THREE-DIMENSIONAL SLOPE STABILITY ANALYSIS USING AN EXTENDED SPENCER METHOD

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ABSTRACT

A search procedure is presented to locate 3D critical slip surfaces by combining dynamic programming with a column method extended from the Spencer safety factor equation for 2D analysis. The procedure was applied to typical 3D problems (conical heaps) in order to make a meaningful comparison with variational approaches. Both minimum safety factors and their associated slip surfaces predicted using the two analyses agreed well for all cases investigated. As variational analysis is a rigorous limit equilibrium method without priori statical assumptions, this good agreement adds confidence to the simplifying assumptions of intercolumn forces for the 3D Spencer method. The comparative study presented in this paper strongly supports a recommendation of the use of a statically rigorous limit equilibrium approach satisfying both force and moment equilibrium for a realistic 3D analysis of the slope stability.

Key words: 3D analysis, critical slip surface, limit equilibrium, minimum safety factor, slope stability, variational approach (IGC: E6)

INTRODUCTION

Failure of natural and man-made slopes occurs in three dimensions. Consequently, it is necessary to develop and apply three-dimensional (3D) theories in order to evaluate the stability of slopes more reliably. In practice, limit equilibrium approaches of columns are commonly used for 3D stability analyses of slopes. Limit equilibrium analysis for slope stability requires determination of the critical slip surface that yields the minimum factor of safety. The authors have developed an effective method based on dynamic programming to search for the 3D critical slip surface of a general slope (Yamagami and Jiang, 1997). Major features of this method are the absence of arbitrary assumptions with respect to the shape of potential slip surfaces, the ability to deal with geometrically and geologically complicated slopes, and the applicability of the proposed search scheme to most existing column methods. The search scheme presented by Yamagami and Jiang (1997) was also applied to determine sliding direction in 3D slope stability analysis (Jiang and Yamagami, 1999) and to locate 3D critical slip surfaces based on a nonlinear strength envelope (Jiang et al., 2003).

While the search procedure proposed by Yamagami and Jiang (1997) has the potential to combine with other existing methods of columns (e.g. Hung et al., 1989), the factor of safety equation of the 3D simplified Janbu method (Ugai, 1988) has been employed in previous studies where effort was mainly devoted to the development of an effective search scheme for locating critical slip surfaces. In the 3D Janbu method by Ugai, however, moment equilibrium for the entire sliding mass is not satisfied although the equilibrium conditions for horizontal and vertical forces are both satisfied. Owing to this, it was found that in some cases, in particular for slopes in cohesive soil with limited friction, critical slip surfaces located using the 3D Janbu method were significantly deep compared with those obtained from variational limit equilibrium analyses (Baker and Leshchinsky, 1987; Leshchinsky and Huang, 1992). Also, the values of minimum safety factors calculated using the 3D simplified Janbu method were more conservative than variational solutions.

In this paper, a more rigorous method of columns satisfying both force and moment equilibrium is incorporated into dynamic programming to locate the 3D critical slip surface for a general slope. This is an extension of the search procedure presented by Yamagami and Jiang (1997), with all the characteristic features being inherited from the original method. Both 3D critical slip surfaces and associated minimum safety factors for typical slopes agree well with those using variational analyses (Baker and Leshchinsky, 1987). As the variational approach and the extended Spencer method employ the same definition of safety factor, such a comparison of the two methods under the most critical conditions is meaningful. It has been shown that variational analysis, as a rigorous limit equilibrium method without priori static assumptions (Leshchinsky and Huang, 1992), can yield nearly the same safety factors and the corresponding critical slip surfaces as those from finite element computations for

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Manuscript was received for review on July 28, 2003.

Written discussions on this paper should be submitted before March 1, 2005 to the Japanese Geotechnical Society, 4-38-2, Sengoku, Bunkyo-ku, Tokyo 112-0011, Japan. Upon request the closing date may be extended one month.
vertical cuts (Ugai and Leshchinsky, 1995). Therefore, the comparison presented in this paper can be used to judge the validity of simplified assumptions about intercolumn forces made in the 3D Spencer method. The good agreement of results by these two analyses strongly supports a recommendation of the use of a statically rigorous limit equilibrium method satisfying both force and moment equilibrium for a realistic 3D stability analysis of slopes.

**AN EXTENDED SPENCER METHOD FOR 3D SLOPE STABILITY ANALYSIS**

A 3D slip surface within a slope is shown schematically in Fig. 1. In deriving the factor of safety equations, existing limit equilibrium methods of columns, with the exception of Huang and Tsai (2000) and Huang et al. (2002), employ the same slip mechanism where the entire sliding mass is assumed to move in a single direction (i.e. the direction of sliding is uniform for the whole soil mass over a potential slip surface), e.g. Hung et al., 1989; Lam and Fredlund, 1993. This slip mechanism is also employed in this paper.

Figure 2 depicts forces acting on a typical column taken from Fig. 1: \( W \) = weight of a column; \( T \) and \( N \) = shear force and total normal force acting on the column base; \( Q \) = resultant of all intercolumn forces acting on the column sides; \( \Delta x \) and \( \Delta y \) = width of the column divided in the \( x \)- and \( y \)-directions; \( \alpha_{xz} \) and \( \alpha_{yz} \) = inclination angles of the column base to the horizontal direction in the \( xz \) and \( yz \) planes, respectively.

By assuming an applying direction of \( Q \) as shown in Fig. 3(a), a 3D method for slope stability analysis was presented by Ugai and Hosobori (1989) which could be considered partly as an extension of the Spencer method (Spencer, 1967) for 2D analyses. It is seen from Fig. 3(a) that \( Q \) consists of two components, i.e. \( Q_x \) in the \( xz \) plane and \( Q_y \) in the \( yz \) plane. The former is inclined at an angle of \( \delta \) (an unknown constant for all columns) to the \( x \)-axis (sliding direction) as in the Spencer method (1967), and the latter is assumed to be parallel to the \( y \)-axis. The normal force \( N \) and shear force \( T \) acting on the column base can be derived by considering force equilibrium in the direction perpendicular to the \( Q, QQ \), plane and the Mohr-Coulomb failure criterion at the column base. The factor of safety with respect to force equilibrium, \( F_t \), was given by summing forces for all columns over the whole slip surface in the horizontal direction (the \( x \)-axis):

\[
F_t = \frac{\sum \left[(c' - u \tan \phi') \sec \alpha_{xz} \Delta x \Delta y + W[(\sec \alpha_{xz} - \tan \delta \sin \alpha_{xz}) \tan \phi' + F \tan \delta \tan^2 \alpha_{xz}/J]\right] / m_a}{\sum W \tan \alpha_{xz}}
\]  

(1)

where \( c', \phi' = \) effective strength parameters of soil, \( u = \) pore water pressure at the column base, and \( J = (1 + \tan^2 \alpha_{xz} + \tan^2 \alpha_{yz})^{1/2} \).

The summation of moments of all columns about an axis of rotation parallel to the \( y \)-axis was used to derive the factor of safety \( F_m \) with respect to moment equilibrium:

\[
F_m = \frac{\sum R \left[(c' - u \tan \phi') \sec \alpha_{xz} \Delta x \Delta y \sin(\theta + \delta) / \cos \delta + W \cos(\theta + \alpha_{xz})[(\tan \phi' \tan(\theta + \alpha_{xz}) + F \tan \delta \tan^2 \alpha_{xz}/J]\right] / m_a}{\sum WR \cos \theta}
\]  

(2)

in which \( R = \) distance from the axis of rotation to the base centre of a column in the \( xz \) plane, and \( \theta = \) angle between the horizontal direction and the \( R \) direction in the \( xz \) plane.

The symbol \( m_a \) in Eqs. (1)-(2) is given by

\[
m_a = (1 + \tan \delta \tan \alpha_{xz})/J + (\sin \alpha_{xz} - \tan \delta \cos \alpha_{xz}) \tan \phi'/F
\]
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![Diagrams of Extended Spencer method and 3D simplified Janbu method](image)

Fig. 3. Assumption of acting direction of intercolumn resultant \( Q \)

where \( F \) is taken as \( F_1 \) when \( m_a \) is used in Eq. (1) and as \( F_m \) when \( m_a \) is used in Eq. (2).

The factor of safety of a given slip surface may be obtained by solving the simultaneous equations, Eqs. (1) and (2). For several appropriately given angles (\( \delta \)), values of \( F_1 \) and \( F_m \) can separately be computed from Eqs. (1) and (2) for a given slip surface. Then, two curves showing the \( F_1 - \delta \) and \( F_m - \delta \) relationships can be plotted. The point of intersection of these two curves leads to a required value of the factor of safety. Obviously, the factor of safety so obtained satisfies both force equilibrium and moment equilibrium. It should be noted that in deriving Eqs. (1) and (2) the simplifying assumptions are made not for individual forces acting on the side faces of each column but for the resultant \( Q \) of intercolumn forces. This avoids difficulties in making the problem determinate and significantly simplifies the solution procedure. Nevertheless, sufficiently accurate critical slip surfaces and minimum factors of safety for 3D slopes can be determined based on Eqs. (1) and (2), as will be shown later.

A 3D simplified Janbu method for slope stability analysis was also presented by Ugai (1988). In this method, component \( Q_1 \) of \( Q \) in the \( xz \) plane was assumed to be parallel to the \( x \)-axis and component \( Q_2 \) of \( Q \) in the \( yz \) plane to be inclined to the \( y \)-axis at an angle of \( \tan^{-1}(\eta \tan \alpha_{cz}) \) (\( \eta \) is an unknown constant) as shown in Fig. 3(b). Two safety factors, \( F_h \) and \( F_v \), are derived based on the force equilibrium of all columns in horizontal and vertical directions, respectively. Both factors depend on \( \eta \) values and they become equal to each other at a certain \( \eta \) value. It has been shown that as a required solution, the equation \( F = F_h = F_v \) was usually obtained at very small \( \eta \) values, e.g., \( \eta = 0.0082 \) for an actual 3D failure surface (Ugai, 1987) and \( \eta = 0.0075 \) for the critical 3D slip surface in a non-symmetrical slope (Jiang and Yamagami, 1999). Since very small \( \eta \) values (close to zero) are reached as computational results and a value of \( \eta \approx 0.0 \) leads to a nearly horizontal resultant \( Q \) of intercolumn forces that is clearly inconsistent with actual situations, the acting direction of \( Q \) shown in Fig. 3(b) is, in principle, not a suitable assumption. Possible errors caused by this assumption will be discussed later together with example problems.

It has been shown that safety factor, \( F_h \), calculated for very small \( \eta \) values including \( \eta = 0.0 \) shows little change with variation in \( \eta \) (Ugai, 1987; Jiang and Yamagami, 1999). In other words, safety factors obtained by solving the simultaneous \( F_h \) and \( F_v \) equations almost equal to \( F_h \) values calculated using \( \eta = 0 \). Therefore, the safety factor of the 3D simplified Janbu method can be defined by the following \( F_h \) equation (\( \eta = 0 \)).

\[
F_h = \sum [(c' - \mu \tan \phi')A_\alpha x A_y y + W \tan \phi'[(1/J + \sin \alpha_{cz} \tan \phi'/F_h) \cos \alpha_{cz}]] \sum W \tan \alpha_{cz} \tag{3}
\]

DYNAMIC PROGRAMMING SEARCH PROCEDURE

The search scheme for finding the minimum factor of safety of a given 3D slope was developed by incorporating the extended Janbu method (Eq. (3)) into dynamic programming (Yamagami and Jiang, 1997). This scheme is first outlined herein, and then a search procedure is presented based on the 3D Spencer Eqs. (1)–(2) to determine critical slip surfaces.

Application of dynamic programming to a particular problem requires a stage-state system (Baker, 1980). Figure 4 illustrates such a stage-state system for a 3D slope (Yamagami and Jiang, 1997). In the present case, 'stages' are vertical planes (sections) perpendicular to the \( x \)-axis (sliding direction), and a 'state' in a stage is represented by a curve in the stage plane. Note that for convenience only one state for each stage is shown in Fig. 4, though a large number of state curves in each stage plane are necessary in order to search for a smooth 3D critical slip surface. State curves in each stage plane are produced by making use of a random number generation technique (Yamagami and Jiang, 1997). Field observations of actual 3D slip surfaces suggest that state curves are concave thus satisfying the \( \partial^2 z/\partial^2 y \geq 0 \) condition. This constraint was enforced during generation of state curves to exclude highly undulating slip surfaces.

In order to obtain a sufficiently accurate solution for a general slope, a few hundred state curves usually need to be produced in each stage plane prior to the dynamic
programming search. If a state curve $k$ in a stage $i$ and a state curve $kk$ in stage $i+1$ are selected, as shown in Fig. 4, the shaded concave segment sandwiched between these two state curves constitutes a part of the potential 3D slip surface. When a state curve is selected from each stage plane, connecting these state curves in turn will therefore result in a possible 3D slip surface. The factor of safety of the slip surface can be calculated using Eqs. (1) and (2) by dividing the soil mass into columns. The objective is, of course, to find the critical slip surface giving the minimum factor of safety. This is a typical optimization problem to which dynamic programming can effectively and efficiently be applied. The details of the dynamic programming search based on the 3D simplified Janbu method were presented in the previous paper (Yamagami and Jiang, 1997). It has been shown through several examples that using this search method a smooth 3D critical slip surface can always be obtained provided that a sufficient number of state curves have been generated in each stage plane prior to the dynamic programming search.

As mentioned previously, the search scheme presented by Yamagami and Jiang (1997) can also be incorporated into other existing column methods as long as their safety factor equation is expressed with a ratio of two summations as described by Eq. (3). It is noted that both $F_i$ in Eq. (1) and $F_m$ in Eq. (2) have a similar form to the factor of safety formula of the simplified Janbu method i.e. Eq. (3). This means that the search scheme by Yamagami and Jiang (1997) can also be coupled with Eqs. (1) and (2) separately to find minimum values for $F_i$ and $F_m$. The computation procedure is shown as follows.

1. Several appropriate values of $\delta$ are first specified.
2. For each $\delta$ value, the search scheme by Yamagami and Jiang (1997) is applied to Eq. (1), so that minimum values of the factor of safety $F_i$ are obtained.
3. If the above-mentioned search scheme is combined with Eq. (2), then minimum values of the factor of safety $F_m$ can also be determined using the assumed $\delta$ values.
4. A curve can then be plotted showing the relationship between $F_i$ and $\delta$, and on the same graph, another curve can also be plotted showing the $F_m-\delta$ relationship (see Fig. 7 in the next section). The intersection of the two curves gives a required value of $\delta=\delta_0$.
5. Substituting the above $\delta_0$ value into Eqs. (1) or (2) and performing the proposed search procedure again, a required solution, i.e. the critical slip surface and the associated minimum factor of safe-
ty, $F_0$, are determined.

It has been shown through several example problems that critical slip surfaces from the $F_I$ equation are nearly the same as those from the $F_m$ equation for cases of $\delta = \delta_0$. However, when different $\delta$ values from $\delta_0$ are used, critical slip surfaces and associated minimum safety factors predicted using the $F_I$ and $F_m$ equations usually differ. Moreover, it has also been examined that when the search procedure is performed together with the $F_m$ equation, i.e., Eq. (2), critical slip surfaces obtained are hardly changed with variation in positions of the rotation centre.

COMPARISON WITH VARIATIONAL SOLUTIONS

The ability of the proposed search procedure to deal with a variety of 3D problems including geometrically and geologically complicated slopes has been shown by Yamagami and Jiang (1997) and Jiang and Yamagami (1999). Herein, the method is applied to typical homogeneous slopes, i.e., 3D conical heaps with an inclination of $30^\circ$, which were previously analyzed using the variational limit equilibrium analyses (Baker and Leshchinsky, 1987). As the variational approach employs the same definition of safety factor as in the 3D Spencer method, the two analyses are compatible. It should be noted that minimum factors of safety for both methods are needed to make the comparison meaningful (e.g., Cavounidis, 1987; Duncan, 1996). The variational analyses under most critical conditions resulted in extended log-spiral slip surfaces, as shown in Fig. 5 (Baker and Leshchinsky, 1987).

A search for critical slip surfaces was first carried out based on the 3D simplified Janbu method using the same soil parameters as those shown in Fig. 5. Results obtained are illustrated in Fig. 6. It should be noted that coordinates are indicated by non-dimensional parameters $x/H$, $y/H$ and $z/H$ ($H$ is cone height) in Fig. 5, while cone height for the present analyses is taken as 5.0 m in Fig. 6. In addition, values of cohesion $c$ for the present analyses are calculated from a non-dimensional parameter $\lambda = c/(\gamma H \tan \phi)$ used in Baker and Leshchinsky’s paper (1987) by assuming unit weight of soil to be $\gamma = 17.64 \text{kN/m}^3$. It has been shown (Yamagami and Jiang, 1997) that the analyses by changing $c-\phi$ values but keeping $\lambda$ constant result in the same locations of critical slip surfaces with different values of minimum safety factors.

In the case of a purely cohesive soil (i.e., $\phi = 0$), the critical slip surface located from the 3D Janbu method, as shown in Fig. 6(b), is significantly deeper than the extended log-spiral surface in Fig. 5(b), and the minimum safety factor ($F_m = 0.865$) was much smaller than the variational value ($F = 1.00$). In order to assess the static feasibility of the deep-seated surface shown in Fig. 6(b), normal forces acting on the critical slip surface were checked (these are not given due to the limitation of space). It has been shown that distribution of normal stresses along the critical slip surface was feasible and no negative normal forces were found. In addition, inspection of the force polygon for each of randomly selected columns indicates that the intercolumn resultant $Q$ and the resultant shear and normal forces on the column base completely balance the weight force of the column. Consequently, it may be concluded that the critical slip surface shown in Fig. 6(b) is a statically acceptable solution which satisfies all the force equilibrium conditions involved in the 3D Janbu method. The difference between results by the 3D simplified Janbu method and variational analysis probably stems from the difference in the static formulation used in the two approaches. This will be discussed later.

By choosing $\delta = 0^\circ - 40^\circ$ at $5^\circ$ intervals, the search procedures based on the extended Spencer method, i.e., $F_I$ in Eq. (1) and $F_m$ in Eq. (2) were performed for the conical heaps as shown in Fig. 5. The $F_I-\delta$ and $F_m-\delta$ relationships obtained are presented in Fig. 7. Values of $\delta_0$ given by the intersections of $F_I-\delta$ and $F_m-\delta$ curves are also presented in the same figure. Substituting these $\delta_0$ values into Eqs. (1) or (2), the search method proposed in this paper is carried out again, and results are given in Fig. 8. Note that the critical slip surfaces and minimum factors of safety in Fig. 8 are found based on the $F_I$ equation, which are identical to those from the $F_m$ equation. In other words, the solutions shown in Fig. 8 satisfy both force and moment equilibrium conditions involved in the extended Spencer method. It can be seen from Figs. 8 and 5 that both critical slip surfaces and minimum factors of safety from the extended Spencer method agree well with those from the variational analysis.
Using different combinations of strength parameters, Figs. 9(a) and (b) show traces of the 3D critical slip surfaces on the plane of symmetry obtained from the 3D Janbu method and the extended Spencer method. Results from the variational analysis (Baker and Leshchinsky, 1987) are also plotted in these two figures. It is seen from Fig. 9(a) that in every case critical slip surfaces from the 3D Janbu method are deeper than log-spiral ones. This discrepancy tends to be larger with an increase in \( \lambda = c / (\gamma H \tan \phi) \) (i.e. in cohesion of soil), reaching a maximum for the case of a purely cohesive soil (i.e. \( \lambda = \infty \) or \( \phi = 0 \)). The results shown in Fig. 9(b) indicate, however, that the critical slip surfaces located from the extended Spencer method are in good agreement with those by variational analyses, especially for cases of smaller \( \lambda \) values.

DISCUSSION

On Simplifying Assumption of Intercolumn Forces in Extended Spencer Method

Variational analysis is a rigorous limit equilibrium method without priori static assumptions which can produce nearly the same safety factors and the associated critical slip surfaces as those from finite element computations for vertical cuts (Ugai and Leshchinsky, 1995). Therefore, the comparison presented above can be used to judge the validity of simplifying assumptions of intercolumn forces for the 3D Janbu and extended Spencer methods used in this paper. As mentioned before, the critical slip surfaces shown both in Figs. 6 and 8 are statically acceptable solutions which satisfy all force and/or moment equilibrium conditions involved in the 3D Janbu method and extended Spencer method, respectively. Naturally, one may question why the results obtained using the 3D simplified Janbu method are so different from those by the variational computation? Error of the 3D Janbu method may arise from an inappropriate assumption about resultant \( Q \) of intercolumn forces acting on each column. Previous studies (Ugai, 1987; Jiang and Yamagami, 1999) have shown that solutions of the 3D simplified Janbu method were usually obtained at very small \( \eta \) values (close to zero), and as a result, the safety factors obtained by solving combined \( F_s \) and \( F_t \) equations almost equal to the \( F_s \) values calculated from Eq. (3). This is also true for the cases investigated in this paper and therefore Eq. (3) rather than the simultaneous \( F_s \) and \( F_t \) equations is used for calculation of safety factors. However, a value of \( \eta = 0.0 \) means that resultant \( Q \) of intercolumn forces of each column acts in a horizontal direction, and this is clearly inconsistent with actual situations. Consequently, the acting direction of \( Q \) shown in Fig. 3(b) is, actually, not a suitable assumption. Another source of the error may be due to unsatisfaction of overall moment equilibrium in the 3D Janbu method. It is of interest to note that although a simplifying assumption regarding internal forces as shown in Fig. 3(a) is made for the extended Spencer method, both critical 3D slip surfaces and associated safety factors agreed well with variational solutions. This adds reliability to the simplifying assumptions in Fig. 3(a) and indicates that the method of columns satisfying both force and moment equilibrium could provide a more realistic evaluation of the slope stability.

Validity of Single Sliding Direction Assumption in Existing Column Methods

Almost all existing column methods for 3D slope stability analysis (e.g. Ugai, 1988; Hungr et al., 1989; Lam and Fredlund, 1993) employ a simple slip mechanism in which the entire sliding mass is assumed to move in a single direction (i.e. the direction of sliding is uniform for the whole soil mass over a potential slip surface). In these methods, force and/or moment equilibrium conditions are satisfied only in the sliding direction (i.e. in the \( xz \) plane) but are ignored in the transverse direction (i.e. in the \( yz \) plane). Hence, they are sometimes referred to as ‘one-directional force and moment equilibrium’ methods (Huang and Tsai, 2000). The variational analyses (e.g. Leshchinsky and Huang, 1992) and the ‘two-directional force and moment equilibrium’ method by Huang et al. (2002) consider variable direction of movement; namely the direction of sliding in these methods is obtained as a part of the analytical solution rather than being assumed in advance. As a result, the computed direction of sliding (i.e. direction of mobilized resultant of shear force on the slip surface) may vary from place to place over the potential slip surface.
It seems that the assumption of a uniform movement-direction is appropriate for analysis of an approximately symmetrical 3D sliding mass (Jiang and Yamagami, 2004). In such a case, the distribution of shear force acting on the slip surface should also be symmetrical. This means that the transverse-direction components of the shear force at two symmetrical points on the slip surface have the same magnitude but act in the opposite direction. When the limit equilibrium equations of the whole sliding mass are considered, therefore, their effects in the transverse direction will be cancelled out. In other words, transverse force and moment equilibrium conditions are automatically satisfied due to symmetry of the problem. Consequently, little inaccuracy due to the single sliding-direction assumption and neglect of equilibrium conditions in the transverse-direction may occur for 3D slopes which can approximately be regarded as symmetrical. This is strongly supported by the good agreement of the results using the variational computation which considers variable direction of sliding and the extended 3D Spencer method which assumes a single movement direction. As unbalanced forces and moments in the transverse direction of sliding may account for the errors in the calculated value of safety factors (Huang and Tsai, 2000), the ‘two-directional force and moment equilibrium’ method by Huang et al. (2002) may be expected to make more reliable prediction of the stability of asymmetrical 3D slopes.

Comparisons of Different 3D Methods for Slope Stability Analysis

In an attempt to create a different perspective for the stability analysis using variational analyses and 3D column methods, the authors produced a sliding surface (Fig. 10) based on the critical slip surface shown in Fig. 8(b) by adjusting the trace of the extended Spencer method to coincidence with the trace from the variational analysis on the symmetry plane and by making similar adjustment on the other longitudinal sections. The surface so generated is an approximation of the variational sliding surface shown in Fig. 5(b). On this approximate surface the safety factor of the 3D Janbu method and the extended Spencer method was computed to be 0.973 and 1.024 (at $\delta_0=5.8^\circ$), respectively, com-
pared with their minimum values of 0.865 and 0.980 for the critical slip surfaces. The value of 0.973 for the 3D Janbu method is very close to the variational factor of safety, 1.00. These results indicate that if safety factors of different methods are compared for arbitrary given slip surfaces rather than critical ones, a misunderstanding may occur. Therefore, comparisons of results using different methods should be made only for most critical conditions, i.e. comparing minimum factors of safety and associated critical slip surfaces. This was emphasized by Duncan (1996) for 2D analyses and also by Cavounidis (1987) when comparing 3D and 2D factors of safety for a particular slope. Consequently, the search method described in this paper provides a useful tool to make a meaningful comparison i) among different 3D limit equilibrium methods of columns and ii) between 2D and 3D analyses for a slope. Such comparative studies are necessary and useful to investigate computational accuracy of individual slope stability analysis methods. It should be noted that safety factors computed using different methods for an actual failed slope can be compared as it has a well-defined 3D failure surface for which the minimum factor of safety is known and equal to unity (Stark and Eid, 1998).

CONCLUSIONS
A search procedure has been presented to locate the 3D critical slip surface by combining dynamic programming with an extended Spencer method satisfying both force and moment equilibrium. This is an extension of the search scheme presented by Yamagami and Jiang (1997) where a 3D simplified Janbu method satisfying only force equilibrium is employed. The following conclusions are based on the 3D stability analyses for typical slopes (conical heaps) and a comparison with variational results carried out in this study.

Minimum safety factors and associated 3D critical slip surfaces determined for conical heaps using the extended Spencer method agreed well with variational solutions. However, discrepancy was exhibited between results by the 3D simplified Janbu method and variational computation. As variational analysis is a rigorous limit equilibrium approach and can predict nearly the same safety factors and the associated critical slip surfaces as those by finite element computation for vertical cuts, the good agreement obtained from the problems investigated indicate that the 3D Spencer method can be used with increased reliability. The comparative study presented in this paper strongly supports a recommendation of the use of a statically rigorous limit equilibrium method satisfying both force and moment equilibrium for a realistic 3D stability analysis of slopes.

In most of existing column methods for 3D slope stability analysis, the entire sliding mass is assumed to move in a single direction and force and/or moment equilibrium is satisfied only in the direction of movement (equilibrium conditions in the transverse direction are ignored). Variational approach by Leshchinsky and Huang (1992) and generalized method of columns by Huang et al. (2002) consider variable direction of movement along the potential slip surface and satisfies global force and moment equilibrium for a sliding mass. Huang et al. (2002) reported that safety factors for symmetrical slopes computed using their generalized method were rather close to values from rigorous 'one-directional force and moment equilibrium' methods. This together with the good agreement obtained for the cases analyzed in this paper demonstrates that existing 'one-directional force and moment equilibrium' methods can be used for analysis of approximately symmetrical slopes with increased confidence.

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