EXPERIMENTAL EVALUATION OF THE VISCOUS PROPERTIES
OF SAND IN SHEAR

HASBULLAH NAWIR(i), FUMIO TATSUOKA(ii) and REIKO KIWANO(iii)

ABSTRACT

Effects of wet conditions (air-dried or saturated), pressure level and dry density on the viscous properties of sand were evaluated by performing unconventional triaxial compression tests on Toyoura sand. Effects of stress path were studied to a limited extent. During otherwise monotonic loading at a constant strain rate, the strain rate was changed stepwise and/or creep loading was performed after different strain rate histories. The test results were analysed in the framework of a three-component model, called the TESRA model, that had been developed based on the results from plane strain compression tests on dense Hostun and Toyoura sands at a confining pressure of 400 kPa. The manner of viscous stress change by a change in the irreversible shear strain rate observed under the various test conditions could be represented by the same viscosity function representing the viscous properties. The viscous stress decayed with an increase in the irreversible shear strain in all the tests, while the rate of decay was noticeably larger with air-dried sand than with saturated sand. The TESRA model using the same viscosity and decay functions simulates very well the effects of viscous properties observed in the whole tests.

Key words: constitutive modelling, creep, deformation, sand, triaxial compression tests, viscous properties (IGC: D6/D7)

INTRODUCTION

Viscous properties of a given material can be evaluated based on the observed loading rate effects, including: a) different stress-strain relations for different constant strain rates; b) responses of stress to step and gradual changes in the strain rate; c) creep deformation; and d) stress relaxation. In this paper, viscous properties of geomaterials that are not due to delayed dissipation of excess pore water pressure are dealt with. Kuwano and Jardine (2002) thought such viscous properties as above as "the gradual stabilization of a process in which the most critically loaded columns continue to buckle and fail, but at a rate that decreases continuously as the out of balance loads are transferred to more optimally arranged force columns". They also thought that "particle shape, packing and yielding at particle asperities are important factors". A number of researchers have pointed out that the effects of viscous properties on the stress-strain behaviour of sand could be significant (e.g., Murayama et al., 1984; Lade et al., 1997; Mejia et al., 1988; Yamamuro and Lade, 1993; Nakamura et al., 1999; Lade and Liu, 1998, 2001; Howie et al., 2001; Nawir et al., 2002; Kuwano and Jardine, 2002). It is essential therefore to properly formulate the viscous properties, in addition to elastic and plastic properties, when developing a realistic constitutive model that can simulate the stress-strain-time behaviour of sand subjected to a wide variety of loading histories. Matsushita et al. (1999), Di Benedetto et al. (2002) and Tatsuoka et al. (2002) evaluated the viscous properties of sand in shear by performing a series of plane strain compression tests (PSC tests) on air-dried and saturated specimens of Hostun and Toyoura sands. The tests on saturated specimens were performed under both drained and undrained conditions. With the same objective, Di Benedetto et al. (2001) performed torsional shear tests (TS tests) on air-dried Hostun sand, and Matsushita et al. (1999) performed drained triaxial compression tests (TC tests) on air-dried and saturated Toyoura sand.

Di Benedetto et al. (2002) and Tatsuoka et al. (2002) started developing a constitutive model based on the results from the PSC tests while referring to those from the TS and TC tests within the framework of the general three-component constitutive model (Fig. 1). The model is called the TESRA (temporary effects of strain rate and strain acceleration) model. The model has been validated for the shear deformation characteristics of poorly graded sand with a single stress variable at a constant confining pressure (Di Benedetto et al., 2002; Tatsuoka et al.,

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elastic model having a set of elastic modulus (or more generally the elastic stiffness matrix) that is a function of the instantaneous stress state (and strain history when relevant).

3. A given stress, \( \sigma \), is decomposed into the inviscid (or rate-independent) component, \( \sigma' \), and the viscous component, \( \sigma'' \), as:

\[
\sigma = \sigma' + \sigma''
\]  

(2)

4. The inviscid stress \( \sigma' \) is a unique function of irreversible strain \( \varepsilon'' \) in the case of monotonic loading (ML), for which the rate \( \dot{\varepsilon}'' = \partial \varepsilon'' / \partial \tau \) is always positive irrespective of the sign of the stress rate \( \dot{\sigma} \). The \( \sigma' - \varepsilon'' \) relation, which becomes hysteretic under cyclic loading conditions, is modelled similarly as the conventional elasto-plastic theories.

5. The basic variable for the viscous stress \( \sigma'' \) is not "time \( t'' \)"; because it is not possible to define the origin in an objective way. In the ML case, \( \sigma'' \) is always positive and a unique function of the instantaneous value of \( \varepsilon'' \) and its rate \( \dot{\varepsilon}'' \) with some geometrical materials including sedimentary soft rock (Hayano et al., 2001) and some types of clay (Tatsuoka et al., 2002). On the other hand, with poorly graded sands, such as Hostun and Toyoura sands, \( \sigma'' \) decays with an increase in \( \varepsilon'' \) when ML continues at a constant \( \varepsilon'' \). As a result, the current viscous stress \( \sigma'' \) is controlled not only by the instantaneous values of \( \varepsilon'' \) and its rate \( \dot{\varepsilon}'' \) but also recent strain history, while \( \sigma'' \) could be either positive or zero or negative.

6. A viscous stress increment \( d\sigma'' \) is caused by an irreversible strain increment or its rate or both and it is always proportional to the instantaneous inviscid stress \( \sigma' \).

In this study, the relationships between the stress ratio \( R = \sigma' / \sigma'' = \sigma' / \sigma'' \) and the shear strain \( \gamma = \gamma_1 - \gamma_3 = \gamma_1 - \gamma_3 \) are simulated by the TESRA model. Equation (2) is then rewritten as:

\[
R = R'(\gamma'') + \gamma''
\]  

(3)

where \( R'(\gamma'') \) is the relationship between the inviscid stress ratio \( R' \) and \( \gamma'' \). It is called the reference stress-strain relation. \( \gamma'' \) is the viscous stress ratio. Referring to Fig. 1(b), the current value of \( \gamma'' \) (when \( \gamma'' = \gamma'' \)) is obtained as:

\[
\gamma'' = \gamma'' + \int_{\gamma''}^{\gamma''} [dR'']_{\gamma''} g_{\text{decays}}(\gamma'' - \tau)
\]

\[
= \int_{\gamma''}^{\gamma''} d\{R'(\gamma'') \cdot g_{\text{decays}}(\gamma'')\}_{\gamma''} g_{\text{decays}}(\gamma'' - \tau)
\]

\[
= \int_{\gamma''}^{\gamma''} [d\{R'(\gamma'') \cdot g_{\text{decays}}(\gamma'')\}_{\gamma''} g_{\text{decays}}(\gamma'' - \tau)
\]

\[
= \int_{\gamma''}^{\gamma''} [d\{R'(\gamma'') \cdot g_{\text{decays}}(\gamma'')\}_{\gamma''} g_{\text{decays}}(\gamma'' - \tau)
\]

(4)

where \( \tau \) is the irreversible shear strain \( \gamma'' \) when the specific viscous stress ratio increment \( [dR'']_{\gamma''} \) develops; \( \gamma'' \) is the irreversible shear strain at the start of integration where \( R'' = 0 \) and \( \gamma'' = 0 \); \( g_{\text{decays}}(\gamma'' - \tau) = \gamma'' - \gamma'' \) is the decay function that controls the manner of the decay of \( R'' \) with an

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**A BRIEF SUMMARY OF TESRA MODEL**

The model is described in greater detail by Di Benedetto et al. (2002) and Tatsuoka et al. (2002). Thus, only the basic framework is presented below referring to Figs. 1(a) and (b):

1. A given strain rate increment, \( d\varepsilon \), is decomposed into an elastic (rate-independent and reversible) component, \( d\varepsilon' \), and a rate-dependent and irreversible component, \( d\varepsilon'' \), as:

\[
d\varepsilon = d\varepsilon' + d\varepsilon''
\]  

(1)

2. Elastic strain increments are obtained by a hypo-
increase in $\gamma^v$, for which the parameter $r_1$ is a positive constant less than unity such that the value of $R^{\gamma^v - \tau}$ decreases with the increase in the strain difference $\gamma^v - \tau$; and $g_r(\dot{\gamma}^v)$ is the viscosity function, given as:

$$g_r(\dot{\gamma}^v) = \alpha \left[ 1 - \exp \left( 1 - \left( \frac{\dot{\gamma}^v}{\dot{e}_i^v} + 1 \right)^m \right) \right] \quad (\geq 0) \quad (5)$$

where $|\dot{\gamma}^v|$ is the absolute value of $\dot{\gamma}^v$ (so this value is positive even when $\dot{\gamma}^v$ is negative), and $\alpha$, $m$ and $\dot{e}_i^v$ are positive constants. The value of $\alpha$ is the upper limit of $g_r(\dot{\gamma}^v)$ when $\dot{\gamma}^v$ becomes infinity, which can not be evaluated experimentally. The value of $\alpha$ has minor effects on results of the simulation of such tests as performed in the present study, where the strain rate was maintained within a relatively small range. The value of $\dot{e}_i^v$ directly controls the magnitude of $R^v$ when $R^v$ does not decay (i.e., when $r_1=1.0$); i.e., a smaller value of $\dot{e}_i^v$ results into a larger value of $R^v$. With an increase in the decay rate of $R^v$ (i.e., with an increase in $r_1$ from unity), the effects of $\dot{e}_i^v$ on the present value of $R^v$ decrease. The values, $\alpha=0.35$ and $\dot{e}_i^v=0.00006$%/minute, which were found relevant to Toyoura sand in PSC tests, reported by Di Benedetto et al. (2002), were also used throughout the present study.

**TEST METHODS**

An automated triaxial apparatus (Fig. 2(a)) developed by Ling and Tatsuoka (1994) and Roh and Tatsuoka (2002) was modified and used. Axial loading was performed by using a gear-type loading system driven by an analogue servo-motor (Tatsuoka et al., 1994; Santucci de Magistris et al., 1999). This loading system can control axial displacements to an accuracy of less than 1 $\mu$m and exhibits essentially no backlash upon load reversal. The lateral stress $\sigma_l$ was applied using air-pressure to act on the surface of the cell water and was controlled by using an electro-pneumatic transducer. An electric-resistant strain gauge-type load cell, designed based on the original idea of Tani et al. (1983), was placed inside the triaxial cell to measure the axial load without piston friction. As shown in Fig. 2(b), axial and lateral strains that were free from the effects of bedding error at the top and bottom ends and lateral surfaces of the specimen were measured by using, respectively, a pair of vertical LDTs (Goto et al., 1991) and four pairs of lateral (or horizontal) LDTs (Hoque et al., 1997). The strain values presented in this paper are the average of readings from the respective set of LDT.

Rectangular prismatic specimens (18 cm in height and 11 cm x 11 cm in cross-section) were prepared to use the lateral LDTs. This specimen shape was employed to use lateral LDTs first by Hoque et al. (1997) for sand and then by Hayano et al. (1997) for sedimentary soft rock, by Jiang et al. (1997) and Ahn Dan et al. (2002) for well-graded gravel and by Kongskpraseit et al. (2001) for cement-mixed sand. A pair of vertical LDTs were set on a narrow (7 mm-wide) vertical flat belt at both ends of one diagonal in the cross-section of the specimen (Fig. 2(b)) to avoid an interaction between the vertical and lateral LDTs. To this end, a pair of slender Acrylic bar having a half rectangular (i.e., triangular) cross-section with a
length equal to the specimen height was prepared. Before pluviating air-dried sand particles into the inside of the specimen mould, these bars were vertically set at two opposite corners of the specimen between the membrane and the mould. It was confirmed that axial strains obtained by this method are essentially the same as those obtained using vertical LDTs set at the central part of each flat lateral surface (Nawir, 2002).

The cap and pedestal, which were made of duralumin, had enlarged ends having a polished stainless steel platen. Their surfaces were lubricated by using a 0.3 mm-thick latex rubber disk smeared with a 50 μm-thick Dow High-Vacuum silicon grease layer (Tatsuoka et al., 1984). Drainage was made through a porous stone disk installed in a 3 mm-diameter drainage hole at the center of the cap and pedestal.

The specimens were produced by using a single batch of Toyoura sand having a specific gravity of 2.65; a mean diameter of 0.21 mm; a coefficient of uniformity of 1.20; and the maximum and minimum void ratios of 0.980 and 0.600. Except for two specimens prepared to evaluate the effects of dry density, specimens having an initial void ratio around 0.69 were prepared by the air-pluviation method using four fixed sieves having an opening of 1.0 mm with a falling height of 23 cm (Miura and Toki, 1982). A loose specimen with an initial void ratio of 0.925 was prepared by another air-pluviation method using a single nozzle with an inner diameter of 1.0 cm with a fall height of about 1 cm. On the other hand, a dense specimen with an initial void ratio of 0.69 was prepared by the same air-pluviation as the medium dense ones followed by tapping the specimen mould. Except for specimens that were air-dried during TC loading, the specimens were then saturated by the dry setting method (Ampadu and Tatsuoka, 1993). This method consisted of applying vacuum, flushing with de-aired water and finally

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**Table 1. Test conditions to evaluate the viscous properties of Toyoura sand**

<table>
<thead>
<tr>
<th>Test name</th>
<th>Stress path</th>
<th>Initial void ratio at point A, ( e_0 )</th>
<th>Loading history</th>
<th>Stress states of creep loading: ((\sigma_u'; \sigma_v') (\text{kPa}))</th>
<th>Sequences of axial strain rate (as a ratio to ( \varepsilon_0 = 0.08 % / \text{min} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC100r</td>
<td>(100,100)</td>
<td>0.686</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CC200r</td>
<td>(200,200)</td>
<td>0.697</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CC400r</td>
<td>(400,400)</td>
<td>0.703</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>DN400r (dense)</td>
<td>(400,400)</td>
<td>0.699</td>
<td>-</td>
<td>1(\rightarrow)1/10(\rightarrow)1 (\rightarrow)1/2 (\rightarrow)4(\rightarrow)10(\rightarrow)1(\rightarrow)15 (\rightarrow)...</td>
<td></td>
</tr>
<tr>
<td>LS400r (loose)</td>
<td>(400,400)</td>
<td>0.925</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CC600r</td>
<td>(600,600)</td>
<td>0.690</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>DR200r (air-dried)</td>
<td>(200,200)</td>
<td>0.692</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CC400m</td>
<td>(400,400)</td>
<td>0.690</td>
<td>1 (\rightarrow)1/10(\rightarrow)10 (\rightarrow)1(\rightarrow)4(\rightarrow)10(\rightarrow)1(\rightarrow)15 (\rightarrow)...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CC100c</td>
<td>(100,100)</td>
<td>0.693</td>
<td>1 (\rightarrow)1/10(\rightarrow)10 (\rightarrow)1(\rightarrow)4(\rightarrow)10(\rightarrow)1(\rightarrow)15 (\rightarrow)...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CC200c</td>
<td>(200,200)</td>
<td>0.691</td>
<td>1 (\rightarrow)1/10(\rightarrow)10 (\rightarrow)1(\rightarrow)4(\rightarrow)10(\rightarrow)1(\rightarrow)15 (\rightarrow)...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CC400c</td>
<td>(400,400)</td>
<td>0.697</td>
<td>1 (\rightarrow)1/10(\rightarrow)10 (\rightarrow)1(\rightarrow)4(\rightarrow)10(\rightarrow)1(\rightarrow)15 (\rightarrow)...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CC600c</td>
<td>(600,600)</td>
<td>0.690</td>
<td>1 (\rightarrow)1/10(\rightarrow)10 (\rightarrow)1(\rightarrow)4(\rightarrow)10(\rightarrow)1(\rightarrow)15 (\rightarrow)...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CR3r</td>
<td>(30,114)</td>
<td>0.690</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CP600r</td>
<td>(600,600)</td>
<td>0.684</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
applying a back pressure of 60 kPa (except when 30 kPa was used in the tests with a cell pressure of 600 kPa).

**Stress Paths and Typical Test Results**

The stress path in each test was comprised of two straight lines (see Table 1). That is, except for one continuous anisotropic compression test, the specimen was first isotropically compressed at a constant axial strain rate \( \varepsilon \) equal to 0.0025% / minute from the initial isotropic stress of \( \sigma'_i = \sigma''_i = 30 \) kPa to the respective target stress state. This was followed by drained TC loading at a constant effective lateral stress \( \sigma''_i \), as shown in Fig. 3(a), or at a constant mean effective stress, \( p' = (\sigma'_i + 2\sigma''_i) / 3 \) equal to 600 kPa in one test. Several times of stepwise change in the axial strain rate \( \varepsilon \), or creep loading tests or both were performed during otherwise ML at a constant strain rate. More details of the testing method are described in Nawir (2002). Figure 3(b) shows the measured relationships between the deviator stress \( q \) (or the stress ratio \( R = \sigma'_i / \sigma''_i = \sigma_i / \sigma'_i \)) and the total and irreversible shear strains \( \gamma \) and \( \gamma'' \) for air-dried Toyoura sand (test DR200r, performed following the stress path depicted in Fig. 3(a)).
irreversible strains were obtained by integrating their increments; $de^e = de - de^p$. The elastic strain increment, $de$, for a given stress increment was obtained based on a hypo-elastic model having inherently anisotropic, stress state-dependent Young's moduli and Poisson's ratios (Hoque and Tatsuoka, 1998). The vertical arrows shown in Fig. 3(b) indicate the values of $\gamma$ at which the axial strain rate was changed stepwise by a factor of between 2 and 150. In this figure and other similar ones in this paper, axial strain rates are expressed as a ratio to the basic value, $e_0$, equal to 0.08%/minute. Figures 3(c-1) and 3(c-2) show the measured time histories of axial strain rate, $e_0 = de/dt$, and irreversible shear strain rate $\dot{\gamma}^e (= \dot{e}^e - \dot{e}^p)$. This and other measured time histories of $\dot{\gamma}^e$ were used in the strain control simulation by the TESRA model (presented later in this paper). On the other hand, the creep behaviour was simulated by stress control for the same period as the respective experiment. The simulation method is explained in detail by Di Benedetto et al. (2002) and Tatsuoka et al. (2002).

TEST RESULTS

Effects of Wet Conditions on the Viscous Properties

Despite being much less pronounced compared with soft clay, sand can exhibit noticeable drained creep deformation (e.g., Murayama et al., 1984; Lade et al., 1997). The amount of drained creep deformation increases with pressure level, becoming particularly large at high pressures, where particle crushing becomes significant (Yamamuro and Lade, 1993). It also increases with shear stress level (e.g., Matsushita et al., 1999; Di Benedetto et al., 2002; Tatsuoka et al., 2002). Due to a low hydraulic conductivity, most of the creep strain that develops during early stages of creep loading on saturated clay could be attributed to delayed dissipation of excess pore pressure. On the other hand, Matsushita et al. (1999), Di Benedetto et al. (2001, 2002) and Tatsuoka et al. (2002) showed that loading rate effects, including creep deformation, that were observed in their plane strain compression tests on saturated specimens of sand were due primarily to the viscous properties of sand with negligible effects of delayed dissipation of excess pore pressure. They also showed that air-dried sand exhibited noticeable loading rate effects as did drained saturated sand.

To re-confirm this behaviour, the following two pairs of drained TC tests were performed on air-dried and fully saturated specimens (see Table 1):

a) The axial strain rate was changed stepwise by a factor of between 2 and 150 during otherwise ML at $\sigma_0^k =$
VISCOSOUS PROPERTIES OF SAND

200 kPa; tests DR200r (air-dried) and CC200r (saturated).

b) Drained creep loading was performed at three stages, each lasting for five hours, during otherwise ML at \( \sigma' = 400 \) kPa; tests DR400m (air-dried) and CC400m (saturated).

Figure 3(d) shows the measured and simulated \( q \) (and \( R \)) - \( y^p \) relations from the first pair of tests on the air-dried and saturated specimens. Figure 3(e) shows the measured total and irreversible strain paths. Figure 4(a) shows the measured and simulated \( R - y^p \) relations from the second pair of tests, in which creep loading tests were performed at three deviator stress levels, \( q = 600, 900 \) and 1200 kPa (denoted by letters \( b, d \) and \( f \)). Each creep loading lasted for five hours. To systematically evaluate effects of strain rate history (or initial strain rate) on the creep behaviour, the strain rate was changed stepwise at points \( c \) and \( e \), each located between consecutive creep loading stages. Figure 4(b) shows the time history of \( \dot{y}^p = \dot{\varepsilon}^p - \dot{\varepsilon}^c \) in test DR400m (air-dried). Figure 4(c) shows the irreversible strain paths from the two tests.

The following trends of behaviour could be noted from Figs. 3 and 4:

1) The overall pre-peak stiffness and peak strength of the saturated specimens are noticeably smaller than those of the air-dried ones. More initial contractive behaviour could be seen with the saturated specimens, while the dilative behaviour is similar. These trends of behaviour are slightly different from those observed in the drained TC tests on Toyoura sand at \( \sigma' = 10 \) and 78 kPa (Tatsuoka et al., 1986), in which the strength and deformation characteristics of saturated and air-dried specimens were essentially the same. These different trends were primarily due to: a) different confining pressures; and b) the use of different batches of sand (Yasin et al., 1999).

2) The air-dried and saturated specimens exhibit the following similar trends of loading rate effect:
   a) Creep deformation is noticeable irrespective of wet conditions and it increases with the shear stress level in a similar manner.
   b) The effects of step change in the strain rate on the shear stress-shear strain relations are similar. Similar very stiff behaviour is observed when ML is restarted at a constant strain rate following the respective creep loading.
   c) The stress change that has taken place with a step change in the strain rate decays in a similar manner with the increase in the strain when monotonic loading continues at a constant strain rate.

3) The pattern of irreversible strain paths is similar between the saturated and air-dried specimens.

The trends of loading rate effects, including creep deformation, observed in these tests are essentially the same with those observed in the PSC tests on air-dried and saturated specimens of Hostun and Toyoura sands performed by Matsushita et al. (1999) and Di Benedetto et al. (2002). The facts listed above indicate that these two tests on saturated specimens (and the other ones described in this paper) were performed essentially under fully drained conditions, while the loading rate effects observed with the saturated specimens were not due to the development of a gradient of excess pore water pressure within the specimen. It is likely that the origin of such global viscous properties of sand as described above is time-dependent slipping at inter-particle contacts caused by viscous deformation inside particles at and near the inter-particle contacts (and particle crushing when the confining pressure is high enough; Yamamuro and Lade, 1993). It seems that, inside a sand specimen subjected to creep loading, such local viscous behaviour as described above is active only in limited zones at every moment and these zones are continuously moving while the overall viscous activities decay with time.

**Simulation by TESRA Model**

The parameter \( m \) of the viscosity function (Eq. (5)) was determined as follows. Matsushita et al. (1999) and Di Benedetto et al. (2002) showed that the following empirical equation is valid for Hostun and Toyoura sands in PSC:

\[
\frac{\Delta R}{R} = \beta \cdot \log_{10} \left( \frac{\dot{y}_{after}}{\dot{y}_{before}} \right) = b \cdot \ln \left( \frac{\dot{y}_{after}}{\dot{y}_{before}} \right)
\]  

(a)
where $\Delta R$ is the change in the stress ratio taking place with a step change in the irreversible shear strain rate from $\dot{\gamma}^r_{\text{before}}$ to $\dot{\gamma}^r_{\text{after}}$, as illustrated in the figure inset in Fig. 5(a). $R$ is the stress ratio before the step change in the strain rate and $b$ is the coefficient, equal to $\beta/\ln 10$. The value of $\Delta R$ is defined for a fixed value of $\dot{\gamma}^r$ before $\Delta R$ starts decaying. Equation (6) can be rewritten into an incremental form:

$$\frac{dR}{R} = d \ln R = b \cdot d(\ln \dot{\gamma}^r)$$  \hspace{2cm} (7a)

Since $dR$ in Eq. (7a) is the stress ratio increment that has not started decaying, we obtain the following equation by referring to Eq. (4):

$$d \ln (R^i \cdot (1 + g_i(\dot{\gamma}^r))) = b \cdot d(\ln \dot{\gamma}^r)$$  \hspace{2cm} (7b)

Since $R^i$ is constant in this incremental form of Eq. (7b), we obtain:

$$d[\ln (1 + g_i(\dot{\gamma}^r))] = b \cdot d(\ln \dot{\gamma}^r)$$  \hspace{2cm} (7c)

By integrating Eq. (7c) with respect to $\dot{\gamma}^r$, we obtain:

$$1 + g_i(\dot{\gamma}^r) = c_i \cdot (\dot{\gamma}^r)^b$$  \hspace{2cm} (8)

where $c_i$ is a constant. The parameters of the viscosity function (Eq. (5)) should be defined to fit Eq. (8).

Figure 5(a) shows the measured relationships representing Eq. (6) obtained from the four tests that are described in Figs. 3 and 4. The following trends of behaviour may be seen from Fig. 5(a):

1) The relationship is linear, showing that Eq. (6) is relevant to the present case as the PSC test case (Di Benedetto et al., 2002; Tatsuoka et al., 2002). The average slope $\beta$ is equal to 0.024, thus the parameter $b$ of Eq. (8) is equal to $0.024/\ln 10 = 0.0105$.  

<table>
<thead>
<tr>
<th>Test name</th>
<th>$\epsilon_0$</th>
<th>No. of figure showing the simulation</th>
<th>$R^i = R_0 + A_1 \cdot \left(1 - \exp \left(\frac{-\dot{\gamma}^r}{c_1}\right)\right) + A_2 \cdot \left(1 - \exp \left(\frac{-\dot{\gamma}^r}{c_2}\right)\right)$</th>
<th>$R_0$</th>
<th>$A_1$</th>
<th>$c_1$</th>
<th>$A_2$</th>
<th>$c_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC100r</td>
<td>0.686</td>
<td>6b</td>
<td>2.280 \cdot 2.423 \cdot 2.102 \cdot 0.272</td>
<td>2.280</td>
<td>2.423</td>
<td>2.102</td>
<td>0.272</td>
<td></td>
</tr>
<tr>
<td>CC200r</td>
<td>0.697</td>
<td>3d</td>
<td>2.464 \cdot 2.423 \cdot 1.372 \cdot 0.352</td>
<td>2.464</td>
<td>2.423</td>
<td>1.372</td>
<td>0.352</td>
<td></td>
</tr>
<tr>
<td>CC400r</td>
<td>0.703</td>
<td>6b and 13</td>
<td>1.369 \cdot 0.443 \cdot 1.984 \cdot 2.763</td>
<td>1.369</td>
<td>0.443</td>
<td>1.984</td>
<td>2.763</td>
<td></td>
</tr>
<tr>
<td>CC600r</td>
<td>0.690</td>
<td>6b</td>
<td>4.590 \cdot 0.520 \cdot 1.775 \cdot 2.670</td>
<td>4.590</td>
<td>0.520</td>
<td>1.775</td>
<td>2.670</td>
<td></td>
</tr>
<tr>
<td>CC400m</td>
<td>0.690</td>
<td>4a</td>
<td>1.848 \cdot 4.510 \cdot 2.019 \cdot 0.885</td>
<td>1.848</td>
<td>4.510</td>
<td>2.019</td>
<td>0.885</td>
<td></td>
</tr>
<tr>
<td>DR200r</td>
<td>0.692</td>
<td>3d</td>
<td>2.246 \cdot 1.600 \cdot 1.471 \cdot 0.203</td>
<td>2.246</td>
<td>1.600</td>
<td>1.471</td>
<td>0.203</td>
<td></td>
</tr>
<tr>
<td>DR400m</td>
<td>0.706</td>
<td>4a</td>
<td>1.863 \cdot 3.460 \cdot 2.019 \cdot 0.665</td>
<td>1.863</td>
<td>3.460</td>
<td>2.019</td>
<td>0.665</td>
<td></td>
</tr>
<tr>
<td>DN400r</td>
<td>0.690</td>
<td>13a</td>
<td>2.488 \cdot 1.962 \cdot 1.304 \cdot 0.244</td>
<td>2.488</td>
<td>1.962</td>
<td>1.304</td>
<td>0.244</td>
<td></td>
</tr>
<tr>
<td>LS400r</td>
<td>0.925</td>
<td>13a</td>
<td>0.550 \cdot 0.413 \cdot 1.633 \cdot 4.313</td>
<td>0.550</td>
<td>0.413</td>
<td>1.633</td>
<td>4.313</td>
<td></td>
</tr>
<tr>
<td>CC100c</td>
<td>0.693</td>
<td>8b</td>
<td>3.230 \cdot 2.140 \cdot 1.205 \cdot 0.274</td>
<td>3.230</td>
<td>2.140</td>
<td>1.205</td>
<td>0.274</td>
<td></td>
</tr>
<tr>
<td>CC200c</td>
<td>0.691</td>
<td>8b</td>
<td>1.548 \cdot 0.394 \cdot 2.525 \cdot 3.055</td>
<td>1.548</td>
<td>0.394</td>
<td>2.525</td>
<td>3.055</td>
<td></td>
</tr>
<tr>
<td>CC400c</td>
<td>0.697</td>
<td></td>
<td>2.234 \cdot 2.482 \cdot 1.392 \cdot 0.653</td>
<td>2.234</td>
<td>2.482</td>
<td>1.392</td>
<td>0.653</td>
<td></td>
</tr>
<tr>
<td>CC600c</td>
<td>0.690</td>
<td></td>
<td>0.507 \cdot 0.192 \cdot 2.439 \cdot 1.257</td>
<td>0.507</td>
<td>0.192</td>
<td>2.439</td>
<td>1.257</td>
<td></td>
</tr>
<tr>
<td>CC401m</td>
<td>0.690</td>
<td>10a</td>
<td>1.848 \cdot 4.510 \cdot 2.019 \cdot 0.885</td>
<td>1.848</td>
<td>4.510</td>
<td>2.019</td>
<td>0.885</td>
<td></td>
</tr>
<tr>
<td>CC402m</td>
<td>0.695</td>
<td>10b</td>
<td>2.250 \cdot 3.034 \cdot 1.369 \cdot 0.500</td>
<td>2.250</td>
<td>3.034</td>
<td>1.369</td>
<td>0.500</td>
<td></td>
</tr>
<tr>
<td>CC403m</td>
<td>0.688</td>
<td>10c</td>
<td>2.212 \cdot 1.883 \cdot 1.498 \cdot 0.351</td>
<td>2.212</td>
<td>1.883</td>
<td>1.498</td>
<td>0.351</td>
<td></td>
</tr>
<tr>
<td>CC400a</td>
<td>0.687</td>
<td>12a</td>
<td>1.269 \cdot 0.328 \cdot 2.260 \cdot 2.478</td>
<td>1.269</td>
<td>0.328</td>
<td>2.260</td>
<td>2.478</td>
<td></td>
</tr>
<tr>
<td>CC400h</td>
<td>0.681</td>
<td>12b</td>
<td>1.638 \cdot 0.528 \cdot 2.094 \cdot 3.457</td>
<td>1.638</td>
<td>0.528</td>
<td>2.094</td>
<td>3.457</td>
<td></td>
</tr>
<tr>
<td>CC400l</td>
<td>0.693</td>
<td>12c</td>
<td>2.214 \cdot 2.832 \cdot 1.371 \cdot 0.392</td>
<td>2.214</td>
<td>2.832</td>
<td>1.371</td>
<td>0.392</td>
<td></td>
</tr>
<tr>
<td>CP600r</td>
<td>0.684</td>
<td>15c</td>
<td>1.681 \cdot 0.589 \cdot 2.192 \cdot 2.685</td>
<td>1.681</td>
<td>0.589</td>
<td>2.192</td>
<td>2.685</td>
<td></td>
</tr>
</tbody>
</table>

Not simulated
VISCOSITY OF SAND

2) The parameter $\beta$ of Eq. (6) (thus the parameter $b$) for Toyoura sand does not depend on the wet conditions within the limits of the test conditions employed in the present study.

3) The effects of confining pressure are not noticeable.

This issue is examined more in the next section.

Figure 5(b) shows the relationship between $1.0 + \cdot$ the viscosity function, $g(\gamma')$ (Eq. (5))" and $\gamma'$ (in a full-logarithmic plot). This relation has a linear part that fits to Eq. (8) with $b = 0.0105$. A parameter $m$ equal to 0.035 was obtained by trial and error and was used throughout the simulation in the present study. The values of $b_{max}$ and $b_{min}$ indicated in this figure are explained later in this paper.

It can readily be seen from Figs. 3 and 4 (and other figures shown below) that the viscous stress decays in a similar manner with air-dried and saturated specimens. However, the parameter $r_i$ for the decay function $g_{decay} (\gamma' - \tau)$ (Eq. (4)) that was obtained by trial and error was 0.017 for the air-dried specimens and 0.17 for the saturated specimens. The reason for this difference is not known to the authors. The different values indicate that the viscous stress decays at a larger rate with air-dried sand than with saturated sand. The value 0.17 for the saturated specimens in TC at $\sigma' = 200$ kPa is only slightly smaller than the value 0.20 defined for dense saturated Toyoura sand in the PSC tests at $\sigma' = 400$ kPa (Di Benedetto et al., 2002).

The reference stress-strain relation (i.e., the $R^t - \gamma''$ relation) was obtained by fitting the following equation to the respective deduced overall stress-strain curve that would have been obtained if ML had been performed at an extremely low constant strain rate from the beginning of loading. In this case, the $R^t$ value becomes negligible throughout the ML.

$$ R^t = R_0 + A_1 \left( 1 - \exp \left( - \frac{\gamma'}{c_1} \right) \right) + A_2 \left( 1 - \exp \left( - \frac{\gamma'}{c_2} \right) \right) $$

(9)

where $R_0$ is the initial value of $R = \sigma' / \sigma'_{ref}$ (= 1.0 in the present case). Table 2 lists the parameters of the reference relation obtained by this fitting procedure. Since a unique elastic strain increment is obtained for a given stress increment in the present ML, a unique reference relation is obtained for each relation fitted by Eq. (9). An equation other than Eq. (9) could be employed if it fits the experimental result.

The variations seen in the values of the parameter of Eq. (9) listed in Table 2 are due to:

1) the different confining pressures, dry densities and wet conditions that were made purposely;
2) a scatter in the dry density among the specimens that took place unintentionally; and
3) an inevitable scatter in the data that took place even under nominally the same test conditions.

In the present study, the respective deduced inviscid stress-strain relation was best fitted by Eq. (9) without referring to the other test results for the following reasons. Firstly, the first and second types of data variation could be accounted for by determining the parameters based on relevant empirical equations developed to express the effects of these influencing factors, as attempted by Tatsuoka et al. (1993). However, it was not possible to derive such empirical equations relevant to the test conditions of the present study based on only the data from the present study. Even when such empirical equations were available, the predicted relation would somehow deviate

Fig. 6. (a) Measured stress paths ($\sigma'_i = 100-600$ kPa; saturated), (b) measured and simulated $q (= \sigma'_i - \sigma'_s) - \gamma''$ relations and (c) irreversible strain paths
from the respective experimental relation, which will make it difficult to evaluate whether a given model could accurately predict the viscous properties. The main objective of the present study was to find a relevant model that could accurately evaluate the viscous properties of sand. Discussions on the general modelling of inviscid stress-strain relation are beyond the scope of the present study. Secondly, the third type of data variation is difficult to deal with by any theoretical or empirical equation.

The measured and simulated stress-strain relations are compared in Figs. 3(d) and 4(a). In this simulation, slightly different reference stress-strain relations were defined for the saturated and air-dried specimens and at different confining pressures. The same parameters of the viscosity function (Eq. (5)) were used for all the tests. In comparison, the different decay parameters \( r_i \) were employed for the saturated and air-dried specimens. It may be seen that the loading rate effects observed with the air-dried and saturated specimens are simulated similarly well by the TESRA model, showing that the viscous properties of saturated and air-dried Toyoura sand are basically the same. Figure 4(d) compares the measured stress-strain curve from test DR400m (on an air-dried specimen) with the simulated relation obtained by using \( r_i = 0.17 \), which is relevant to the saturated specimens. It may be seen that \( r_i = 0.17 \) is too large for the air-dried specimen of Toyoura sand.

Effects of Pressure Level on the Viscous Properties

Figures 6(a), (b) and (c) show the stress paths, \( R - \gamma^h \) relations and irreversible strain paths from tests performed to evaluate the effects of pressure level on the viscous properties. The axial strain rate was changed stepwise several times during otherwise ML in each test. The numerals indicated next to the respective stress-strain curve in Fig. 6(b) mean the ratios of axial strain rate to the basic axial strain rate, \( \dot{\varepsilon}_a = 0.08\%/\text{minute} \). It may be seen from these figures that: 1) the sand is more contractive at higher pressures; and 2) the change of stress ratio associated with a step change in the axial strain rate becomes larger with an increase in the pressure level and in the stress ratio.

Figure 7 shows the results from these four tests, similar to Fig. 5(a). A unique relationship may be seen in this figure, which confirms the second feature above. This result indicates that the viscous stress ratio increment \( \dot{R}^h \) is always proportional to the instantaneous inviscid stress ratio \( R^h \) irrespective of confining pressures and, therefore, the viscosity function (Eq. (5)) is independent of pressure level. Figure 6(b) also compares the stress-strain relations measured at different pressures with those simulated by using the same viscosity and decay functions. The good simulation seen in this figure indicates that the
TESRA model, which was formulated based on the results from PSC tests along a single constant $\sigma_1$ stress path, could be applied to more general stress conditions.

To reconfirm the above, another series of TC tests were performed at different confining pressures. In each test, one or two creep loading stages were included during otherwise ML at a constant axial strain rate equal to 0.08% /min. Figure 8(a) shows the stress paths. Figure 8(b) compares the measured stress-strain relations with the simulated relations obtained using the same viscosity function and $n = 0.17$ as used in the simulations presented in Figs. 3(d), 4(a) and 6(b). It is important to note that, at each pressure level, the creep behaviour is predicted very well using the model parameters determined from the behaviour during ML with step changes in the strain rate.

It may be seen from Fig. 8(c) that, at each confining pressure, the trend of irreversible strain path during creep loading becomes more contractant than during ML at a constant strain rate. The effects of viscous properties on the flow behaviour are beyond the scope of this paper; Nawir et al. (2001) discussed this complicated flow behaviour.

**Table 3. Tests to evaluate the effects of initial strain rate on the creep behaviour**

<table>
<thead>
<tr>
<th>Path</th>
<th>Initial axial strain rate at the start of creep loading</th>
<th>States of creep loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>a-b-c</td>
<td>$\dot{\varepsilon}_0$</td>
<td>$1/10\dot{\varepsilon}_0$</td>
</tr>
<tr>
<td>c-d-e</td>
<td>$(1/10)\dot{\varepsilon}_0$</td>
<td>$10\dot{\varepsilon}_0$</td>
</tr>
<tr>
<td>c-f-g</td>
<td>$10\dot{\varepsilon}_0$</td>
<td>$\dot{\varepsilon}_0$</td>
</tr>
</tbody>
</table>

* $\dot{\varepsilon}_0 = 0.08$/minute

**Effects of Strain History on the Creep Behaviour**

Tatsuoka et al. (2002) showed that the creep deformation of sand is controlled by the initial strain rate. To reconfirm the above, three TC tests were performed along the same stress path at a constant confining pressure of 400 kPa (Fig. 9(a)). In each of these tests, creep loading tests (each lasting five hours) were performed at different initial axial strain rates $\dot{\varepsilon}_0$, equal to $\dot{\varepsilon}_0/10$, $\dot{\varepsilon}_0$ or $10\dot{\varepsilon}_0$ ($\dot{\varepsilon}_0 = 0.08%$/minute) (see Table 3). Figures 9(b) and 9(c) show the test results. At points c and e, which were located between two consecutive creep loading stages, the axial
strain rate was changed stepwise to achieve various initial strain rates at the start of creep loading stage. In Fig. 9(d), the creep shear strains observed at the end of each five hour creep loading stage are plotted against the initial irreversible shear strain rate. The following trends of behaviour can be seen from Fig. 9(d):
1) The creep shear strain increases with an increase in the initial strain rate. This result is consistent with that obtained by Tatsuoka et al. (2002).
2) The creep shear strain at a given stress state is a rather unique function of the initial strain rate at the start of the respective creep loading stage, but irrespective of the different strain histories. These unique relations are represented by lines B, D and F.
3) The slope is largest with relation F, at the highest stress ratio, and smallest with relation D, at the lowest stress ratio.

Figures 10(a), (b) and (c) compare the measured stress and strain relations with their simulations that were obtained by using the same viscosity and decay functions as used in the preceding sections. It may be seen that the test results are well simulated. Figure 9(d) compares the measured and simulated creep strains. It may be seen that the trends 1), 2) and 3) summarised above are also well simulated.

Another series of TC tests were performed in order to evaluate the effects of the history of irreversible shear strain rate immediately before the start of creep loading on the creep deformation (Table 4). At a common stress state ($\sigma_1 = 1,520$ kPa and $\sigma_3 = 400$ kPa) along the stress path shown in Fig. 11(a), creep loading tests were performed following the different strain histories shown in Fig. 11(b):
1) Test CC400l: A relatively low constant axial strain rate, equal to 0.08%/min. (i.e., or an irreversible shear strain rate of about 0.135%/min.) was applied for about 20 minutes during ML until the start of the creep loading stage. Figure 11(c) shows the measured time histories of irreversible shear strain rate for a wider range of period than in Fig. 11(b) from this test. The solid curve until the start of the creep test represents the average of the measured relation, which was used in the strain control simulation. On the other hand, the solid curve after the start of creep loading is the result of the stress control simulation by the TESRA model.

### Table 4. Tests to evaluate the effects of recent strain history on the creep behaviour

<table>
<thead>
<tr>
<th>Test code</th>
<th>Initial void ratio, $c_0$</th>
<th>Stress path</th>
<th>Axial strain rate history before the start of creep loading at point c</th>
<th>Axial strain rate in ML after creep stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC400l</td>
<td>0.693</td>
<td></td>
<td>$\sigma_{1}^{c}, \sigma_{3}^{c}$</td>
<td>0.08 %/minute</td>
</tr>
<tr>
<td>CC400m</td>
<td>0.681</td>
<td></td>
<td>A: 30 30</td>
<td>1.2 %/minute</td>
</tr>
<tr>
<td>CC400a</td>
<td>0.687</td>
<td></td>
<td>B: 400 400</td>
<td>0.08→1.2 %/minute</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>C: 1520 400 (kPa)</td>
<td></td>
</tr>
</tbody>
</table>
VISCOS PROPERTIES OF SAND

2) Test CC400h: A relatively high constant axial strain rate, equal to 1.2%/minute (i.e., an irreversible shear strain rate of about 1.75%/min.) was applied for a period of about 0.7 minutes (i.e., for an irreversible shear strain increment of 1.2%) until the start of the creep test.

3) Test CC400a: The axial strain rate was increased gradually from 0.08%/minute towards 1.2%/minute, which is nearly the same as the value at the start of creep loading in test CC400h.

Figure 11(d) shows the measured deviator stress-reversible shear strain relations from the three tests. Although the effects of recent strain rate history on the creep behaviour are rather subtle when compared with those of the initial strain rate, the following trends of behaviour may be noted from this figure:

1) The creep shear strain is smallest in test CC400l, which had the lowest initial shear strain rate. This result is consistent with those presented in Fig. 9.

2) The creep shear strain is noticeably larger in test CC400a, in which the strain rate increased at a high rate until the start of creep, than in test CC400h, which had a relatively high constant strain rate immediately before the start of creep loading. This result means that the creep shear strain increases with an increase in the rate of strain rate (i.e., strain acceleration) at the start of creep loading under otherwise the same conditions.

The trends of behaviour described above are well simulated by the TESRA model using the same viscosity and decay functions as before, as seen from Figs. 12(a), (b) and (c). This result indicates that the TESRA model is able to simulate delicate features of the viscous properties of sand.

Effects of Dry Density on the Viscous Properties

Three specimens having three different dry densities were subjected to a similar TC loading history at the same confining pressure, $\sigma^c = 400$ kPa. In each test, the axial strain rate was changed stepwise several times during otherwise constant ML. Figures 13(a) and 13(b) show the test results. Figure 14 shows the relationship between $\Delta R/R$ and $\log(Y_{\text{after}}/Y_{\text{before}})$, similar to Figs. 5(a) and 7. It may be seen from Fig. 14 that this relationship is independent of the dry density of specimen. It may also be seen from Fig. 13(a) that the test results for different dry
Fig. 12. Measured and simulated $q - \gamma'$ relations: (a) test CC400a, (b) test CC400b and (c) test CC400I

densities could be simulated similarly well by using the same viscosity and decay functions.

**Effects of Stress Path on the Viscous Properties**

The following two tests were performed along stress paths shown in Fig. 15(a), other than the TC stress path at a constant confining pressure, to evaluate the effects of stress path on the viscous properties:

1) Test CP600r: the specimen was sheared at constant $p' = 600$ kPa from the isotropic stress state.

2) Test CR38r: the specimen was sheared at constant confining pressure from the initial isotropic stress state ($\sigma_i = \sigma_{ii} = 30$ kPa) by increasing the stress ratio $R = \sigma_i' / \sigma_{ii}'$ towards 3.8. Then, anisotropic compres-

sion loading was performed at a nominally constant value of $R$ equal to 3.8 towards the final stress state, where $\sigma_{ii}' = 570$ kPa.

In these tests, the axial strain rate was changed stepwise several times during otherwise constant ML at a constant strain rate, as shown in Fig. 15(b).

In test CP600r (at a constant $p'$), the deviator stress $q$ and stress ratio $R$ sharply changed corresponding to step
changes in the strain rate (Figs. 15(b) and 15(c)). The effective stress path was maintained at a rather constant $p'$ value when the strain rate was changed stepwise (Fig. 15(a)). Figure 16 compares the $\Delta R/R - \log(\bar{\gamma}_{after}/\bar{\gamma}_{before})$ relation from the $p'$ constant test with the one from a typical TC test at a constant confining pressure $\sigma_3'$ (test CC200r). The solid line represents the average relationship for all the TC tests performed at constant confining pressures (Fig. 17: the details of Fig. 17 are explained later in this paper). It may be seen from Fig. 16 that the relation from the $p'$ constant test is consistent with the one from test CC200r and the other similar $\sigma_3'$ constant tests. Figure 15(c) compares the measured stress-strain relation from the $p'$ constant test with its simulation obtained by using the same viscosity and decay functions as those used for the $\sigma_3'$ constant tests. A good agreement seen in this figure suggests that the TESRA model could be applied to more general stress paths other than $\sigma_3'$ constant stress paths.

In test CR38r, a feedback system was used to keep the stress ratio $R = \sigma_1'/\sigma_3'$ constant, equal to 3.8. Despite the above, the stress ratio changed noticeably upon a step change in the strain rate due to the viscous properties of sand, as seen from Fig. 15(c). It appears that the capacity of the feedback system was not enough to suppress the change in the stress ratio. Therefore, the stress ratio changed noticeably upon the respective step change in the strain rate (Fig. 15(c)). The following trends of behaviour...
may be seen from these figures:
1) The stress ratio change, $\Delta R$, associated with the respective step change in the strain rate during this test was rather constant. This would be due to the fact that the stress ratio was basically constant, equal to 3.8, in this test.

2) Corresponding to the above, the $\Delta R / R - \log (\dot{\gamma}_{\text{after}} / \dot{\gamma}_{\text{before}})$ relation from this test is similar to the one from the $\dot{\gamma}_i' = \text{constant}$ and $p' = \text{constant}$ tests. These results suggest that the viscosity function (Eq. (5)) could be applied to more general stress paths other than $\dot{\gamma}_i' = \text{constant}$ and $p' = \text{constant}$ stress paths.

3) A close examination of Fig. 16 reveals that the deviation of the data in this test from the average relation for the other $\dot{\gamma}_i' = \text{constant}$ and $p' = \text{constant}$ tests is noticeably large, showing generally smaller values of $\Delta R / R$. This would be due to the specific loading method in this test, in which the change in the stress ratio was suppressed to an unknown extent by the feedback system. Therefore, simulation of the test result was not performed.

4) Development of large irreversible shear strain during ML along a stress path at a nominally constant stress ratio indicates that the stress ratio is not constant along a shear yield locus, but it should decrease with pressure level, as shown by Tatsuoka and Ishihara (1974).

With respect to the item 4) above, Fig. 18 shows segments of shear yield locus and deduced yield loci obtained from a series of special TC tests on Toyoura sand (Nawir, 2002; Nawir et al., 2003). In the respective test, as shown in the figure inset in Fig. 18, the specimen was first loaded to some stress state by TC loading at a certain confining pressure (path A–B). Then, the deviator stress $q$ decreased at the same confining pressure to some lower level (path B–C). Then, the mean principal stress $p'$ increased or decreased at constant $q$ (path C–D), followed by the restart of TC loading at an increased or decreased confining pressure (path D–E) to find a shear yield point (Y). The yield stress point is defined as the stress point where the tangent stiffness in the stress-strain relation when loaded along a fixed straight stress path decreases suddenly and largely, not as the boundary between the purely elastic behaviour and the elasto-plastic behaviour. Then, a segment of shear yield locus (B–Y) is obtained. A set of broken curves presented in Fig. 18 represent the following inviscid shear yield function that was determined to fit these measured segments of shear yield locus:

$$\frac{q' / p'_s}{(p' / p'_s)^{\frac{1}{n}}} - X'(\kappa_s) = 0$$

(10)

where $p'_s$ is a constant pressure, equal to 98 kPa. $q'$ and $p'$ are the inviscid components of $q$ and $p'$, which are equal
to $q$ and $p'$ for the plot of the inviscid shear yield loci based Eq. (10) presented in Fig. 18. $\beta_q$ is the positive exponent, which is equal to 0.84 for the data presented in Fig. 18. Therefore, the stress ratio $q/p'$ decreases with an increase in $p'$ along each inviscid shear yield locus. $X'(\kappa_s)$ is the inviscid shear hardening function (not including the effects of viscous properties), and $\kappa_s$ is the shear hardening parameter, which is constant along each inviscid shear yield locus. Many of the shear yield locus segments presented in Fig. 18 noticeably deviate from the respective inviscid shear yield locus (Eq. (10)). This deviation was essentially due to effects of the viscous properties, caused by reloading at a strain rate different from the one during primary loading, or by performing creep loading before the start of unloading, or by performing both. More detail is reported by Nawir et al. (2003).

The inviscid reference stress-strain relations in the single-stress formulation employed in the present study should be considered as a simplified version of the hardening function $X'(\kappa_s)$, which is formulated to be independent of stress path. In the present study, different reference relations are introduced for different pressure levels and different stress path directions ($\sigma = \text{constant}$ and $p' = \text{constant}$ paths) while using the irreversible shear strain $\gamma'$ as $\kappa_s$. The simulation of experimental results based on Eq. (10) is described in Nawir (2002) and will also be reported by the authors in the near future.

Some Discussions on the Viscosity Function

Figure 17 summarises all the data of $\Delta R/R - \log_{10}(\gamma_{\text{after}}/\gamma_{\text{before}})$ relation obtained in the present study (as presented in Figs. 5(a), 7 and 16). It may be seen that the relationship is basically unique for different wet conditions, confining pressures, specimen densities and stress paths. The data from a $R = \text{constant}$ test (test CR38r) deviate most from the average relation for the reason cited above. This result indicates the following:

1) A stress ratio increment $\Delta R$ caused by an irreversible shear strain increment $\gamma'$ or an increment of irreversible shear strain rate $d\gamma'$ or both was always essentially proportional to the instantaneous stress ratio $R$ under the various test conditions employed in the present study. This result validates Eq. (4), which is one of the cores of the TESRA model.

2) The formulation of the viscosity function in terms of the stress ratio $R = \sigma/\sigma_s$ (Eq. (5)) is relevant within the various test conditions employed in the present study.

With respect to the second item, Fig. 19 shows the relationships between $\Delta R/\eta = q/p'$ and $\log_{10}(\gamma'_{\text{after}}/\gamma'_{\text{before}})$ for the data presented in Fig. 7. It may be seen that the $\Delta R/\eta = \log_{10}(\gamma'_{\text{after}}/\gamma'_{\text{before}})$ relation is slightly less unique than the $\Delta R/R - \log_{10}(\gamma'_{\text{after}}/\gamma'_{\text{before}})$ relation. Despite being difficult to obtain a conclusion in this respect based on the results from the present study, Eq. (5), which is expressed in terms of the stress ratio $R$, is used in the present study following Di Benedetto et al. (2002) and Tatsuoka et al. (2002). To obtain a conclusion, the viscous properties should be evaluated at low $R$ values. It is not known whether another stress parameter, other than $R = \sigma/\sigma_s$, should be used, such as the one showing a similar curvature as the yield loci presented in Fig. 18, in the formulation of the viscous component in the form of Eqs. (3) and (4) for application to more general stress paths.

It is also true that, even when excluding test CR38r, the
data in Fig. 17 scatter noticeably. It seems that the scatter does not reflect some specific effects of any of the factors examined in the present study. The two curves labelled \( b_{\text{max}} \) and \( b_{\text{min}} \) in Fig. 5(b) correspond to the upper and lower bounds of slope shown in Fig. 17. To obtain the three curves presented in Fig. 5(b), different parameters \( \alpha \) and \( \beta \) are used, while the same value of \( \sigma_s' \) is used. Figures 20(a) and (b) show the simulations of the data from test DN400r obtained by using these three curves presented in Fig. 5(b). It may be seen that the effects of these variations in parameter \( b \) are noticeable. The simulation using the mean curve, as used for all the other simulations, is the most relevant.

CONCLUSIONS

From the results from triaxial compression (TC) tests and the simulation of the test results by the TESRA model presented in this paper, the following conclusions can be derived:

1) Noticeable loading rate effects due to the material viscous properties were observed by changing step-wise the strain rate and performing creep loading during otherwise monotonic loading triaxial compression (TC) at a constant strain rate on both air-dried and water-saturated Toyoura sand under drained conditions.

2) The loading rate effects due to the viscous properties decayed with an increase in the irreversible shear strain when monotonic loading continued at a constant strain rate. The manner of stress decay observed in the TC tests was essentially the same as the one observed in the plane strain compression (PSC) tests. Despite the fact that the general trend of the rate of decay was similar with air-dried and saturated specimens, the rate of decay was noticeably larger with the air-dried specimens than the saturated specimens.

3) The change in the principal stress ratio, \( R = \sigma_1 / \sigma_3 \), upon a step change in the irreversible shear strain rate was always proportional to the instantaneous stress ratio during a single TC test at a fixed confining pressure and in different TC tests at different confining pressures. This trend of behaviour was very similar with air-dried and wet specimens and those having different dry densities and for different stress paths within the limits of test conditions in the present study.

4) The viscosity function represented in terms of principal stress ratio, \( R = \sigma_1 / \sigma_3 \), which had been proposed based on the PSC tests at a fixed confining pressure (400 kPa), could be applied to more general stress paths under the TC stress conditions.

5) The creep shear strain increased with an increase in not only the initial shear strain rate but also the rate of shear strain rate at the start of creep loading under otherwise the same testing conditions.

6) The loading rate effects under various testing conditions in terms of wet condition, pressure level, dry density and stress path observed in the present study could be simulated by the TESRA model using the same functions and parameters for the viscous properties.

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