SIMPLIFIED METHOD FOR ANALYSIS OF PILES UNDERGOING LATERAL SPREADING IN LIQUEFIED SOILS

Misko Cubrnovski and Kenji Ishihara

ABSTRACT

The analysis of piles subjected to lateral spreading is burdened by the uncertainties associated with the spreading of liquefied soils. Namely, it is very difficult to reliably predict the magnitude and spatial distribution of post-liquefaction ground displacements and also there are uncertainties regarding the stiffness and strength of liquefied soils undergoing lateral spreading. In view of the unknowns involved, there is a need for a sufficiently accurate, yet simple and rational method of analysis that will permit to efficiently evaluate the pile response for various magnitudes of ground displacements and stiffness properties of the spreading soils.

In this paper a simple analytical solution is presented for evaluating the pile response to lateral spreading with due consideration of the kinematic mechanism associated with spreading of liquefied soils and the need to estimate the inelastic response and damage to piles. Particular attention is given to the loads arising from a non-liquefied crust layer at the ground surface and to the kinematic effects at the interface between the liquefied layer and underlying non-liquefied layer. A closed-form solution for linear behavior is first derived based on the classical equation for beam on elastic foundation. The method is then extended over the range of nonlinear behavior using the equivalent linear approach for simplified modeling of the deformational behavior of the pile and soil. Key parameters influencing the pile response and being associated with intrinsic uncertainties are identified and discussed. The proposed method permits to estimate the inelastic response of piles, yet it is a simple analytical solution that requires a few conventional engineering parameters as input. The method is envisioned as a rational tool that will be of practical use in the preliminary assessment of piles and uncertainties involved.

Key words: analysis, inelastic deformation, lateral spreading, liquefaction, pile (IGC: E4/E13)

INTRODUCTION

Soil liquefaction has been a major cause of damage to pile foundations in many strong earthquakes. The numerous case histories of damaged piles due to liquefaction in the 1995 Kobe earthquake (JGS, 1998) have instigated intensive research on the subject in an effort to better understand soil-pile interaction in liquefied soils and to improve the seismic performance of pile foundations. The kinematic interaction or effects of ground displacements on the pile response have received particular attention, for two reasons. One is because it has been found that the excessive lateral movement of liquefied soils was a key factor in the damage to piles, and the other is the fact that, by and large, the effects of lateral ground displacements had been either ignored or crudely approximated in the seismic design codes for piles.

When analyzing the response of piles in liquefiable soils, it is useful to distinguish between two different phases in the soil-pile interaction during or after an earthquake: (1) cyclic phase in the course of intense ground shaking and consequent development of liquefaction, and (2) the phase of lateral spreading following the liquefaction. As illustrated in Fig. 1(a), cyclic ground displacements are accompanied by respective inertial forces from the superstructure and the combination of these oscillatory kinematic and inertial loads determines the critical load for the integrity of the pile during the cyclic loading phase. Lateral spreading, on the other hand, is characterized by very large uniaxial ground displacements and relatively small inertial effects (Fig. 1(b)). The inertial loads may play an important role at the outset of lateral spreading, but as the spreading progresses and large ground displacements develop, the intensity of shaking diminishes and its effects become relatively small. Thus, the kinematic load resulting from the lateral ground movement is the key factor controlling the pile response to lateral spreading.

There are several methods available for analysis of
piles in liquefiable soils including sophisticated finite-element analysis based on the effective stress principle and Winkler-type models as a typical approach in modeling pile behavior (e.g., Finn and Thavaraj, 2001; Gazetas and Mylonakis, 1998; Hamada, 2000; O'Rourke et al., 1994). Closed-form solutions based on the classical equation for beam on elastic foundation would be an alternative approach to the analysis of piles (e.g., Nishimura, 1978; Wang and Reese, 1998; Mylonakis, 2001). Irrespective of the adopted analytical method, however, the analysis of piles subjected to lateral spreading is burdened by the uncertainties associated with the spreading of liquefied soils. Namely, it is very difficult to reliably predict the magnitude and spatial distribution of post-liquefaction ground displacements, and also, there are uncertainties regarding the stiffness and strength of liquefied soils undergoing lateral spreading. Thus, the key issue is not the methodology itself, but rather how to deal with unknowns in the analysis while capturing the essential features of pile behavior. In this context, it is desirable to have available a method of analysis which is sufficiently accurate, yet simple and rational, so as to allow for variations in parameters and assessment of the uncertainties involved.

Based on these premises, the aim of this paper is to present a simple method for analysis of piles that permits to efficiently examine the pile response for a wide range of ground displacements and stiffness characteristics of the spreading soils. Particular attention is given to the kinematic mechanism associated with the spreading of liquefied soils and to the need to estimate the inelastic response and damage to piles. The adopted mathematical model is first introduced and closed-form solution for linear behavior is derived. The method of analysis is then extended over the range of nonlinear behavior using the equivalent linear approach. Key parameters influencing the pile response and being associated with intrinsic uncertainties are identified and discussed.

**ADOPTED SOIL-PILE MODEL.**

The most frequently encountered soil profile for piles in liquefied deposits is schematically illustrated in Fig. 2 where the liquefied layer is sandwiched between a non-liquefied crust layer at the ground surface and non-liquefied base layer. The identification of these distinct soil layers is an important consideration in the assessment of pile behavior because the kinematic loads associated with lateral spreading are directly related to this kind of soil stratification, as discussed below.

In most cases of liquefiable field deposits, a layer of soil at the ground surface does not liquefy during the earthquake. During lateral spreading, the non-liquefied crust layer is carried along with the underlying spreading soil, and when driven against embedded piles, the crust layer is envisioned to exert large lateral loads on the piles. Berrill et al. (2001), for example, presented a case history from the 1987 Edgecumbe earthquake in which the pile foundation of a bridge pier was subjected to large lateral loads from a 1.5 m thick non-liquefied crust. Large lateral loads from a non-liquefied surface layer have also been measured in full-scale tests on piles in laterally spreading soils (Suda et al., 2002; Yamamoto et al., 2002).

One important observation from the 1995 Kobe earthquake was that piles have been consistently damaged in the zone of the interface between a liquefied layer and an underlying non-liquefied layer (Fuji et al., 1998; Tokimatsu and Asaka, 1998; Cubrinovski and Ishihara, 2001). It is now well recognized and supported both experimentally and analytically that large kinematic bending moments could develop at an interface of two soil layers where sharp change in stiffness occurs. In the case of liquefaction, the stiffness contrast between the liquefied soil and underlying non-liquefied layer is markedly pronounced, thus leading to a large increase in the bending strains of the pile near this interface. The ground displacement profile, which is characterized by very large displacements in the liquefied layer and much
smaller deformation in the underlying base layer, is another factor contributing to the development of large interface bending moments. Clearly, the pile response at the interface between the liquefied layer and the underlying non-liquefied layer is of primary importance in the case of lateral spreading.

In the light of the liquefaction characteristics and kinematic mechanism as above, a three-layer soil model is considered to represent the most frequently encountered soil profile in the simplest yet sufficiently accurate way for developing in-depth understanding of pile behavior in laterally spreading soils. As illustrated in Fig. 2, in the adopted pseudostatic approach the spreading is represented by a horizontal displacement of the liquefied soil whereas effects of the surface layer are modeled by an earth pressure and lateral force at the pile head. Here, the earth pressure represents the loads that act directly on the pile while the lateral force approximates the loads that are transferred to the pile through the upper foundation. With reference to the general classification of analytical soil-pile models into earth-pressure and ground-displacement models (Yasuda and Berrill, 2000), the model depicted in Fig. 2 is of a hybrid type since it employs both soil displacement and pressure to specify the kinematic loads due to lateral spreading.

Figure 3 shows the adopted analytical model together with its computational parameters. It is assumed that the pile is vertical, that the soil in each of the three layers is homogeneous, and that both soil and pile exhibit linear behavior. For the soil-pile system shown in Fig. 3, the problem is now reduced to determining the pile response due to lateral displacement of the liquefied soil and loads at the pile head. In what follows, a closed-form solution is derived for the adopted soil-pile model based on the classical differential equation for beam on elastic foundation.

**CLOSED-FORM SOLUTION FOR LINEAR BEHAVIOR**

The governing differential equations for the soil-pile model shown in Fig. 3 are

\[ EI \frac{d^4y_i}{dz_i^4} = P_y z_i + P_q \quad (0 \leq z_i \leq H_1) \]  
\[ EI \frac{d^4y_2}{dz_2^4} = \beta_2 k_2 D_0 (U_{z_2}(z_2) - y_2) \quad (0 \leq z_2 \leq H_2) \]  
\[ EI \frac{d^4y_3}{dz_3^4} = -\beta_3 k_3 D_0 y_3 \quad (0 \leq z_3 \leq H_3) \]

where \( D_0 \) and \( EI \) are the diameter and bending stiffness of the pile, \( y_i(z_i) \) is the horizontal pile displacement, \( z_i \) and \( H_i \) denote, respectively, depth in local coordinates and thickness of \( i \)-th layer (\( i = 1, 3 \)), \( P_q \) and \( P_y \) are earth pressure parameters of the surface layer, and \( k_2 \) and \( k_3 \) are the horizontal subgrade reaction coefficients while \( \beta_2 \) and \( \beta_3 \) are respective stiffness reduction coefficients for the liquefied layer and non-liquefied base layer. Note that the product \( \beta k \) specifies the soil stiffness and that \( \beta \) indicates the degree of reduction in stiffness due to liquefaction and nonlinear behavior.

With respect to the distribution displacement in the liquefied layer, model tests have shown that it can be approximately represented by a cosine function. Thus, assuming a cosine function for the distribution of lateral ground displacement throughout the depth of the liquefied layer as

\[ U_{z_2}(z_2) = U_{z_2} \cos \left( \frac{\pi \ z_2}{2 H_2} \right) \quad (0 \leq z_2 \leq H_2) \]

the general solution of the differential equations takes the following form

\[ y_1(z_1) = \frac{P_y}{120EI} z_1 + \frac{P_q}{24EI} z_1 + A_1 z_1 + B_1 z_1^3 + C_1 z_1 + D_1 \]

\[ y_2(z_2) = e^{x(z_2/H_2)} [A_2 \sin (\lambda z_2/H_2) + B_2 \cos (\lambda z_2/H_2)] + e^{-x(z_2/H_2)} [C_2 \sin (\lambda z_2/H_2) + D_2 \cos (\lambda z_2/H_2)] + G_2 \cos \left( \frac{\pi z_2}{2 H_2} \right) \]

\[ y_3(z_3) = e^{x(z_3/H_3)} [A_3 \sin (\lambda z_3/H_3) + B_3 \cos (\lambda z_3/H_3)] + e^{-x(z_3/H_3)} [C_3 \sin (\lambda z_3/H_3) + D_3 \cos (\lambda z_3/H_3)] \]

Here, \( U_{z_2} \) is the horizontal ground displacement at the top of the liquefied soil and \( L = H_1 + H_2 + H_3 \) denotes the length of the pile (Fig. 3). The parameter \( G_2 \) is defined as
\[ G_2 = \frac{U_{\alpha 2}}{1 + \frac{\pi^4}{64\lambda_i^4}} \]  

whereas \( \lambda_i \) is a dimensionless parameter expressing the flexibility of the pile relative to the soil and is given by

\[ \lambda_i = H \left( \frac{\beta_i k_i D_i}{4EI} \right)^{1/4}, \quad (i = 2, 3) \]

Finally, \( A_i, B_i, C_i \) and \( D_i \) \((i = 1, 3)\) are twelve constants to be determined from continuity and boundary conditions, as described in APPENDIX A. Once the above constants are determined and introduced into the general solution, pile displacements can be readily computed using Eqs. (3a) through (3c) whereas bending moments, shear forces and lateral loads can be evaluated using the following expressions;

\[ M(z) = Ely''(z) \quad (i = 1, 3) \]

\[ Q(z) = Ely''(z) \]

\[ q(z) = Ely''(z) \]

The above solution for the linear behavior can be applied to piles under various boundary conditions including free-head piles, piles restrained from rotation at the top, and piles either hinged or fixed at the base. Details of the evaluation of the integration constants \((A_i, B_i, C_i, D_i; i = 1, 3)\) including compatibility equations and boundary expressions are given in APPENDIX A for piles fixed at the base and restrained from rotation at the top.

**MODELING OF NONLINEAR BEHAVIOR**

The phenomenon of lateral spreading involves very large ground displacements and the deflection of piles embedded in such soil deposits does induce bending moments in the pile body which are in excess of the cracking or yield level. Thus, it is necessary in the analysis of piles to have means for examining the pile response in the range of inelastic deformations. To this goal, the use of the analytical solution for linear behavior will be extended over the range of nonlinear behavior.

The liquefaction causes almost a complete loss of strength and stiffness in the soil, and therefore, the exact nonlinear behavior of liquefied soils undergoing spreading is very complex and difficult for precise modeling. For this reason, it was adopted in this study to simply represent the nonlinear behavior of the liquefied layer by a degraded linear stiffness \( \beta_i k_i \) in which \( \beta_i \) incorporates the reduction in stiffness due to liquefaction and subsequent spreading of sandy soil deposits. In the analysis of a given pile, it is envisioned that \( \beta_i \) will serve as a parameter that will be varied over a relevant range of values thus permitting the pile response to be evaluated by assuming different properties of the liquefied soil. For modeling the nonlinear behavior of the pile and non-liquefied soils, an iterative scheme of analysis was developed using the closed-form linear solution as a basis. In the following sections, simplified methods for modeling nonlinear behavior of the pile, yielding in the base layer and nonlinear effects in the surface layer are first introduced separately and then they are incorporated into an integral computational procedure for evaluating the inelastic pile response.

**Nonlinear Behavior of the Pile**

In the closed-form solution, a linear moment-curvature relationship of the pile is assumed by way of the bending stiffness \( EI \). Instead of using the initial stiffness, however, it is possible to employ in the solution an equivalent linear or secant stiffness \((EI_{\alpha})\) so as to approximately model the nonlinear moment-curvature relationship of the pile. Based on this reasoning, an equivalent linear model was developed for the pile in which the closed-form linear solution is used in an iterative manner.

The calculation method will be illustrated in the example of a 30 cm-diameter PHC pile (prestressed high-strength concrete pile) embedded in a two-layer deposit consisting of an upper liquefied layer and a base non-liquefied layer having thicknesses of 10 m and 12 m, as shown in Fig. 4(a). Note that the crust layer has been omitted here. Subgrade reaction coefficients and stiffness reduction factors of the liquefied layer and base layer are assumed as \( k_t = 13 \text{ MN/m}^2 \), \( \beta_t = 1/100 \) and \( k_t = 27 \text{ MN/m}^2 \), \( \beta_t = 1 \), respectively. Thus, it is assumed that the liquefied soil has a reduced stiffness by a factor of 1/100 while the initial stiffness is adopted for the base soil. The trilinear moment-curvature relationship of the pile is shown in Fig. 5 where point \( C \) denotes concrete cracking, point \( Y \) indicates yielding of steel reinforcement and the ultimate level or concrete crushing is denoted by point \( U \). The pile, which is fixed at the base and restrained from rotation at the top, is assumed to be subjected to a lateral
displacement of the liquefied soil of $U_{G2} = 50$ cm at the ground surface, as illustrated in Fig. 4(a).

The calculation consists of an iterative procedure in which a series of equivalent linear analyses are made using the closed-form solution, as follows. First, an elastic analysis is carried out using the initial bending stiffness of the pile, $EI_1 = EI_a$ (Fig. 5). The pile response computed in the elastic analysis is shown with dashed lines in Figs. 6(a) and 6(b) where pile displacements and bending moments throughout the depth are shown respectively. Focusing on the maximum bending response, it is seen in Fig. 6(b) that the peak bending moment, $|M|_{max,1}$, is obtained just below the interface of the two layers. Using this maximum bending moment and the cracking moment, $M_C$, an average bending moment $M_i$ can be evaluated as

$$M_i = \frac{1}{2} (M_C + |M|_{max,1})$$

The maximum bending moment and average bending moment obtained as above are indicated by an open square mark and solid circle respectively in the $M-\phi$ diagram shown in Fig. 7.

Next, the curvature corresponding to the average bending moment is determined as

$$\phi_i = \frac{M_i}{EI_i}$$

and using this curvature and the trilinear $M-\phi$ relationship of the pile, an equivalent (secant) bending stiffness is identified

$$EI_2 = \frac{M(\phi_i)}{\phi_i}$$

as illustrated in Fig. 7. This secant stiffness is then used as the new bending stiffness of the pile and the linear calculation (second iteration) is performed again using the closed-form solution.

The calculation as above is repeated stepwise until convergence is achieved or the difference between the values of the curvature in two successive iterations becomes less than $1\%$, i.e., $|\phi_{i+1} - \phi_i| < |\phi_i|/100$.

The steps to be followed in the $i$-th iteration can be summarized as below:

**Step 1** Compute linear pile response using the analytical solution and identify the peak bending moment $|M|_{max,i}$ which is the largest absolute moment throughout the pile length.
Step 2 Calculate an average value of the bending moment between $|M|_{\text{max}, i}$ and $M_C$, and determine its respective curvature $\phi_i$.

Step 3 Identify new secant stiffness, $EI_{i+1}$, using $\phi_i$ and $M - \phi$ relationship of the pile.

As illustrated in Fig. 7, the procedure consists of a sequence of linear analyses in which different secant stiffness is used for the pile and eventually it converges toward a solution in which the average bending moment $M$ and its corresponding curvature computed in the $n$-th iteration are compatible with the actual $M - \phi$ relationship of the pile. The equivalent bending stiffness used in the last iteration is denoted as $EI_{\text{eq}}$, and the pile response computed in this last iteration is taken as the final output of the equivalent linear calculation.

Note that the cracking moment $M_C$ is continuously used as a reference bending moment when evaluating the equivalent bending stiffness of the pile throughout the iterations. Since $M_C$ denotes the threshold bending level at which inelastic deformation is initiated, the equivalent bending stiffness evaluated based on $M_C$ and $|M|_{\text{max}}$ as above in effect represents an estimate for the average stiffness of the pile over the portion where inelastic behavior occurs. The equivalent stiffness defined above was found to provide a reasonable degree of approximation for the deformational behavior of the pile as well as good estimate for its maximum response.

In order to examine the accuracy of the equivalent linear calculation, a more rigorous nonlinear analysis was conducted using the multiple beam-spring model (BS model) depicted in Fig. 4(b). In this finite-element model, the pile is represented by a series of beam elements while the surrounding soil is modeled by means of horizontal springs. Each beam element is formulated as a nonlinear member with a trilinear $M - \phi$ relationship as shown in Fig. 5. The response of the pile computed using the BS analysis is shown with circular symbols in Figs. 6(a) and 6(b). It may be seen in these figures that the pile response computed by way of the equivalent linear procedure is nearly the same as the nonlinear response calculated using the more accurate BS model. In particular, the pile displacement and the maximum bending moment both in terms of its magnitude and location are seen being in good agreement. On the other hand, it is noted that the bending moment near the pile head shows a value which is lower than that obtained by the BS analysis.

Yielding in the Base Layer

The pile response induced by lateral spreading invariably involves some relative displacement between the pile and the base soil. This relative displacement would be the largest at the top of the base layer and gradually decrease downwards as the pile deflection diminishes with depth. Thus, yielding of the base soil may occur over a certain depth below its interface with the overlying liquefied layer. The softening of the uppermost portion of the base layer has important bearing since it changes the feature of the stiffness contrast between the liquefied layer and base layer, and this in turn affects more or less the bending moment of the pile at the interface.

The soil-pile model shown in Fig. 8(a) will be used to illustrate the effects of soil yielding in the base layer and the secant stiffness approach adopted for its modeling. The soil-pile model is very similar to the one analyzed in the preceding section (Fig. 4(a)), except that now the focus in the modeling has been shifted from the pile nonlinearity to the nonlinear behavior of the base soil. Thus, it is assumed that the pile behavior is linear elastic whereas the non-liquefied base layer is characterized by a bilinear $p - \delta$ relationship as defined by the coefficient of subgrade reaction of $k_1 = 6.8$ MN/m$^3$ and ultimate lateral pressure of $p_{\text{ ultimate}} = 60$ kPa, as shown in Fig. 9. The threshold relative displacement at which the behavior changes from elastic to plastic is evaluated as $\delta_{1-2} = p_{\text{ ultimate}}/k_1 = 60/6800 = 0.0088$ m = 0.088 cm.

In the adopted analytical model shown in Fig. 3, the stiffness of the base soil is defined by the product $\beta_1 k_1$, and therefore when using $\beta_1 < 1$ in effect a secant (degraded) stiffness is specified for the base layer. This secant stiffness can be used to approximately model the nonlinear $p - \delta$ relationship of the base soil, as suggested by Matlock et al. (1978), Dobry and O'Rourke (1983) and Byrne et al. (1984). The calculation procedure described below provides a method for evaluating the secant stiffness of the base layer or the appropriate value of $\beta_1$.

In principle, the equivalent linear model for the base soil is analogous to that proposed for modeling the pile behavior. The analysis procedure consists of a sequence of linear calculations using the closed-form solution and starts with an elastic analysis in which the initial stiffness is used for the base soil with $\beta_1 = 1$. Results of such elastic analysis for the pile in question are shown with dashed
lines in Fig. 10. Focusing on the response in the top part of the base layer, it is seen in Fig. 11(a) that the pile displacement computed in the elastic analysis exceeds the yield displacement of \( \delta_{y} = 0.88 \text{ cm} \), from 10 m to 11.56 m depth, thus indicating that plastic flow occurs in the base soil over this depth. Note that, here, the pile displacement is equal to the relative displacement between the pile and the soil since no movement of the base soil has been assumed in the analysis. Next, the displacement \( |\delta_{3,1}| \) at the mid height of the yielded portion (\( H_{3,1}/2 \)) is adopted as a reference, as indicated in Fig. 11(a). Using this reference displacement and the bilinear \( p-\delta \) relationship of the base soil, new stiffness ratio \( \beta_{3,j} \) is estimated as

\[
\beta_{3,j} = \frac{k_{3,j}}{k_{1}|\delta_{3,j}|} = \frac{P_{3{-}\text{max}}}{k_{1}|\delta_{3,j}|} (j = 1, k; \text{iteration number}) \quad (10)
\]

and new secant stiffness (\( \beta_{3,k} \)) is thus identified for the base layer, as illustrated in Fig. 9. The new value of \( \beta_{3} \) is then used in the subsequent linear calculation (second iteration) and the above procedure is repeated stepwise until eventually convergence is achieved in which the difference between reference displacements in two successive iterations becomes less than 1%, i.e., \( |\delta_{3,j+1} - \delta_{3,j}| < |\delta_{3,j}|/100 \). The identification of new secant stiffness in successive iterations is illustrated in the \( p-\delta \) diagram in Fig. 9 whereas Figs. 11(a)-11(d) display input \( \beta_{3} \) values, identification of reference displacements and computed \( \beta_{3} \) values in four consecutive iterations. The value of \( \beta_{3} \) obtained in the last iteration of the equivalent linear calculation is denoted as \( \beta_{3{-}\text{eq}} \).

In order to examine the accuracy of the pile response computed by the equivalent linear calculation as above, a more rigorous nonlinear analysis for the pile in question was conducted using the multiple beam-spring model (BS model) depicted in Fig. 8(b). In this finite-element model, the pile is represented by a series of beam elements, each formulated as a linear elastic member, while the base soil is represented by a series of bilinear springs. It may be seen in Figs. 10(a) and 10(b) that the response of the pile computed by way of the equivalent linear procedure is approximately the same as that calculated using the more rigorous BS analysis.

A key feature of the above model is that the behavior
of the top part of the base layer where soil yielding occurs is used to characterize the base layer. The evaluated secant stiffness of the base layer $\beta_{3,eq}k_3$ in effect represents an estimate for the average stiffness of the soil over the depth of the base layer where yielding of soil occurs. In principle, this approach is equivalent to the concept of utilizing the average soil stiffness over the active length of laterally loaded piles suggested by Randolph (1981).

**Lateral Loads from the Surface Layer**

The last simplified model to be introduced is that for the nonlinear behavior of the surface layer. During spreading of liquefied soils, large lateral loads on the pile may arise from the non-liquefied surface layer. To examine these loads, the soil-pile interaction in the surface layer is assumed to be characterized by a bilinear pressure-displacement relationship as defined by a subgrade reaction coefficient $k_1$ and ultimate pressure $p_{1-max}$. The ultimate pressure is assumed to be given by

$$p_{1-max}(z_i) = \alpha_0K_0\sigma(z_i)$$

where $\sigma$ is the effective overburden stress, $K_0$ is the Rankine passive earth-pressure coefficient and $\alpha_0$ is a scaling factor to account for the difference in the ultimate pressure between a single pile or pile in a group as opposed to that of a wall. For a subgrade reaction coefficient assumed to be constant throughout the depth of the layer ($k_1(z_i) =$ const.), the threshold relative displacement at which yielding is initiated in the crust soil would be proportional to the depth

$$\delta_{1-y}(z_i) = \frac{p_{1-max}(z_i)}{k_1} = \alpha_0 K_0 \gamma_i z_i, \quad \left(\frac{\alpha_0 K_0 \gamma_i}{k_1}\right) = \text{const.}$$

(12)

As illustrated in Fig. 12(a), the yield displacement defined as above marks off two zones of different soil behavior on each side of the deformed ground profile: a zone of elastic behavior, for $|\delta(z_i)| < \delta_{1-y}(z_i)$, and a zone of plastic flow in the soil, for $|\delta(z_i)| \geq \delta_{1-y}(z_i)$. If the pile in its deformed state falls within the shaded area in Fig. 12(a), then the soil behavior is elastic and the pressure applied to the pile is proportional to the initial soil stiffness

$$p_i(z_i) = k_1\delta(z_i) = k_1[U_A(z_i) - U_B(z_i)]$$

for $|\delta(z_i)| < \delta_{1-y}(z_i)$

(13)

This case is illustrated in Fig. 12(b) where lateral pressure, deformed configuration of the soil-pile system and $p - \delta$ relationships at two arbitrarily selected depths, denoted as A and B, are schematically shown. On the other hand, Fig. 12(c) depicts the case when plastic flow is induced throughout the crust layer and the pile is subjected to the ultimate lateral pressure of $p_{1-max}(z_i)$. Finally, Fig. 12(d) displays respective plots for an intermediate case in which the upper part of the crust layer undergoes plastic flow.
while the bottom part exhibits elastic behavior.

It is obvious that the magnitude and direction of lateral pressure from the crust layer on the pile depend on the relative displacement between the crust soil and the pile. Hence, this interaction load can not be known in advance, i.e., before the pile response is evaluated. For this reason, the values of the earth pressure parameters \( P_0 \) and \( P_0 \), and lateral force, \( F \), in the adopted analytical model, as introduced in Fig. 3, would have to be determined through an iterative procedure as described below.

The employed load-adjustment procedure consists of a sequence of calculations using the analytical solution in which the applied pressure \((P_a, P_b)\) is incrementally modified and systematically adjusted until eventually the difference between the applied (guessed) resultant pressure from the surface layer \((P_i)\) and the pressure resulting from the computed relative displacements \((\delta_i)\) becomes less than 5%, i.e., \(|P_i - P_b| < 0.05|P_i|\). In other words, the adopted convergence criterion is to achieve coincidence between the applied resultant pressure

$$P_i = \left( P_a + \frac{P_b H_i}{2} \right) H_i \quad (14)$$

and the displacement-compatible pressure resulting from the calculation

$$P_i = k_i D_0 \int_{0}^{\delta_i} \bar{b}_i(z) dz,$$

$$\bar{b}_i = \begin{cases} \frac{\delta_i}{\delta_{i-y}} & \text{for } |\delta_i| < \delta_{i-y} \\ \frac{\delta_{i-y}}{|\delta_i|} & \text{for } |\delta_i| \geq \delta_{i-y} \end{cases} \quad (15)$$

within a prescribed tolerance of ±5%. In this way, lateral loads \((P_a, P_b, F)\) can be evaluated that are consistent with the relative displacement between the pile and crust soil resulting from the analysis. It is to be mentioned here that in the case when the lateral pressure changes its sign (direction) throughout the depth of the layer, the above criterion is to be implemented separately for the positive and negative pressure components.

It is generally the case that the pile top is anchored rigidly into the bottom of a footing which is embedded into the surface layer. In the course of the spreading, the footing will be therefore subjected to a lateral load from the surface layer, as discussed earlier and represented by the force \( F \) in the model. This lateral force is evaluated according to the same rules as described above for the loads on the pile from the surface layer and is therefore proportional to the relative displacement between the surface layer and footing. The maximum lateral force that can be transferred to the pile through the footing is roughly approximated as \( F = P_b B/n \) where \( P_b \) is the resultant Rankine passive pressure per unit width of the footing, \( B \) is the width of the footing (slab) and \( n \) is the total number of piles in the group. In the above, it is apparently assumed that the total lateral force acting on the footing will be uniformly distributed to the piles under the footing, each of the individual piles thus receiving an amount of \( 1/n \) of the total force.

**Calculation Flow**

The simplified models discussed above separately for the nonlinear behavior of the pile, base layer and surface layer will now be combined to develop an integral computational procedure. As outlined in the calculation flow shown in Fig. 13, the overall procedure consists of two computational loops. The internal loop is a computational scheme in which the equivalent linear models of the pile and base soil are combined to evaluate the pile response for a given ground displacement and guessed lateral loads from the surface layer. In the external
computational loop, the lateral loads from the surface layer are systematically modified so that the applied loads become eventually compatible with the computed pile response.

The calculation within the inner loop is executed as follows. For an assumed lateral ground displacement, an elastic linear calculation is first carried out while ignoring the effects of the surface layer on the pile, that is, by assuming initial values of \( P_0 = 0 \), \( P_a = 0 \) and \( F = 0 \). Using the equivalent linear models of the pile and base soil, secant bending stiffness \( EI_{01} \) and base-layer stiffness \( \beta_1 \alpha k_3 \) respectively are evaluated through successive linear calculations as described in the preceding sections. The procedure involving simultaneous adjustments for the stiffness of the pile and the base layer is stable and typically converges within several iterations.

The computation depicted by the outer loop in Fig. 13 is performed as follows. Using the converged pile response obtained in the previous iteration as above, a displacement-compatible resultant pressure of the surface layer \( \bar{P}_r \) is calculated using Eq. (15) and then compared to the applied resultant pressure \( P_{a1} \) calculated by Eq. (14). The obtained difference between the computed and applied pressures \( (\bar{P}_r - P_{a1}) \) is then used to gradually modify the values of \( P_0 \), \( P_a \) and \( F \) by adding small increments of \( \Delta P_0 \), \( \Delta P_a \) and \( \Delta F \) respectively. Using the renewed values of \( P_0 \), \( P_a \) and \( F \), a new series of linear analyses is executed including additional iterations for \( EI \) and \( \beta \alpha k_3 \), if needed, and further adjustments of the lateral load from the surface layer. The above procedure is repeated until essentially the difference between the guessed and computed lateral loads becomes less than 5%. As a final output of the computation, a pile response is evaluated that incorporates the effects of nonlinear behavior of all three layers as well as the pile itself.
Accuracy and Limitations

While the above computational procedure permits to estimate the nonlinear pile response, the result is in effect obtained from an equivalent linear calculation in which secant stiffness is used for the pile, liquefied soil and base soil. Needless to say, when using a single flexural property along the entire length of the pile, it is not possible to provide accurate representation of bending stresses and strains at all depths. Recognizing this limitation, the primary objective in exploring the use of the equivalent-linear approach in the modeling was to achieve a reasonably good level of accuracy in the estimate of the peak bending moment along the pile and also for the pile displacements especially at the pile head. Thus, the determination of the peak bending moment and pile-head displacement by means of the model is considered of key importance because these parameters are critical in the assessment of the overall pile response including damage to piles and its effects on the superstructure.

Comparisons of pile responses shown in Figs. 6 and 10 are illustrative of the performance and overall accuracy of the simplified method of analysis. In order to examine more closely the accuracy of the equivalent linear calculation for various pile characteristics, parametric analyses were conducted using the soil-pile model shown in Fig. 14(a). PHC piles with diameters of 40, 60 and 80 cm, prestressed to 4 MPa (A-type) and 10 MPa (C-type), with axial loads of 0, 300 and 500 kN were taken up in these analyses. These piles have markedly different bending stiffness (EI = 40–600 MN·m²) and yield moments (Mₚ = 0.07–1.24 MN·m) as depicted in Fig. 14(b) where trilinear $M-\phi$ relationships of the piles are shown. As indicated in Fig. 14(a), the thickness of the crust layer was assumed to be $H₁ = 0$, 1 or 2 m. For each numerical case, two series of analyses were made: one using the equivalent linear method proposed in this study and the other using a more rigorous nonlinear analysis with a multiple beam-spring model in which bilinear springs were used for the surface
layer and base layer, and trilinear $M - \phi$ relationship was used for each beam element representing the pile. Each of the above series consisted of several sets of analyses in which different magnitude of ground displacement was applied thus inducing different level of bending deformation and pile responses ranging from the elastic behavior to failure.

Results of the analyses are summarized in Figs. 15(a)–15(c) and 15(d)–15(f) where computed maximum bending moments and pile-head displacements are shown respectively. Here, the maximum moment is the peak absolute bending moment that was computed along the entire length of the pile. The plots in both axes are given in terms of values normalized to the yield moment $M_y$ and pile displacement at yielding $U_{p-y}$ respectively. Thus, the value of unity indicates the yield level in these figures. It is seen in Figs. 15(a)–15(c) that the maximum bending moment is well predicted up to the yield level by way of the equivalent linear method with the deviation being typically within a few percents from the bending moment computed by the more exact nonlinear analysis. Very good accord is also seen in Figs. 15(d)–15(f) for the pile-head displacements. Beyond the yield level, the maximum bending moment is consistently overpredicted by the simplified analysis, with a gradually increasing deviation as the response progresses from the yield level towards the ultimate level.

MODEL PARAMETERS

To summarize the whole procedure of computation, the parameters required as an input for the simplified analysis are depicted in Fig. 16 and summarized in Table 1. All parameters are of clear physical definition and can be evaluated using expressions at the convenience of the user. Some comments on the parameter determination and useful expressions are given below.

$M - \phi$ relation . . . . A user-defined analytical function can be used to specify the moment-curvature relationship of the pile. The conventional trilinear $M - \phi$ relationship of reinforced concrete piles is used as an example in Fig. 16 and Table 1 where the $(M, \phi)$-values at cracking $(M_C, \phi_C)$, yielding $(M_y, \phi_y)$ and ultimate level $(M_u, \phi_u)$ serve as input parameters.

$D_o, H_s, H_l, H_o, B, n, \ldots$ . . . The geometry of the soil-pile system is defined by the pile diameter $(D_o)$ and thicknesses of the soil layers $(H_i, i = 1, 3)$ corresponding to the embedded pile. As shown in Fig. 16, the surface layer is divided into two sub-layers, one from the ground surface to the pile top $(H_1)$, and the other from the pile top to the interface with the liquefied layer $(H_2)$. An equivalent width of the footing $(B)$ and total number of piles $(n)$ are required to be specified in the case of piles in a group.

$k_1, k_2, k_3, \ldots$ . . . The subgrade reaction coefficients can be evaluated using empirical expressions based on the SPT blow count such as

$$k_o = 56N D_o^{-3/4} \text{ (MN/m}^3)$$

which is stipulated in the Japanese Design Code for Building Foundations (AIJ, 2001). Here, $N$ is the representative SPT blow count of the layer and $D_o$ is the pile diameter in cm. It is to be noted that the $N$-value for the base soil to be used in Eq. (16) is not the average blow count of the base layer, but rather it is a representative blow count for the top part of the base layer. Since $k_o$ specifies the stiffness for a single pile at a relative displacement of about 1 cm, the subgrade reaction coefficients of the surface layer and base layer may need to be modified to allow for pile-group effects and $k_i$ may need to be additionally reduced if the yield displacement in the surface layer is greater than 1 cm. Thus, the following generalized expressions can be used for estimating the subgrade reaction coefficients of the three layers

$$k_i = \xi \alpha k_o, i = \begin{cases} 1, & \xi \leq 1, \quad \alpha \leq 1 \\ 2, & \xi = 1, \quad \alpha = 1 \\ 3, & \xi = 1, \quad \alpha \leq 1 \end{cases}$$

where $\alpha_i$ is pile-group stiffness factor and $\xi$ denotes the ratio of the yield stiffness and reference stiffness $k_o$. $p_{1-max}, p_{3-max}, \ldots$ The ultimate soil pressure can be estimated as
\[ p_{1-\text{max}} = \alpha_p p_0(z_i) \]

where \( p_0(z_i) \) is the Rankine passive pressure at the bottom of the surface layer \( (z_i = H_i) \) or at the top of the base layer \( (z_i = 0) \) while \( \alpha_p \) is a scaling factor to account for the difference in the lateral pressure between a single pile and pile in a group. Experimental data from full-scale tests on piles (Cubrinovski et al., 2004) and scaled-down model tests on piles (Poulos et al., 1995; Chen et al., 1997) indicate that \( \alpha_p \) can take a value as high as 4 to 5 for a single pile. The parameter \( \alpha_p \) can also be used to consider possible reduction in the mobilized pressure in the surface layer due to sand boils, fissuring of the ground or impediment of ground deformation by adjacent foundations. In the case of base-layer of cohesive soils, the ultimate lateral pressure for a single pile can be given as \( p_{1-\text{max}} = 9s_u \) in which \( s_u \) denotes the undrained shear strength.

The factor \( \beta_2 \), which specifies the stiffness degradation in the liquefied soil is affected by a number of factors including density of sand, excess pore pressures, magnitude and rate of ground displacements and drainage conditions. Thus, there is a great deal of uncertainty and hence difficulty in the selection of its most appropriate value. Typically, \( \beta_2 \) may take a value in the range between 1/50 and 1/10 for cyclic liquefaction (e.g., Tokimatsu and Asaka, 1998) and a value between 1/1000 and 1/50 in the case of lateral spreading (e.g., Ishihara and Cubrinovski, 1998; Yasuda and Berrell, 2000; Cubrinovski et al., 2004). Nevertheless, the value of \( \beta_2 \) has to be assumed duly.

The lateral displacement of the spreading soil \( U_{p2} \) has also to be assumed properly. Here, empirical correlations for estimating lateral ground displacements due to spreading of liquefied soils may be used (e.g., Ishihara et al., 1997; Tokimatsu and Asaka, 1998; Hamada et al., 2001; Youd et al., 2002).

**Key Parameters and Uncertainties Involved**

Key parameters influencing the pile response and hence requiring special attention are the magnitude of ground displacement \( U_{p2} \), ultimate pressure from the crust layer \( p_{1-\text{max}} \) and stiffness degradation parameter \( \beta_2 \) for the liquefied layer. The lateral ground movement is in effect the sole external agency to be specified in the pile analysis, and clearly the level of the pile response will directly depend on the magnitude of the applied ground displacement. The lateral load from the surface layer, on the other hand, may often be the critical load for the integrity of the pile because of its large magnitude and unfavorable condition of a top-heavy load acting above a laterally unsupported portion of the pile in the liquefied soil. Finally, the stiffness reduction factor \( \beta_2 \) affects the relative flexibility of the pile as represented by the non-dimensional factor \( \lambda \) in Eq. (5), and in the extreme case this influence may result either in a relatively flexible pile that moves together with the surrounding soil or a relatively stiff pile that does not follow the ground movement.

The intrinsic uncertainties associated with the spreading of liquefied soils are directly reflected on the values of \( U_{p2} \), \( p_{1-\text{max}} \) and \( \beta_2 \). Hence, these parameters can not be uniquely determined, but rather a range of values for these parameters needs to be considered in the analysis of piles. The simplified method of analysis presented herein is envisioned as a rational tool that will be of practical use in addressing the uncertainties and needs for parametric studies as above. A comprehensive method for assessment of piles utilizing the proposed simplified analysis within a demand-capacity framework is presented in Cubrinovski and Ishihara (2004).

**CONCLUSIONS**

A simple method for analysis of piles undergoing lateral spreading in liquefied soils has been presented. The key features of the theoretical procedure can be summarized as follows:

1. It is postulated that kinematic loads resulting from large horizontal ground displacements control the pile response to lateral spreading. On this basis, a pseudo-static approach was adopted for the analysis. A three-layer soil model is used to represent the soil stratification typical for piles embedded in liquefiable soils and to capture the kinematic mechanism associated with lateral spreading. Particular attention is given to the loads arising from a non-liquefied crust layer at the ground surface and to the kinematic effects at the interface between the liquefied layer and underlying non-liquefied layer.

2. Assuming linear properties for the soil and pile, a closed-form solution was derived based on the classical differential equation for beam on elastic foundation. Simplified models for nonlinear behavior of the pile and soil were developed using the equivalent linear approach and these models were thereafter combined with the analytical solution via an iterative scheme to establish an integral computational procedure. The analysis method permits to estimate the inelastic response of piles, yet it is a simple closed-form solution that requires a small number of conventional engineering parameters as input.

3. Important modeling features that need to be emphasized are: (a) a single bending stiffness corresponding to the average bending moment between the cracking moment and the computed maximum moment is used for the pile along its entire length; (b) a resultant load which is compatible with the relative displacements between the pile and soil specifies the lateral pressure from the surface layer on the pile; (c) properties and deformation of the top part of the base layer are used to characterize the behavior of the non-liquefied base layer; (d) the method aims at achieving accurate predictions for the maximum bending moment of the pile and displacement at the pilehead; (e) results of the simplified analysis are shown to be in good agreement with those of a more sophisticated nonlinear analysis for bending responses up to the yield level.

4. Key parameters influencing the pile response are
identified to be the magnitude of ground displacement, ultimate lateral pressure from the surface layer and stiffness degradation of the liquefied soil. It is of importance to note that these parameters do involve inherent uncertainties about their values, making it necessary to evaluate the pile response by considering a relatively wide variation in these parameters. A performance-based method for assessment of pile response to lateral spreading by taking into account various magnitudes of ground displacement and stiffness properties of the spreading soils is presented in Cubrinovski and Ishihara (2004).

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REFERENCES


APPENDIX A: EXPRESSIONS FOR PILLES FIXED AT THE BASE AND RESTRAINED FROM ROTATION AT THE TOP

The continuity conditions for pile deflection, slope, moment and shear force at the interface between the surface layer and liquefied layer are given by the following equations

\[ y(H_e) = y_e(0) \]  \hspace{1cm} (A-1a)

\[ y'(H_e) = y'_e(0) \]  \hspace{1cm} (A-1b)

\[ y''(H_e) = y''_e(0) \]  \hspace{1cm} (A-1c)

\[ y'''(H_e) = y'''_e(0) \]  \hspace{1cm} (A-1d)

Similarly, the continuity equations at the interface of the
liquefied layer and base layer are
\begin{align*}
y_s(H_z) &= y_s(0) \\
y'_s(H_z) &= y'_s(0) \\
y''_s(H_z) &= y''_s(0) \\
y'''_s(H_z) &= y'''_s(0)
\end{align*}
(A-2a)
(A-2b)
(A-2c)
(A-2d)

The boundary conditions for piles restrained from rotation at the top and fixed at the base are given by
\begin{align*}
y'_t(0) &= 0 \\
EIy''_t(0) &= F
\end{align*}
(A-3a)
(A-3b)

and
\begin{align*}
y_b(H_z) &= 0 \\
y'_b(H_z) &= 0
\end{align*}
(A-4a)
(A-4b)

respectively, where \( F \) is the lateral force at the pile top.

Using the eight continuity conditions (Eqs. (A-1) and (A-2)) and four boundary conditions (Eqs. (A-3) and (A-4)), twelve simultaneous algebraic equations are obtained for determining the twelve constants \( A_1, B_1, C_1, D_1, A_2, B_2, C_2, D_2, A_3, B_3, C_3 \) and \( D_3 \), as follows. Differentiating Eqs. (3a) and (3b), and then substituting into the continuity equations for the interface of the surface layer and liquefied layer (Eqs. (A-1a) through (A-1d)), the following expressions are obtained:
\begin{align*}
\frac{P_h}{120EI} H^2 + \frac{P_q}{24EI} H^4 + A_1 H^4 + B_1 H^4 + C_1 H^4 + D_1 \\
&= B_2 + D_2 + G_2
\end{align*}
(A-5)

\begin{align*}
\frac{P_h}{24EI} H^4 + \frac{P_q}{6EI} H^4 + 3 A_1 H^4 + 2 B_1 H^4 + C_1 \\
&= \frac{\lambda^2}{H_z} (A_2 + B_2 + C_2 - D_2)
\end{align*}
(A-6)

\begin{align*}
\frac{P_h}{6EI} H^4 + \frac{P_q}{2EI} H^4 + 6 A_1 H^4 + 2 B_1 \\
&= 2 \left( \frac{\lambda^2}{H_z} \right) \left( A_1 - C_2 - G_2 \pi^2 \frac{\lambda^2}{8\pi^2} \right)
\end{align*}
(A-7)

\begin{align*}
\frac{P_h}{2EI} H^4 + \frac{P_q}{EI} H^4 + 6 A_1 = 2 \left( \frac{\lambda^2}{H_z} \right)^3 (A_2 - B_2 - C_2 + D_2)
\end{align*}
(A-8)

In a similar manner, the continuity equations for the interface of the liquefied layer and base layer (Eqs. (A-2a) through (A-2d)) provide the following set of expressions:
\begin{align*}
e^{i\lambda} (A_2 \sin \lambda_z + B_2 \cos \lambda_z) + e^{-i\lambda} (C_2 \sin \lambda_z + D_2 \cos \lambda_z) \\
&= B_1 + D_1
\end{align*}
(A-9)