 16 shows the stress and stress-history path of element 423. The stress at the end of excavation has already exceeded the residual line, while the stress history is under the line, showing that it is stable at the moment. Then the stress history and the stress move towards the failure line and finally they reached the line and failed. In present case, the initial stress-history is near to the residual line. If the initial stress-history is under isotropic condition, which means the stress state is far from residual failure, the strain softening of the element may also happen, as shown in Fig. 5, in which compression is started from isotropic stress condition. The time and the degree of the excavation at which the strain softening may occur, however, are dependent on the initial stress condition.

Figure 17 shows the time history of the stress ratio, the stress-history ratio, the volumetric strain and the E.P.W.P. of element 423. The figure gives a clear description of the change in these variables. The strain softening of the element always accompanied with dilatancy, resulting in an increase of negative E.P.W.P. and an acceleration of the strain rate. It is also known from Fig. 17(a) that in post-peak process, the stress ratio decreased while the stress history ratio increase and finally reached the state of $\eta^* - M^*_f = 0$, which represents the typical post-peak behavior of the model.

**CONCLUSION**

Based on the numerical analysis of progressive failure in a cut slope conducted in this paper, the following conclusions can be obtained.
NOTATION

\( G \): elastic shearing modulus
\( K \): elastic volumetric modulus
\( r \): parameter of stress history tensor
\( M^2_1 \): stress ratio at residual state
\( G^* \): strain hardening parameter
\( b \): plastic potential parameter (tensile strength)
\( \sigma_{pl} \): plastic potential parameter
\( M^2_{ex} \): boundary surface parameter
\( c \): strain measure
\( \sigma \): stress tensor
\( \sigma_{ex}, \sigma_{ex}^* \): mean stress and effective mean stress
\( \sigma_{ex} \): stress history tensor
\( e \): deviatoric strain tensor
\( s \): deviatoric stress tensor
\( s_{ex} \): deviatoric stress history tensor
\( c_{ex}, e_{ex} \): volumetric strain and its rate
\( I_2 \): second invariant of stress tensor
\( J_2 \): second invariant of stress tensor
\( \gamma \): equivalent plastic shear strain
\( f \): plastic potential
\( f_{ex} \): yielding function
\( f_{ex} \): overconsolidated boundary surface
\( \eta \): stress history ratio
\( \eta \): stress ratio
\( k \): plastic strain hardening factor
\( u_{ex} \): excessive pore water pressure (E.P.W.P.) of an arbitrary element in FEM
\( k \): coefficient of permeability
\( \gamma \): floating unit weight of soil
\( [B] \): geometry function of an arbitrary element in FEM
\( [N] \): shape function of an arbitrary element in FEM
\( [D] \): elastic stiffness matrix
\( (da) \): incremental total stress vector of an arbitrary element in FEM
\( (da') \): incremental effective stress vector of an arbitrary element in FEM
\( (da_s) \): incremental relaxed stress vector due to plastic strain
\( (db) \): incremental strain vector of an arbitrary element in FEM
\( (df) \): the incremental nodal force vector
\( (dT) \): incremental external force vector acting on the stress boundary
\( (d\beta) \): incremental nodal displacement vector of an arbitrary element in FEM
\( q \): quantity of water drained from a element to its neighboring elements
\( V \): volume of an arbitrary element in FEM
\( \gamma \): unit weight of water
\( b \): drainage area of an arbitrary element in edge \( i \)
\( s \): center-to-center distance of an arbitrary element to its \( r \)th neighboring element
\( S_0 \): boundary where a stress is given
\( S \): boundary where a displacement is given
\( S_{ud} \): undrained boundary
\( S_d \): drained boundary

REFERENCES

APPENDIX

The incremental form of the equilibrium equation in a finite element scheme under plane-strain condition can be given as follow,

\[ \int_V \{ B \}^T (d\sigma) dV = \{ dF \} \quad (A-1) \]

Where

\[ \{ dF \} = \sum_i \left\{ \begin{bmatrix} N_i \times 0 \\ 0 \ N_i \times \end{bmatrix} \{ dT \} dS_i, \right. \]

\[ [B.] = \begin{bmatrix} N_{i,x} \\ 0 \ N_{i,y} \end{bmatrix}, \quad [B.] = [B. \cdots B. \cdots B.] \quad (A-2) \]

\[ [N] \] is the shape function of the isoparametric element. \( \{ dF \} \) is the incremental nodal force vector and \( \{ dT \} \) is the incremental external force vector acting on the stress boundary as shown in Fig. 2. An initial strain method is adopted in the numerical procedure related to a boundary value problem,

\[ \{ d\sigma \}' = [D]\{ d\varepsilon \} = [D]\{ d\varepsilon \} - \{ d\sigma \} \]

\[ \{ d\sigma \} = [D]\{ d\varepsilon \}, \quad \{ d\varepsilon \} = [B]\{ d\varepsilon \} \quad (A-3) \]

where \( \{ d\sigma \}' \) represents the incremental effective stress vector and \( \{ d\varepsilon \} \) represents the incremental strain vector in an element. It should be pointed out that in the content of the paper, all stresses are discussed in effective stresses. In the appendix, however, total stresses and effective stresses are treated simultaneously, so is necessary to use the symbols \( \{ d\sigma \}' \) and \( \{ d\sigma \} \) to distinguish the effect stresses and total stresses. \( \{ d\varepsilon \} \) represents the displacement vector at the nodes of the element. If taking into consideration the following relations in plane strain condition,

\[ \{ d\sigma \} = \{ d\sigma \}' + \{ M \} du_e, \]

where \( \{ M \} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \) and 

\[ \{ d\sigma \} = \{ d\sigma_{11} \} \{ d\sigma_{22} \} \{ d\sigma_{12} \} \quad (A-4) \]

The equilibrium equation can be written as,

\[ \int_V [B]^T \{ d\sigma \}' + \{ M \} du_e dV = \{ dF \} \quad (A-5) \]

Substituting Eq. (A-2) into above equation, the following equation can be obtained.

\[ [K]\{ d\varepsilon \} + [L] du_e = \{ dF \} + \{ dF_k \} \quad (A-6) \]

where

\[ [K] = \int_V [B]^T [D] [B] dV, \]

\[ [L] = \int_V [B]^T \{ M \} dV, \]

\[ [dF_k] = \int_V [B]^T \{ d\sigma_k \} dV \quad (A-7) \]

Combining Eq. (17) and neglecting the compressibility of the water and the soil particles, the volumetric change of the element will be equal to the quantity of the drained water, that is,

\[ dV = -\frac{\kappa}{\gamma_w} \left( \sum_i \frac{\kappa u_{ie} - u_{in}}{b_i} s_{di} - c u_e + \sum_i \alpha_i u_{in} \right) \quad (A-8) \]

\[ \alpha = \frac{k}{\gamma_w} \int dt \frac{s_i}{b_i}, \quad \alpha = \frac{k}{\gamma_w} \int dt \frac{\gamma_i}{b_i} \quad (A-9) \]

\[ dV = V_{|t+\Delta t} - V_{|t} = -\alpha u_{e|t+\Delta t} + \sum_i \alpha_i u_{i|t+\Delta t} \quad (A-10) \]

\[ dV = -\int_V \epsilon_0 dV = -\int_V \{ M \}^T [B] dV \{ d\varepsilon \} = -[L]^T \{ d\varepsilon \} \quad (A-11) \]

Combining Eqs. (A-10) and (A-11), it is easy to obtain the continuity equation,

\[ [L]^T \{ d\varepsilon \} - \alpha u_{e|t+\Delta t} + \sum_i \alpha_i u_{i|t+\Delta t} = 0 \quad (A-12) \]

Substituting the relation \( du_e = u_{e|t+\Delta t} - u_{e|t} \) into Eq. (A-6), the following equation can be obtained:

\[ [K]\{ d\varepsilon \} + [L](u_{e|t+\Delta t} - u_{e|t}) = \{ dF \} + \{ dF_k \} \quad (A-13) \]

\[ [K]\{ d\varepsilon \} + [L](u_{e|t+\Delta t}) = \{ dF \} + \{ dF_k \} + [L](u_{e|t}) \quad (A-14) \]

Combining Eqs. (A-13) and (A-14), Eq. (18) can be obtained immediately.