MODELING OF THE STEADY-STATE RESPONSE OF GRANULAR SOILS

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ABSTRACT

The paper presents a study of the static liquefaction of granular soils, particularly the existence and uniqueness of the so-called steady-state and the conditions leading to the steady-state. The study is carried out using a simple constitutive model for granular soils established using a few well-known relationships based on results of drained triaxial tests on sands and silts. The paper clarifies the necessary theoretical and experimental conditions in determining whether a soil has achieved the steady-state. The results of the analytical study strongly emphasize the importance of dilatancy and deformability on the steady-state response of granular soils. It is shown that, in the absence of instability, liquefaction should be treated simply as another aspect of the stress-strain response of granular soils, and liquefaction should be treated not just as a strength problem but also as a deformation problem.

Key words: critical-state, dilatancy, liquefaction, sand, shear strength, steady-state, stress-strain (IGC: D6/E13)

INTRODUCTION

The two important aspects related to the liquefaction of soils according to Seed (1987) are: 1) the stress conditions that trigger liquefaction, and 2) the consequences of the liquefaction. The first one requires the determination of the liquefaction shear strength, and the second one the residual or post-liquefaction shear strength. Early studies of the shear strength of granular soils by Castro (1969) showed that even after liquefaction (i.e., after the maximum undrained shear strength of the soil has been mobilized), granular soils retain significant resistance to shear deformation. It is now increasingly being recognized that the determination of the residual shear strength could be more important than the determination of the stress conditions that trigger the liquefaction itself (Ishihara, 1993).

The residual or post-liquefaction strength of liquefiable soils is the main factor determining whether a soil mass will experience flow failure and large deformations due to earthquake loading. The cost and extent of measures required to remediate liquefiable soil deposits are, therefore, greatly influenced by the magnitude of the liquefied shear strength. As pointed out by Seed (1987), it may be adequate and economically advantageous simply to ensure the stability of an earth deposit or structure against flow failure after strength loss has been triggered than to prevent the triggering. The use of residual shear strength provides a very simple and attractive method to assess liquefaction potential. Since the residual shear strength should correspond to the minimum shear strength, its use predicts the worst-case scenario. Another advantage is

that for soils that undergo true liquefaction (i.e., soils not exhibiting cyclic mobility), both dynamic and static liquefaction can be predicted since the residual shear strength is the same whether the loading is monotonic or cyclic.

Poulos et al. (1985) have developed a laboratory procedure for estimating the post-liquefaction shear strength of soil deposits using the results of monotonically loaded, consolidated-undrained triaxial tests on reconstituted soil samples. This procedure is based on the so-called steady-state strength and the existence of a unique line relating void ratio and steady-state strength. The main limitations of the procedure stem mainly from the existence and uniqueness of the steady-state strength which is the basis for the laboratory approach.

THE STEADY-STATE OF DEFORMATION

Poulos (1981) defined the steady-state of deformation as “the state at which a soil mass is continuously deforming at constant volume, constant effective stresses and constant velocity.” This state is achieved only after all particle orientations have reached a statistically steady-state configuration and after all particle breakage, if any, is complete, so that the shear stress needed to continue deformation and the velocity of deformation remain constant. The liquefaction evaluation procedure developed by Poulos et al. (1985) assumes that the slope of the steady-state line in the void ratio vs. log (mean stress) plot is unique and is affected mainly by the shape of the grains of a given soil. Due to difficulties in obtaining undisturbed samples, the slope of the steady-state line is

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determined using reconstituted samples in monotonically loaded, consolidated-undrained triaxial tests with pore pressure measurements.

The Poulos et al. (1985) procedure assumes that the steady-state line is the same for reconstituted and undisturbed samples, and is independent of the sample preparation procedure. It is thus postulated that the steady-state strength is an intrinsic soil property and is only a function of the void ratio of the soil. Several recent experimental studies have been performed specifically to investigate the validity of this postulate. These studies appear to indicate that the steady-state strength is not an intrinsic property, but is affected by many variables such as the sample preparation technique and the loading conditions (e.g., Miura and Toki, 1982; Vaid et al., 1990; Reimer and Seed, 1997; Yoshimine and Ishihara, 1998; Høeg et al. 2000), stress path (e.g., Vaid and Chern, 1985; Reimer and Seed, 1997), and effective confining stress (Vasquez-Herrera et al., 1990; Baziar et al., 1992; Ishihara, 1993).

There are relatively a small number of studies on the issue of how the steady-state could be achieved experimentally, and what should be considered as necessary and sufficient criteria to judge that a steady-state has been achieved experimentally. As pointed out by Castro (1999), the most important experimental requirement for determining the steady-state strength is the ability to apply large strains while maintaining sufficient specimen uniformity. The difficulty of achieving a true steady-state (i.e., with constant volume and effective stresses during continuous shearing) at large strain, particularly for contractive-dilative soils, has led to the proposal for the use of the quasi-steady-state as a basis for determining the undrained shear strength of liquefiable soils (*see* for example Ishihara, 1993). The quasi-steady-state occurs at the point where the soil changes from a contractive to a dilative response, and where the shear stress is minimum for contractive-dilative soils (Alarcon-Guzman et al., 1988; Ishihara, 1993). The validity of using the steady-state strength or the quasi-steady-state as basis for determining the residual shear strength of liquefiable soils is still the subject of much discussion and active research.

ASSUMPTIONS AND METHODOLOGY

The objective of the study is to provide a better understanding of the undrained response of granular soils particularly on the existence of the steady-state condition, the parameters that affect the steady-state, and the validity of using the steady-state condition in defining the residual undrained shear strength of liquefied soils. To achieve this objective, a simple constitutive model for granular soils is formulated and used to simulate the undrained response of granular soils. It is hoped that results of the study could be used to improve laboratory-based liquefaction evaluation procedures.

In the analytical modeling of the undrained response of granular soils, the basic postulate taken is that liquefaction is just another aspect of the material response of

soils. This means that stresses and strain increments are uniquely related in all stages of the undrained response of granular soils. Jefferies (1999) advocated a similar philosophy in the formulation of a constitutive model for sand. It is possible, of course, for soils to exhibit unstable response and violate stability postulates in continuum mechanics, in which case stresses and strains are no longer uniquely related. Instability can be due to a negative second increment of work as has been observed in the failure of very loose soil deposits (e.g., Lade, 1992). Another cause of instability is strain localization and shear band formation (e.g., Vardoulakis, 1996). Although unstable response should be included in a complete study of soil liquefaction, such issues are not discussed here.

The study of the steady-state behavior of liquefiable soils consists of the following steps: 1) Develop a model for the undrained monotonic response of granular soils using a number of well-known and well-established empirical relationships that are based on the results of drained tests on granular soils, 2) Validate the model against good quality results on undrained triaxial tests of granular soils, 3) Use the model to investigate the effects of different parameters on the undrained response of granular soils, and 4) Use the model to investigate and extrapolate what a true steady-state would be based on an idealization of granular material response.

The main value of a model is that it could be used to investigate the response of soils to loading conditions that are beyond what could be achieved in laboratory experiments. Also, sensitivity of soil response to variations in material property can be easily investigated. However, to ensure that the predictions and insights gained from the model are valid, it is necessary to show that the general conclusions from the modelling are not significantly dependent on the details of the model, and are not simply artefacts of the model formulation. This can be achieved by making the model consistent, well supported by available data and valid for a wide range of loading conditions.

A SIMPLE CONSTITUTIVE MODEL FOR GRANULAR SOILS

In the following sections, a simple constitutive model for the monotonic behaviour of granular soils is developed. For mathematical simplicity, the model is formulated primarily to predict the undrained response of granular soils over a wide range of initial confining stresses for a given soil density. This means that one set of parameters is required for each density of soil. It is possible to develop a fully integrated model for granular soils requiring only a single set of material parameters over the full range of density values as well as confining stress. However, this will require more data to represent the variation of each constitutive parameter with density. For undrained loading, soil density remains constant so that the density-dependency of the constitutive parameters during loading need not be considered. Another simplifi-

cation made is to assume that elastic deformation is negligible in comparison to the plastic deformation. This allows for the use of a rigid-plastic formulation instead of an elasto-plastic formulation.

The model is based on established relationships on the behavior of granular soils, which are extensively supported by experimental data from drained triaxial tests on sands and silts. The model is formulated in terms of conventional triaxial loading condition using the mean stress p , shear stress q , shear stress ratio η , volumetric strain increment $d\varepsilon_v$ and shear strain increment $d\varepsilon_s$. These are defined as:

$$p = \frac{1}{3} (\sigma_1 + 2\sigma_3), \quad q = \sigma_1 - \sigma_3, \quad \eta = q/p \quad (1)$$

$$d\varepsilon_v = d\varepsilon_1 + 2d\varepsilon_3, \quad d\varepsilon_s = \frac{2}{3} (d\varepsilon_1 - d\varepsilon_3) \quad (2)$$

where σ_1 and σ_3 are the major and minor principal stresses, respectively, and $d\varepsilon_1$ and $d\varepsilon_3$ are the corresponding principal strain increments. All stresses are effective unless otherwise noted. The model can be extended to plane strain or three-dimensional loading conditions, however, the goal of the modeling is to understand the response of granular soils under laboratory triaxial loading conditions. Consequently, extension of the model to more general loading conditions is not included in this paper.

Frictional Resistance

Several experimental results have shown that the peak friction angle ϕ_p of sands increases with increasing relative density D_r and decreases with increasing confining stress p (e.g., Lee and Seed, 1967; Vesic and Clough, 1968; Been and Jefferies, 1985; Bolton, 1986). Using experimental results for several sands, Bolton (1986) showed that the combined effects of relative density D_r and confining stress p on the peak friction angle ϕ_p can be represented by the following expression, which is an extension of the logarithmic relation proposed by Vesic and Clough (1968):

$$\phi_p - \phi_{cr} = C_\phi D_r \ln \left(\frac{p_{cr}}{p} \right) \quad (3)$$

In this equation, C_ϕ is an empirical constant, and $D_r = (e_{max} - e)/(e_{max} - e_{min})$ is the relative density (e_{min} , e_{max} and e are the minimum, maximum and the current void ratios, respectively). Equation (3) indicates that beyond a certain "breakdown pressure" p_{cr} , the peak friction angle is not affected by the initial density of the soil and the stress confinement, and becomes equal to a constant critical-state friction angle ϕ_{cr} .

Analogous to Eq. (3), an expression relating the peak shear stress ratio η_p to confining pressure p is formulated as:

$$\eta_p = \eta_{cr} + CD_r \ln \left(\frac{p_{cr}}{p} \right) \quad (4)$$

where C is a material parameter related to C_ϕ giving the rate of change of the peak shear stress ratio with confin-

ing stress. As will be shown below, η_{cr} and p_{cr} correspond to the shear stress ratio and the mean stress at critical-state, respectively. The peak shear stress ratio η_p and the critical shear stress ratio η_{cr} are related to the peak friction angle ϕ_p and the critical friction angle ϕ_{cr} as:

$$\eta_p = \frac{6 \sin \phi_p}{(3 - \sin \phi_p)}, \quad \eta_{cr} = \frac{6 \sin \phi_{cr}}{(3 - \sin \phi_{cr})} \quad (5)$$

As pointed out by Bolton (1986), the mean stress at critical-state p_{cr} is not a constant for a given soil but should vary with relative density. For undrained loading, p_{cr} will be treated as a constant parameter for a given relative density since the relative density remains constant during undrained loading.

Stress-Dilatancy

Another important equation needed to characterize the response of granular soils is a stress-dilatancy relation linking dilation and shear deformation with the shear stress level. A stress-dilatancy relation will also provide further physical explanation to the stress and density-dependent frictional strength given in Eq. (4). Consider the following stress-dilatancy relation, which can be derived using critical-state soil mechanics (Schofield and Wroth, 1968):

$$\frac{d\varepsilon_{vd}}{d\varepsilon_s} = (\eta_{cr} - \eta) \quad (6)$$

The above relationship gives the ratio of the volumetric strain increment due to dilatancy $d\varepsilon_{vd}$ to the shear strain increment $d\varepsilon_s$ as function of the stress ratio $\eta = q/p$. For $\eta < \eta_{cr}$, it can be seen that $d\varepsilon_{vd}$ is positive and the soil is contracting. For $\eta > \eta_{cr}$, $d\varepsilon_{vd}$ is negative and the soil is dilating. During monotonic shearing under undrained conditions, the mean stress p decreases for $\eta < \eta_{cr}$ and increases for $\eta > \eta_{cr}$.

The maximum positive dilation (i.e., increase in volume) is obtained when $\eta = \eta_p$. Substituting this in Eq. (6) and using Eq. (4), the maximum dilatancy ratio is obtained as a function of stress level and relative density:

$$\left(\frac{d\varepsilon_{vd}}{d\varepsilon_s} \right)_{max} = (\eta_{cr} - \eta_p) = -CD_r \ln \left(\frac{p_{cr}}{p} \right) \quad (7)$$

Equation (7) provides another definition of the parameter C , that is this parameter gives a direct measure of the dilatancy potential of granular soils. For a given relative density D_r and mean stress p , the larger the value of C the larger is the dilation potential of the soil. Equation (7) also indicates that the maximum dilation of soils increases with increasing relative density D_r and decreasing mean stress p , which is consistent with experimental observations. For $p \geq p_{cr}$, relative density and stress confinement have no effect on the peak dilation ratio $\eta_{cr} - \eta_p$, which becomes zero, and the peak shear stress ratio η_p becomes constant and equal to η_{cr} . Once p reaches p_{cr} , the soil can be sheared continuously at a constant volume assuming that most volumetric change is due to shear-induced dilation.

The main deficiency of Eq. (6) is that it yields a non-

zero dilatancy at the start of shear loading at $\eta=0$, and that it predicts a dilatancy ratio $d\varepsilon_{vd}/d\varepsilon_s$ equal to η_{cr} when $\eta=0$. This does not agree with experimental results which show that soils do not immediately dilate when sheared at $\eta=0$. To remedy this deficiency, Eq. (6) is modified and the following stress-dilatancy relation is instead used:

$$\frac{d\varepsilon_{vd}}{d\varepsilon_s} = \frac{\eta}{\eta_p} (\eta_{cr} - \eta) \quad (8)$$

This equation gives a better fit than Eq. (6) of experimental results on sands showing zero dilation at the start of shearing. Validation of Eq. (8) against experimental data is presented in Appendix A. Other than this difference, Eq. (8) predicts the same results as Eq. (6) showing contraction for $\eta < \eta_{cr}$, dilation for $\eta > \eta_{cr}$, zero dilation at $\eta = \eta_{cr}$ and peak dilation at $\eta = \eta_p$. More importantly, the peak dilatancy ratio remains equal to $\eta_{cr} - \eta_p$.

Consolidation Strains

To model the development of volumetric strain due to changes in the mean stress dp , a simple relationship is used assuming linearity between the void ratio e and $\ln(p)$ both during loading and unloading-reloading. This gives a relationship between the change in consolidation strain $d\varepsilon_{vc}$ and the change in mean stress dp of the following form:

$$d\varepsilon_{vc} = \frac{1}{B_0} \frac{dp}{p} \quad (9)$$

where B_0 is a reference bulk modulus related to the slope λ of the consolidation line, the slope κ of the swelling line in the e vs. $\ln(p)$ plot, and the initial void ratio e_0 of the soil. During loading ($dp > 0$) and when $p \geq p_0$ where p_0 is the initial or consolidation stress, B_0 takes a value equal to:

$$B_0 = \frac{(1 + e_0)}{\lambda} \quad (10)$$

During unloading ($dp < 0$) and re-loading ($dp > 0$) when $p < p_0$, the bulk modulus B_0 takes a value equal to:

$$B_0 = \frac{(1 + e_0)}{\kappa} \quad (11)$$

More advanced models for the development of consolidations strains, which are valid for a very wide range of void ratios and mean stresses, can be used. However, in the interest of keeping the analysis simple and to reduce the number of material parameters, Eqs. (10) and (11) are assumed to be sufficient over the range of confining stresses encountered in typical loading situations in the field.

Shear Stress Ratio vs. Shear Displacement

Another well established relation that is used in the analytical study is the hyperbolic stress-strain model (Duncan and Chang, 1970). Here, the hyperbolic relation will be given in terms of the mobilized shear stress ratio $\eta = q/p$ and the shear strain ε_s .

$$\eta = \frac{\eta_p \varepsilon_s}{A + \varepsilon_s} \quad (12)$$

where A is a material parameter related to the initial shear modulus G_{max} , as shown below. A similar mobilized shear stress ratio η vs. shear strain ε_s is used in several models for sand (e.g., Poorooshasb and Pietruszczak, 1986). The above equation models a shear stress ratio η increasing asymptotically towards the peak shear stress ratio η_p with increasing shear strain.

Undrained Response

The total volumetric strain increment $d\varepsilon_v$ is obtained from the superposition of the volumetric consolidation and dilation strains increments:

$$d\varepsilon_v = d\varepsilon_{vc} + d\varepsilon_{vd} \quad (13)$$

Substituting Eqs. (8) and (9) yields:

$$d\varepsilon_v = \frac{1}{B_0} \frac{dp}{p} + \frac{\eta}{\eta_p} (\eta_{cr} - \eta) d\varepsilon_s \quad (14)$$

Using Eqs. (4) and (12) in Eq. (14), an incremental relation giving the total volumetric strain increment $d\varepsilon_v$ as a function of the shear strain increment $d\varepsilon_s$ and mean stress increment dp is obtained.

$$d\varepsilon_v = \frac{1}{B_0} \frac{dp}{p} + \left(\frac{\varepsilon_s}{A + \varepsilon_s} \right) \left[\eta_{cr} \left(\frac{A}{A + \varepsilon_s} \right) - CD_r \ln \left(\frac{p_{cr}}{p} \right) \left(\frac{\varepsilon_s}{A + \varepsilon_s} \right) \right] d\varepsilon_s \quad (15)$$

Assuming the soil is fully-saturated and neglecting pore fluid and soil grain compressibility, the volume of the sample remains constant during undrained loading condition, and $d\varepsilon_v = 0$. Applying this condition in Eq. (15) gives the change in mean stress dp as function of shear deformation as:

$$dp = -B_0 p \left(\frac{\varepsilon_s}{A + \varepsilon_s} \right) \left[\eta_{cr} \left(\frac{A}{A + \varepsilon_s} \right) - CD_r \ln \left(\frac{p_{cr}}{p} \right) \left(\frac{\varepsilon_s}{A + \varepsilon_s} \right) \right] d\varepsilon_s \quad (16)$$

This equation completes the incremental relations needed to describe the undrained monotonic shear behavior of granular soils. For a given stress increment $d\varepsilon_s$, the incremental relations are integrated numerically as follows: 1) the change in mean stress dp is calculated from Eq. (16), 2) p and ε_s are updated as: $p \leftarrow p + dp$ and $\varepsilon_s \leftarrow \varepsilon_s + d\varepsilon_s$, 3) the peak shear stress ratio η_p is updated using Eq. (4) and the new value of p , 4) the shear stress ratio η is updated using Eq. (12) and the new values of p and ε_s . The integration is carried from the initial value of $p = p_0$ as the shear strain ε_s is increased from zero until $p \rightarrow p_{cr}$.

It is important to note the two competing components in the change in mean stress dp in Eq. (16). During an increase in shear strain $d\varepsilon_s$, the first term inside the bracket in Eq. (16) causes a negative change in dp (contraction), while the second term gives a positive change in dp (dilation). These two competing components will determine whether the soil will behave in a contractive or a dilative manner during a load increment.

THE CRITICAL AND STEADY-STATES OF GRANULAR SOILS

Before the model is applied to the simulation of the undrained response granular soils, it is essential to first define the formal conditions corresponding to the critical-state and steady-state, and to show how the model will satisfy these conditions. The critical-state is defined formally by the condition that there are no changes in the parameters describing the state of the soil during an increase in shear distortion $d\epsilon_s$ (e.g., Schofield and Wroth, 1968; Poorooshasb, 1989). Poorooshasb (1989) showed that the same conditions should apply to the steady-state as well. For triaxial loading condition, the state parameters are given by the mean stress p , shear stress q , and by the density of the sample (as measured by the void ratio e or volumetric strain ϵ_v). The mathematical conditions necessary for the occurrence of the critical-state and steady-state are therefore:

$$\frac{dq}{d\epsilon_s} = 0 \quad (17a)$$

$$\frac{dp}{d\epsilon_s} = 0 \quad (17b)$$

$$\frac{d\epsilon_v}{d\epsilon_s} = 0 \quad (17c)$$

As shown in Eq. (14), the total volume changes in a granular soil are due to dilation $d\epsilon_{vd}$ and to consolidation $d\epsilon_{vc}$. Dividing Eq. (14) throughout by $d\epsilon_s$:

$$\frac{d\epsilon_v}{d\epsilon_s} = \frac{1}{B} \frac{dp}{d\epsilon_s} + \frac{\eta}{\eta_p} (\eta_{cr} - \eta) \quad (18)$$

where $B = B_0 p$ is the current value of the bulk modulus. From this equation, it follows that the second condition $dp/d\epsilon_s = 0$ (Eq. (17b)) and the third condition $d\epsilon_v/d\epsilon_s = 0$ (Eq. (17c)) are, in general, not independent of each other, and one of these two must be specified as an external boundary condition (i.e., as a prescribed load or deformation) before the other can be satisfied. Equations (17b) and (17c) can be simultaneously satisfied only in the unlikely condition of $d\epsilon_{vc}$ being exactly equal but opposite $d\epsilon_{vd}$.

For drained loading, Eqs. (17b) and (17c) can be satisfied only during a $p = \text{constant}$ stress path. If $dp = 0$, the consolidation volumetric strain increment $d\epsilon_{vc}$ is equal to zero and Eq. (18) becomes:

$$\frac{d\epsilon_v}{d\epsilon_s} = \frac{\eta}{\eta_p} (\eta_{cr} - \eta); \quad dp = 0 \quad (19)$$

From this equation, the third condition given in Eq. (17c) is satisfied only when dilation is zero. This occurs when $p = p_{cr}$ at which point $\eta = \eta_p = \eta_{cr}$ as can be obtained from Eq. (7).

In the case of undrained condition, the sample volume is constant, hence $d\epsilon_v = 0$, and Eq. (17c) is automatically satisfied. Equation (18) then becomes:

$$\frac{dp}{d\epsilon_s} = -B \frac{\eta}{\eta_p} (\eta_{cr} - \eta); \quad d\epsilon_v = 0 \quad (20)$$

From this equation, the second condition given in Eq. (17b) is also satisfied when $p = p_{cr}$ and where $\eta = \eta_p = \eta_{cr}$.

Since both the shear stress ratio η and the mean stress p are constant at the critical-state, the first condition $dq = 0$ given by Eq. (17a) is also satisfied. Since the dilation is assumed to be zero for $p \geq p_{cr}$, the critical-state is achieved not just at $p = p_{cr}$ but continues with increasing shear deformation for $p > p_{cr}$. More importantly, assuming that Eqs. (17a) to (17c) are also the same conditions for the steady-state to occur, and that critical-state occurs when $\eta = \eta_p = \eta_{cr}$ at $p = p_{cr}$ for both drained and undrained conditions (as given by Eqs. (19) and (20)), it can be concluded that the critical-state and the steady-state are theoretically the same and that both conditions occur at the same stress point at $p = p_{cr}$.

Before the critical-state is reached (i.e., for $p < p_{cr}$), there is another point at which dilation $d\epsilon_{vd}$ is momentarily zero according to Eq. (8). This point occurs at $\eta = \eta_{cr}$ when the soil changes from a contractive or negative dilation ($\eta < \eta_{cr}$) to a positive dilation ($\eta > \eta_{cr}$). This change from contractive to dilative response corresponds to the phase transformation condition (Ishihara et al., 1975), and the $\eta = \eta_{cr}$ line corresponds to the phase transformation line. Since dilation can be zero at only one given shear stress ratio based on Eq. (8), it follows that both phase transformation and the critical-state should both occur at the same critical shear stress ratio η_{cr} . This means that contractive-dilative soils sheared under undrained condition would first contract until it reaches the phase transformation at $\eta = \eta_{cr}$, dilate above the transformation, then gradually return back to the $\eta = \eta_{cr}$ line towards the critical-state. Unlike the peak shear stress ratio η_p , the critical shear stress ratio η_{cr} is a material parameter dependent only on the mineral composition of the soil grains, and is independent of density and confining stress. Finally, another case where Eqs. (17a) to (17c) are satisfied is when $CD_r \approx 0$, which is the case for very loose and non-dilative samples.

MODEL VALIDATION

To illustrate its validity, the constitutive model is used to simulate the triaxial test results obtained by Verdugo and Ishihara (1996) on Toyoura sand. Comparisons of model simulations and experimental results are shown in Figs. 1 to 3. Three sets of test results on Toyoura sand with void ratios at the end of consolidation of 0.933, 0.833 and 0.735 are used in the validation of the model. These void ratios correspond, respectively, to relative densities D_r of 11.6% (loose), 37.9% (medium dense) and 63.7% (very dense). The test results of Verdugo and Ishihara (1996) provide one of the most convincing sets of experimental data on the existence of a true steady-state response. Different series of tests were conducted on identically prepared samples with almost identical void ratios at the end of consolidation. The tendency of each individual test result shown in Figs. 1 to 3 to approach a steady-state can be discerned from the stress-strain curves which tend toward a constant shear stress. Note the large shear

Fig. 1. Comparison of simulated and experimental data on undrained triaxial tests of loose Toyoura sand: SS = steady-state, PT = phase transformation

strains approaching 20% which were used in the experiments to achieve the steady-state.

The strongest evidence for the existence of a steady-state is the tendency of the stress paths and the shear stress-strain curves for each set of relative density to converge towards a single point for all tests starting at different consolidation stresses. The uniqueness of the steady-state for soil samples with the same initial void ratio but different initial confining stresses is one of the most important conditions in the use of the steady-state as the basis for determining the residual shear strength in liquefaction analysis. The uniqueness of the steady-state is obtained even for a very wide range of confining stresses of up to $p = 3$ MPa. Note the very high values of p_{cr} of 1.2 MPa and 3.0 MPa for the medium and dense sands requiring a high-pressure triaxial cell to reach the steady-state.

Table 1 lists the material parameters used in the simulations of the experimental results shown in Figs. 1 to 3. Procedures for determining the model parameters are briefly outlined in Appendix B. Again, it is important to note that for each set of test or initial void ratio, only one set of material parameters is used in the simulation. From Table 1, the only parameter not affected by the initial

Fig. 2. Comparison of simulated and experimental data on undrained triaxial tests of medium dense Toyoura sand

density is the critical shear stress ratio η_{cr} which is equal to 1.25 for the three densities. The results suggest that the critical shear stress ratio η_{cr} is a material property dependent only on the mineral composition of the soil. The shear deformability parameter A , the dilatancy parameter C , the mean stress at critical-state p_{cr} , and the reference bulk modulus parameter B_0 are all dependent on initial density. As can be seen, good agreement between simulated and experimental results is obtained for the individual test results and for the overall response of each set of relative densities over the wide range of consolidation stresses used. The most important result of the simulation is the demonstration that the model can replicate the steady-state response of sands using a model based only on the drained response of sands and not many model parameters. For each test, the stress paths merge and asymptotically approach the critical mean stress at $p = p_{cr}$, and the shear stress q becomes almost constant as the shear strain is increased and as the mean stress p approaches p_{cr} .

PARAMETRIC STUDY

To gain further understanding of the undrained response of granular soils, a parametric study is carried

Fig. 3. Comparison of simulated and experimental data on undrained triaxial tests of very dense Toyoura sand

Fig. 4. Effects of the dilatancy parameter C on simulated undrained triaxial response of granular soils: SS = steady-state, QSS = quasi-steady-state, PT = phase transformation

Table 1. Material parameters used in the simulation of the undrained triaxial behavior of Toyoura sand

Parameter (1)	Loose (2)	Medium dense (3)	Very dense (4)
Initial void ratio, e_0	0.933	0.833	0.735
Relative density, D_r (%)	11.6	37.9	63.7
Mean stress at critical-state, p_{cr} (MPa)	0.05	1.2	3.0
Critical shear stress ratio, η_{cr}	1.25	1.25	1.25
Dilatancy parameter, C	0.13	0.37	0.47
Shear deformability parameter, A (1/MPa)	0.015	0.005	0.002
Compression index, λ	0.013	0.018	0.174
Swelling index, κ	0.003	0.003	0.002

out using the model presented above. As shown in Table 1, the main parameters which control the undrained response of granular soils are the dilatancy parameter C , the shear deformability parameter A , and the reference bulk compressibility parameter B_0 . The effects of these parameters are studied in a series of parametric studies. A parametric study is also carried out to

determine the effects of the initial stress p_0 . Figures 4 to 7 show the results of the parametric study on the effects of these parameters. In all the simulations, the critical shear stress ratio is kept constant at $\eta_{cr} = 1.0$, the mean stress at critical-state at $p_{cr} = 250$ kPa, and the relative density at $D_r = 50\%$. For all the simulations shown in Figs. 4 to 6, shearing started at a consolidation stress of $p_0 = 100$ kPa. To highlight their effects, the different parameters were given a wide range of values, which could be outside their expected normal variations.

Effects of Dilatancy

To investigate the effects of dilatancy on the undrained response of granular soils, a parametric investigation is carried out where different values of the dilatancy parameter C are used while keeping the other parameters constant. As noted above, an increasing value of C increases the peak dilation of the soil for $p < p_{cr}$. The results of the parametric study are shown in Fig. 4 for four different values of C equal to 0.02, 0.1, 0.2 and 1.0. The values of the other model parameters are shown in the figure. The effects of the dilatancy parameter C can be clearly seen in the effective stress paths in the p - q plane. The simulated result for $C = 1.0$ shows an initially con-

Fig. 5. Effects of the shear deformability parameter A on simulated undrained triaxial response of granular soils

Fig. 6. Effects of the reference bulk modulus B_0 on simulated undrained triaxial response of granular soils

tractive response, followed by phase transformation and a very dilative response. In the dilative phase, the effective stress path gradually moves back towards the phase transformation line and the steady-state at $p_{cr} = 250$ kPa with increasing mean stress. The corresponding q vs. ϵ_s also tends towards a steady-state with the shear stress becoming almost constant at $q \approx 250$ kPa as the shear strain is increased. The results for $C = 0.1$ and 0.2 also show contractive response followed by phase transformation then dilative response. However, there is not enough capacity for the soils to dilate for these two results to bring the stresses to the steady-state even after 50% of shear deformation. The simulated curve for $C = 0.02$ exhibits a purely contractive response. The stress path stays close to the phase transformation line with very little change in shear stress, and unable to go beyond phase transformation and dilate due to very low dilatancy potential.

Effects of Shear Deformability

To clarify the physical meaning of the shear deformability parameter A , the derivative of Eq. (12) with respect to the shear strain ϵ_s is obtained.

$$d\eta = d(q/p) = dq - \eta dp = \frac{A\eta_p}{(A + \epsilon_s)^2} d\epsilon_s \quad (22)$$

At the initial state, $\epsilon_s = 0$ and $\eta = 0$, thus

$$\left. \frac{d\eta}{d\epsilon_s} \right|_{\epsilon_s=0, \eta=0} = \left. \frac{dq}{d\epsilon_s} \right|_{\epsilon_s=0} = \frac{\eta_p}{A} \quad (23)$$

Since $dq/d\epsilon_s = 3G_{max}$ for $\epsilon_s = 0$, where G_{max} is the initial or maximum shear modulus, then the shear deformability parameter A is inversely proportional to G_{max} :

$$A = \frac{\eta_p}{3G_{max}} \quad (24)$$

The results of the simulations for four values of the parameter A corresponding to 0.001, 0.005, 0.01 and 0.1/MPa are shown in Fig. 5. As can be seen, for the small value of A of 0.001/MPa (or a large G_{max}), there is very little contractive response. Due to the stiff shear response, an increase in shear strain caused a large change in shear stress dq in comparison to the mean stress change dp . Thus, the soil was able to dilate and asymptotically approach the steady-state at $p_{cr} = 250$ kPa. In contrast, for the high value of $A = 0.1$ /MPa (or a low G_{max}), the shear stress q could not increase much faster than the mean stress p , resulting in a very contractive response. Due to the low shear stiffness, the simulated soil response is unable to mobilize an increase in shear stress with

Fig. 7. Effects of initial confining stresses p_0 on simulated undrained triaxial response of granular soils

increasing shear deformation and counteract the contractive response. As a result, with increasing deformation, the soil stays close to the phase transformation line with very little change in shear stress and the soil is unable to dilate towards the steady-state. The effect of a large initial shear modulus is, therefore, to increase the potential of the soil to dilate under increasing shear deformation.

Effects of Bulk Modulus

In the parametric study involving the bulk modulus parameter B_0 , differences in the response between loading (Eq. (10)) and unloading-reloading (Eq. (11)) are neglected and only a single value of B_0 corresponding to the swelling index κ (Eq. (11)) is used. The employment of a single value of the bulk modulus parameter B_0 will simplify the discussion of the parametric study. The use of B_0 corresponding to the swelling index κ is valid in the contractive-dilative portion of the stress path where the mean stress has not exceeded the initial mean stress (i.e., the over-consolidated region of the stress path when $p < p_0$). The bulk modulus corresponding to the compression index λ (Eq. (10)) is needed only in the purely dilative portion of the effective stress path when the mean stress is increasing and $p > p_0$. Using a bulk modulus corresponding to the swelling index κ instead of λ will affect

only the results for $p > p_0$. However, when the mean stress p has exceeded the initial mean stress p_0 , dilation is expected to dominate the response, and differences between κ and λ are expected not to strongly affect the simulated results. It should be noted, however, that both the compression index λ and swelling index κ should be used in the application of the model to realistic problems.

Four reference bulk moduli are used: $B_0 = 10, 100, 1000$ and 10000 . As can be seen from Eq. (16), for a given change in shear strain $d\epsilon_s$, the change in the mean stress dp is directly proportional to the reference bulk modulus B_0 . As shown in Fig. 6, for a given strain increment $d\epsilon_s$, the larger the value of B_0 the larger is the change in mean stress dp both in the contractive and dilative phases. The most dramatic effect of a high bulk modulus ($B_0 = 10000$) is to cause a very contractive response and reduce the peak shear stress before the phase transformation is reached. For $B_0 = 10000$ the shear stress becomes momentarily close to zero as the stress path crosses the phase transformation line. However, the soil almost immediately dilates recovering from this momentary liquefaction, and the stress path moves to the right towards the steady-state. For $B_0 = 10$, there is very little change in the pore pressure both in the contractive and dilative regimes, and thus the soil is unable to mobilize pore pressure changes and dilate towards the steady-state.

Effects of Initial Stresses

As can be seen for the case of the simulations using $C = 0.02$ in Fig. 4, and $A = 0.01/\text{MPa}$ in Fig. 5, the shear stress-strain curve can exhibit essentially constant shear stresses with increasing shear strain for about 20% of shear deformation. It is tempting to conclude that these simulated curves have attained a true steady-state on the basis of a nearly constant shear response alone. However, as can be seen in the Figs. 4 and 5, the simulated curves for these parameters did not reach the true steady-state at $p_{cr} = 250$ kPa, and the nearly-constant stress-strain curves actually correspond to a quasi-steady-state. The very small dilatancy parameter C used in Fig. 4 prevented the effective stress path from shifting from a contractive response to a dilative response. Similarly for Fig. 5, the low shear modulus (or $A = 0.1/\text{MPa}$) prevented the shear stress from increasing, and the simulated response is unable to dilate past the phase transformation line. For both curves, the contractive response just balanced the dilative response due to the combinations of the parameters used. The simulated curves are then able to stay near the phase transformation, or at a quasi-steady-state, for a long duration of shear deformation.

To show that the simulated curve for $C = 0.02$ in Fig. 7 does not correspond to the true steady-state, simulations are carried out using the same set of parameters as for the $C = 0.02$ curve in Fig. 7 but with three different consolidation stresses. Figure 7 shows the model results for the initial stresses of $p_0 = 50, 100$ and 250 kPa. All the curves show a tendency for the shear stress to become constant with increasing strain. However, the stress paths and the stress-strain curves do not converge to a unique point

even after a shear strain of 50% has been applied, as would have been expected if the curves have reached the steady-state.

CONCLUSIONS

Using a few well-known relationships on the behavior of granular soils based on the results of drained triaxial tests, an analytical study was carried out on the undrained response of granular soils. The study has shown that the undrained behavior and static liquefaction of granular soils can be adequately captured by a simple analytical model and with only a few parameters. It was shown analytically that the true steady-state for granular soils occurs when the soil frictional resistance becomes constant and dilation becomes zero. Upon achieving this state, a granular soil deforms continuously at constant volume and effective stresses. The steady-state and the critical-state were theoretically shown to refer to the same condition.

Some very important conclusions on the undrained behavior of granular soils can be obtained from the results of the parametric study shown in Figs. 4 to 7. The study indicated that the undrained behaviour of granular soils depends not only on the strength parameters and how far the initial stress is from the steady-state, but also on the dilation and deformation parameters. All the simulations shown in Figs. 4 to 7 start from the same stress point at $p_0 = 100$ kPa relative to the steady-state point at $p_{cr} = 250$ kPa. However, only a few of the simulated curves approached the steady-state point.

The dilatancy parameter C together with the relative density of the soil gives a measure of the dilatancy potential of soils. A low C -value for a given relative density means that the soil will behave in contractive manner. If a soil is consolidated at a mean pressure $p_0 < p_{cr}$, a low dilatancy potential due to a low value of CD_r means that the soil may not be able to dilate sufficiently past beyond the phase transformation line and move towards the steady-state. For soils with $CD_r = 0$ (i.e., either $C = 0$ or $D_r = 0$), the soil has no dilatancy at all. As a result, the stress path will stay below the phase transformation line and will contract towards $p = 0$ and $q = 0$ or some very low mean and shear stresses (see for instance Fig. 1).

The parametric study also demonstrated that the maximum shear modulus G_{max} and the reference bulk modulus B_0 have important influence on the liquefaction potential of granular soils. Under strain-controlled condition, the shear modulus controls the development of the shear stress dq for a given shear strain increment. This means, for instance, that even if a soil has a high dilatancy potential, the shear stress may not increase sufficiently if the soil has a low shear modulus. Thus, the shear stress may not increase beyond the phase transformation line and dilate towards the critical/steady-state even if the dilatancy parameter C is large. In other words a dilatant soil may become fully contractive if it has a low shear modulus. The bulk modulus controls the change in the mean stress dp for a given volumetric change both in the contractive

and dilative stages. A high bulk modulus may enhance the contraction of a soil causing it to liquefy momentarily, but it may also enhance dilation causing the mean stress to increase towards the steady-state.

The dependency of the liquefaction potential of granular soils on the deformability parameters G_{max} and B_0 implies that liquefaction should be strictly analyzed both as a deformation and strength problem. At the very least, deformability should be considered in evaluating the undrained shear strength of liquefiable soils. These conclusions support the philosophy stated as the main basis of this study, that liquefaction should be considered as just another aspect of the full constitutive response of granular soils (again barring the occurrence of any form of instability).

As shown by the case of the simulations involving low dilatancy potential and shear modulus, ($C = 0.02$ in Fig. 4, and $A = 0.1$ /MPa in Fig. 5) the shear stress-strain curve can exhibit quasi-steady-state response for long duration of shear deformation. The main conclusion from this result is that the flattening of the shear stress vs. shear strain curve from a single test alone should not be taken as an indication that a true steady-state has been achieved. A necessary condition to show that a granular soil has achieved a steady-state is when the stress paths and the stress-strain curves from tests on samples with the same initial density but with different initial stresses converge to a single constant point (as shown for instance in Figs. 1 to 3).

Finally, almost all the parametric simulations showed that the stress paths and the stress-strain curves can only asymptotically approach the steady-state even after the simulations were carried out to a 50% shear strain. This confirms what is often observed experimentally, that it is difficult to achieve a true steady-state particularly for contractive-dilative soils. There is also a question of the validity of using the true steady-state in cases such as those shown for the curves using $C = 0.02$ in Fig. 4, and $A = 0.1$ /MPa in Fig. 5 where only the quasi-steady-state was achieved. In theory, these two curves could asymptotically approach the steady-state at $p_{cr} = 250$ kPa if the shear strain is increased further. However, the fact that the shear stress remained almost constant for a large amount of shear deformation means that the soil should have already undergone a flow failure before the true steady-state can be achieved. It seems very reasonable, therefore, for cases where a soil is able to exhibit a quasi-steady-state for a large amount of shear deformation that the quasi-steady-state should be used as the basis for selecting the residual shear strength of liquefiable soils. It is important, however, to emphasize that the quasi-steady-state is not unique but is dependent on the magnitude of the initial consolidation stresses. Moreover, the use of the quasi-steady will depend largely on the amount of shear deformation induced while the shear stress stays constant.

It may be argued that some of the details of the simulated behavior from the analytical study are dependent on the elements of the model. Nonetheless, the major con-

clusions from the analytical study are physically sound and consistent, and can be arrived at by physical considerations alone.

REFERENCES

- 1) Alarcon-Guzman, A., Leonards, G. A. and Chameu, J. L. (1988): Undrained monotonic and cyclic strength of sands, *J. Geotech. Engrg.*, ASCE, **116** (6), 1089–1109.
- 2) Baziar, M., Dobry, R. and Elgamal, A. (1992): Engineering evaluation of permanent ground deformations due to seismically-induced liquefaction, Tech. Rep. NCEER-92007, *Natl. Ctr. for Earthquake Engrg. Res.*, State Univ. of New York, Buffalo.
- 3) Been, K. and Jefferies, M. G. (1985): A state parameter for sands, *Géotechnique*, **35** (2), 99–112.
- 4) Bolton, M. D. (1986): The strength and dilatancy of sands, *Géotechnique*, **36** (1), 65–78.
- 5) Casagrande, A. (1936): Characteristics of cohesionless soils affecting the stability of slopes and earthfills, *J. Boston Soci. Civil Engrg.*, **23** (1), 13–32.
- 6) Castro, G. (1969): Liquefaction of sands, *Harvard Soil Mechanics Series No. 81*, Harvard Univ., Cambridge, Mass.
- 7) Castro, G. (1999): Comments on the determination of the undrained steady-state strength of sandy soils, *Physics and Mechanics of Soil Liquefaction*, (eds. by Lade and Yamamuro), Balkema, Rotterdam, 205–212.
- 8) Duncan, J. M. and Chang, C. Y. (1970): Nonlinear analysis of stresses and strains in soils, *J. Soil Mech. Fnd. Div. ASCE*, **96** (SM5), 117–141.
- 9) Gutierrez, M. and Ishihara, K. (2000): Non-coaxiality and energy dissipation in granular materials, *Soils and Foundations*, **40** (2), 49–59.
- 10) Høeg, K. H., Dyvik, R. and Sandbækken, G. (2000): Strength of undisturbed versus reconstituted silt and silty sand specimens, *J. Geotech. Geoenviron. Engrg.*, ASCE, **126** (7), 606–617.
- 11) Ishihara, K. (1993): Liquefaction and flow failure during earthquakes, *Géotechnique*, **43** (3), 351–415.
- 12) Ishihara, K., Tatsuoka, F. and Yasuda, S. (1975): Undrained deformation and liquefaction of sand under cyclic shear stresses, *Soils and Foundations*, **23** (1), 29–44.
- 13) Jefferies, M. G. (1999): A critical-state view of liquefaction. *Physics and Mechanics of Soil Liquefaction*, (eds. by Lade and Yamamuro), Balkema, Rotterdam, 221–235.
- 14) Lade, P. V. (1992): Static instability and liquefaction of loose fine sandy slopes, *J. Geotech. Engrg.*, ASCE, **118** (1), 51–71.
- 15) Lee, K. H. and Seed, H. B. (1967): Drained strength characteristics of sands, *J. Soil Mech. Fnd. Div. ASCE*, **93** (SM6), 117–141.
- 16) Miura, S. and Toki, S. (1982): A sample preparation method and its effect on static and cyclic deformation-strength properties of sand, *Soils and Foundations*, **22** (1), 61–77.
- 17) Poorooshasb, H. B. (1989): Description of flow of sand using state parameters, *Computers and Geotechnics*, **8**, 195–218.
- 18) Poorooshasb, H. B. and Pietruszczak, S. (1986): A generalized flow theory for sand, *Soils and Foundations*, **26** (2), 1–15.
- 19) Poulos, S. J. (1981): The steady-state of deformation, *J. Geotech. Div. ASCE*, **107**, 553–562.
- 20) Poulos, S. J., Castro, G. and France, W. (1985): Liquefaction evaluation procedure, *J. Geotech. Engrg.*, ASCE, **111** (6), 772–792.
- 21) Reimer, M. F. and Seed, R. B. (1996): Factors affecting apparent position of steady-state line. *J. Geotech. and Geoenviron. Engrg.*, **123** (3), 281–288.
- 22) Schofield, A. N. and Wroth, C. P. (1968): *Critical State Soil Mechanics*, McGraw-Hill, London.
- 23) Seed, H. B. (1987): Design problems in soil liquefaction, *J. Geotech. Engrg.*, ASCE, **113** (8), 827–845.
- 24) Vaid, Y. P. and Chern, J. C. (1985): Cyclic and monotonic undrained response of saturated sands, *Advances in the Art of Testing Soils under Cyclic Conditions*, ASCE, 120–147.
- 25) Vaid, Y. P., Chung, E. and Kuerbis, R. (1990): Stress path and steady-state, *Can. Geotech. J.*, **27** (1), 81–91.
- 26) Vardoulakis, I. (1996): Deformation of water-saturated sand: II. Effect of pore water flow and shear banding, *Géotechnique*, **46** (3), 457–472.
- 27) Vasquez-Herrera, A., Dobry, R. and Baziar, M. (1990): Re-evaluation of liquefaction triggering and flow sliding in the Lower San Fernando Dam during the 1971 Earthquake, *4th U.S. Natl. Conf. on Earthquake Engrg.*, EERI, El Cerrito, Calif., 783–792.
- 28) Verdugo, R. and Ishihara, K. (1996): The steady-state of sandy soils, *Soils and Foundations*, **36** (2), 81–91.
- 29) Vesic, A. and Clough, G. W. (1968): Behavior of granular materials under high stresses, *J. Soil Mech. Fnd. Div. ASCE*, **94** (SM3), 661–688.
- 30) Yoshimine, M. and Ishihara, K. (1998): Flow potential of sand during liquefaction, *Soils and Foundations*, **38** (3), 189–198.

APPENDIX A—EXPERIMENTAL VALIDATION OF THE STRESS-DILATANCY RELATION

The model presented in this paper is based on established relationships on the behaviour of granular soils. An exception is the stress-dilatancy relationship given in Eq. (8), which is a modification of the critical-state soil mechanics stress-dilatancy equation (Eq. (6)). As mentioned, the main motivation for introducing Eq. (8) is to provide a better representation of the dilatancy response of granular soils particularly at low shear stress ratios. Equation (8) agrees with experimental results showing very little change in the mean stress at the start of undrained shearing. In comparison, the critical-state soil mechanics stress-dilatancy relationship given in Eq. (6) predicts that granular soils immediately contract upon shearing from $\eta = 0$ with a high contraction rate of $d\epsilon_{vd}/d\epsilon_s = \eta_{cr}$.

To show its validity, Eq. (8) is compared against available experimental results on dilatancy of granular soils. Figure A1 shows a comparison of Eq. (8) with experimental data from Gutierrez and Ishihara (2000) on dense Toyoura sand. The results of Gutierrez and Ishihara (2000) are for p -constant tests. Since there are no changes in mean stress p , the only volumetric changes are due to dilatancy $d\epsilon_{vd}$ during a p -constant loading. Thus, the results can be used to directly verify Eq. (8). Despite the scatter in experimental data, particularly at low shear stress ratio, it can be seen that Eq. (8) fits the available data reasonably well. The scatter at low shear stress ratio, which is also observed in other experimental data reported in the literature, is due to the small deformations at the start of shearing.

APPENDIX B—DETERMINATION OF MODEL PARAMETERS

This section briefly describes suggested procedures for determining the parameters required in the model. As listed in Table 1, the model parameters are the initial void ratio e_0 , the relative density D_r , the mean stress at critical-state p_{cr} , the critical shear stress ratio η_{cr} , the dilatancy parameter C , the shear deformability parameter A , and the consolidation parameters λ and κ . The initial void ratio e_0 and relative density D_r can be determined using standard soil testing procedures. The other parameters

Fig. A1. Stress-dilatancy relation for Toyoura sand during p -constant drained triaxial test (experimental data from Gutierrez and Ishihara, 2000)

can be obtained from the results of at least one isotropic consolidation and at least three triaxial tests. As noted above, all the required parameters can be obtained from the results of drained triaxial tests. However, the procedures outlined here can also be used for determining parameters from the results of undrained tests. It is suggested to conduct the triaxial tests over a wide range of initial confining stresses. From Eqs. (10) and (11), B_0 is related to the compression index λ for $dp > 0$ and $p > p_0$, and the swelling index κ for $dp < 0$ or $p < p_0$. The results of the isotropic consolidation tests are plotted in the void ratio e vs. $\ln(p)$ plot, and the parameters λ and κ are determined, respectively, from the slopes of the loading and unloading/reloading curves.

As noted above, the critical shear stress ratio η_{cr} also corresponds to the shear stress ratio at phase transformation. For drained tests, the critical shear stress ratio η_{cr} can be obtained by plotting shear stress ratio η against volumetric strain ε_v . The critical shear stress ratio η_{cr} corresponds to the point where volumetric strain ε_v changes from contractive (i.e., decreasing volume) to dilative response (i.e., increasing volume). For undrained tests, the critical shear stress ratio η_{cr} is obtained by plotting the effective stress path in terms of shear stress ratio η vs. the mean stress p . The critical shear stress ratio η_{cr} corresponds to the point where the mean stress p changes from contractive (i.e., decreasing mean stress) to dilative response (i.e., increasing mean stress). The critical shear stress ratio η_{cr} should be independent of confining stress, but an average value can be used in case there is a scatter in data from tests at different confining stresses.

The peak shear stress ratio η_p for each individual test is determined from the shear stress ratio η vs. shear strain ε_s plot, or from the procedure described below. The mean stress p corresponding to the peak shear stress ratio is

Fig. B1. Determination of p_{cr} and C from drained and undrained triaxial test

Fig. B2. Determination of η_p and A

noted from the effective stress path. For each test, the maximum dilatancy ratio $(\eta_p - \eta_{cr})$ is calculated and plotted against the mean stress p corresponding to η_p in a semi-logarithmic plot (Fig. B1). Equation (7) is fitted through the $(\eta_p - \eta_{cr})$ vs. $\ln(p)$ points to obtain the parameters C and p_{cr} . The point at which $(\eta_p - \eta_{cr}) = 0$ gives the critical mean stress p_{cr} , while the slope is equal to $0.434C \cdot D_r$. Figure B1 illustrates the determination of the parameters p_{cr} and C for dense Toyoura sand. As can be seen, good agreement is obtained for the maximum dilatancy ratio $(\eta_p - \eta_{cr})$ as function of mean stress p from the results of drained and undrained triaxial tests.

The shear parameter A is obtained by transforming Eq. (12) to the following form:

$$\frac{\varepsilon_s}{\eta} = \frac{A}{\eta_p} + \frac{\varepsilon_s}{\eta_p} \quad (B1)$$

which corresponds to the equation of a straight line $y =$

$mx + b$, where $x = \varepsilon_s$ and $y = \varepsilon_s/n$. Figure B2 shows a plot of the stress-strain response of medium dense Toyoura sand in terms of ε_s/η vs. ε_s . This plot verifies the validity of the linear relation given in Eq. (B1). The slope of the

line is equal to $1/\eta_p$, while the intercept along the vertical axis is equal to A/η_p . The parameter A is relatively unaffected by the effective stress path and initial confining stress.