3D EFFECTS ON EARTH PRESSURE AND DISPLACEMENTS DURING TUNNEL EXCAVATIONS

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ABSTRACT

Three-dimensional model tests of tunnel excavation and the corresponding numerical analyses were carried out to investigate the influence of tunnel excavation on surface settlement and earth pressure surrounding a tunnel. Numerical analyses were performed with the finite element method using elastoplastic subloading $t_b$ model. Two types of apparatuses were used in the model tests, namely-trap door apparatus and pulling out tunnel apparatus. The minimum excavation length along the excavation direction is 8 cm in the trap door apparatus. However, the process of real tunnel excavation is more continuous. For simulating real tunnel excavation in 3D, a new apparatus was developed, which is called as the pulling out tunnel apparatus. This apparatus can simulate tunnel excavation in sequential way. Both experiments and analyses were conducted with various ground depths for simulating the influence of soil cover on tunnel excavations. Surface settlements are measured at the transverse cross-section of the ground. Earth pressures at the top of the tunnel in the trap door apparatus are measured. However, in the tests performed with the pulling out tunnel apparatus, earth pressures are measured adjacent to the tunnel cross section. In this paper, the effects of 3D excavation on surface settlements and earth pressures are discussed. It is revealed that arching is formed in both transverse and longitudinal directions of tunnel excavation. Numerical results show very good agreement with the results of the model tests. In the three-dimensional analyses performed in a sequential way, earth pressure is almost zero at the excavation front, irrespective of the soil cover. In 2D analyses and 3D analyses using the trap door apparatus, the earth pressure at the excavation front does not vanish, as observed in 3D sequential analyses.

Key words: (3D problem), constitutive equation of soil, earth pressure, excavation, finite element method, model test, settlement, (trap door), tunnel (IGC: E2/E5/E14/H5)

INTRODUCTION

In the design of tunnel linings, the design loads are usually computed on the basis that they are independent or nearly independent of the type of construction and even independent of the properties of the soil. However, the design load should reflect all the influences to which it may be subjected. Hence, for designing the tunnel lining one should be concerned with the earth pressures and deformations that develop during tunnel excavation. Usually, earth pressure of tunnel ground is simulated using two-dimensional analysis, because three-dimensional analysis is time consuming and sometimes troublesome. However, to consider the influences of tunnel excavation, both in longitudinal and transverse directions in the real ground conditions, three-dimensional analyses should be carried out. Thus, three-dimensional analyses of tunnel excavations are conducted in this research. A brief description of model tests and the corresponding finite element analyses are presented in this paper.

Generally, results of finite element analyses are very much dependent on the constitutive model. If a model is unable to evaluate stress distribution and density in the ground, proper prediction of deformation is not possible. Subloading $t_b$ model (Nakai and Hinokio, 2004) is one of the constitutive models, which can properly describe the influences of intermediate principal stress, dependence of stress paths on the direction of plastic flow, density and/or confining pressure on the deformation and strength of soils, among other important features. Some important features of this model are reproduced in the APPENDIX of this paper.

Nakai et al. (1997) carried out 3D trap door model tests and the corresponding 3D numerical analyses to investigate the 3D effect and dilatancy effect on the ground movements in tunnel excavation. The trap door apparatus can only approximately simulate the three-dimensional excavation process, because displacements are imposed in a stepwise manner. Later, Nakai et al. (1999) developed another apparatus to simulate the excavation

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sequence in a cross section as well as in the excavation direction and carried out some basic model tests on sandy ground with different construction sequences. Though the apparatus could simulate tunnel excavation in a sequential way, friction developed at the interface of sliding bars and soils. In this research, a new device is used to simulate tunnel excavation, which overcomes the drawback of the previous apparatus.

**LAYOUT OF MODEL TESTS**

*Apparatus of Model Tests*

A continuous trap door apparatus, as shown in Fig. 1(a), is used to investigate the basic mechanism of the ground behavior and earth pressure for tunnel excavation. Details of this apparatus were shown in a previous paper (Nakai et al., 1997). The apparatus consists of 10 brass blocks (blocks A to J) of 8 cm in width each, and set along the centerline of an iron table. These blocks can be moved upwards and downwards individually or simultaneously. There was no measuring device for measuring earth pressures in the previous trap door apparatus. However, in the present study, for measuring earth pressures a block of load cells is placed at the position of block F. The block of load cell is divided into four small parts to measure the earth pressure at four points in each block. Four load cells are set under the four small parts of the load cell block. It can be rotated in the horizontal direction, making possible measurements in both transverse and longitudinal directions, as shown in Fig. 1(b).

Figure 2(a) shows the schematic diagram of the pulling out tunnel apparatus. Figure 2(b) indicates the process of excavations of the pulling out tunnel apparatus. Photo 1 is the picture of this apparatus and Photo 2 is the picture of one model test with this apparatus. A wooden frame
with 80 cm in width, 80 cm in length in the direction of excavation and 8 cm of height is set in the center on an iron table. There are four bars at the top and three bars at each side of the blocks. In the present study, only the top four sliding bars are pulled out to simulate tunnel excavation. Two acrylic plates (Photo 1) are placed at both sides of the apparatus, in order to support the ground in the same elevation as the top of the tunnel block. The acrylic plates covered the side bars of the tunnel block; those are not pulled out in the model tests. The bars are 19.5 mm wide and 4 mm thick. To avoid friction between sliding the bars and soils, a silicon stripe is placed just above the bars and a flexible magnetic tape is glued at the bottom of the silicon stripe. When the bars are pulled out, the silicon stripe fills the position of the bars, and the ground in the interface moves downward to fill the gap (4 mm). In this way, displacements equal to the thickness of the sliding bars are imposed. Since the sliding bars can be pulled out independently, excavation sequence can be changed and model tests can be carried out with any desired excavation sequence. In the pulling out apparatus, there is no device for measuring the earth pressure at the top of sliding bars. Therefore, earth pressure is not measured at the position of tunnel excavation in the model tests. However, the block of load cells is placed adjacent to the tunnel for measuring earth pressures adjacent to the tunnel block.

Surface settlement is measured at the transverse cross section of the ground by using a laser type displacement transducer with accuracy of 0.01 mm in the same way as the previous study (Nakai et al., 1997). The position of the laser transducer is measured by using a supersonic wave transducer to get continuous reading of surface settlements. All data are recorded in a personal computer through a data-logger.

Preparation of Model Ground

Model test can be performed in different depth to width ratio by varying the ground depth. In this research, model tests are conducted for four values of soil covers, $D/B$ equals 0.5, 1.0, 2.0 and 3.0, where $D$ is the depth from the ground surface to the top of the tunnel and $B$ is the width of the tunnel. In the model tests alumina balls, having diameters of 2.0 and 3.0 mm and mixed with a ratio of 1:1 in weight, are used as soil mass. The specific gravity of alumina balls is 3.6. The unit weight of the mass of alumina balls is $22.3 \text{ kN/m}^2$ at model stress level. Model ground of each test is carefully prepared in order to obtain a uniform ground.

NUMERICAL ANALYSES

In the 3D finite element analyses carried out here, half of the ground is analyzed considering symmetry of the problem with respect to the direction of excavation. Isoparametric brick element of different size is used in the mesh of three-dimensional finite element analyses. Figure 3(a) shows the mesh for the ground with $D/B = 2.0$. Smooth boundary conditions are assumed in the lateral faces, while rough boundary condition is assumed in the bottom face of the mesh. To simulate the lowering of the blocks in the numerical analyses, vertical displacements are imposed at the nodal points, which correspond to the top of the lowering blocks in the model tests.

The parameters for materials used in the numerical analyses are shown in Table 1 and are basically the same as those for aluminum rod mass described in another paper (Shahin et al., 2004). With these parameters, stress-strain relations under triaxial condition are drawn as shown in Fig. 4. Solid lines of these plots represent numerical results, which correspond to experimental values. On the other hand, dotted lines represent the numerical results for a confining pressure of 1/100 times the confining pressure of the experiments. This figure shows that these parameters characterize reasonably well the properties of 3D model grounds (alumina balls). It is also noted that the mass of alumina balls shows positive dilatancy similar to the properties of medium to dense sand. In the numerical ground, very low confining pressure ($p_c = 9.8 \times 10^{-6} \text{ kPa}$) with void ratio of $e = 0.35$ are initially considered. Then the stresses correspondent to the geostatic (self-weight) condition are assigned to the ground in all numerical analyses, by applying the corresponding body forces (unit weight of $\gamma = 22.3 \text{ kN/m}^3$).
Fig. 4. Stress-strain curves for alumina balls

Table 2. Patterns of excavation

<table>
<thead>
<tr>
<th>Series</th>
<th>Type of excavations</th>
<th>D/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>3D simultaneous excavation</td>
<td>0.5</td>
</tr>
<tr>
<td>II</td>
<td>3D block by block excavation</td>
<td>0.5</td>
</tr>
<tr>
<td>III</td>
<td>3D sequential excavation</td>
<td>0.5</td>
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PATTERNS OF EXCAVATION

Table 2 shows the patterns of the model tests and numerical analyses. The tests in which a downward displacement of \(d = 4\) mm is imposed to the blocks A to J simultaneously in Fig. 1(a) are called Series I (3D simultaneous excavation). This series corresponds to 2D plane strain condition. Here, measurements of earth pressures are performed in block F. The idea is to simulate the excavation of a cross section and to check the three-dimensional effects of tunnel excavation. Figure 1(a) shows the setup of this type of excavation. The numerical analyses for 3D simultaneous excavation are carried out in two-dimensional condition, where vertical nodal displacements are applied at nodes corresponding to block F. The influence of soil covers is investigated employing four values of ground depths in this series, where the ratios of depth \((D)\) to width \((B = 8\) cm) were \(D/B = 0.5, 1.0, 2.0\) and \(3.0\).

Series II (3D block by block excavation) refers to the tests in which the downward displacement of \(d = 4\) mm is imposed block by block from A to J and measurements of earth pressures are performed in block F. The idea is to simulate the advance process along the excavation direction. Here, excavation is done in every 8 cm interval at a time along the excavation direction. The numerical analyses of these series are carried out applying vertical displacement at the bottom nodes (Fig. 3(b)) along one block length (8 cm) similar to the model tests. Surface settlements in the model tests are measured at the interface of block E and F (middle transverse cross section). Figure 3(b) shows the measurement section of surface settlement in the numerical analyses of this series. The earth pressures in the numerical simulations are calculated from the four elements of excavation direction, because the length of one element in the excavation direction is 2 cm.

Series III (3D sequential excavation) simulates the excavation in a sequential way. In the model tests, the downward displacements of \(d = 4\) mm are imposed sequentially by pulling out the sliding bars of 4 mm thickness. Again, the numerical analyses of this series are carried out imposing the downward displacement of \(d = 4\) mm to the corresponding nodal points in Fig. 3(a) sequentially. The process of applying the displacements to the nodal points in the analyses is illustrated at the bottom of Fig. 3(a). Surface settlements of the model tests and numerical analyses of this series are measured at the place, which is shown in Figs. 2 and 3(a), respectively. Earth pressures in the numerical analyses are calculated along the element shown at the bottom in Fig. 3(a).

Although excavation rate does not affect the behavior of the ground of alumina balls, the lowering blocks are descended in at a slower rate in the model tests of series I and series II. The sliding bars of series III are pulled out in slower rate as well. In this research, the maximum applied displacement (amount of excavation) is 4 mm, which corresponds to 40 cm in real ground assuming similarity ratio 1:100 between model test and prototype. Here, the soil cover is normalized by the width of the tunnel to make the results easily understandable and to use the results in real field condition. In the model tests, the surface settlements are measured for \(d = 1\) mm and 4 mm. However, earth pressures are measured at the interval of very small increment of the applied displacement, and the significant results are chosen for the graphs of earth pressure distributions.

RESULTS AND DISCUSSIONS

Series I (3D Simultaneous Excavation)

Figure 5 shows the surface settlements of the model tests for applied displacements of 1 mm and 4 mm in case of \(D/B = 0.5, 1.0, 2.0\) and \(3.0\). The prescribed displacement pattern of the lowering block and its extent are depicted at the bottom of each figure. Surface settlement of the model tests of this series is consistent with the results of 2D model tests. Thus, this pattern of excavation can be simulated as two-dimensional condition in the finite element analysis. Figure 6 represents the computed profiles of surface settlements of this series. The computed results well support the results of the model tests as a whole, though the computed surface settlements in the shallow ground occur blocky because of the small number of elements above the trap door. From this point it can be said that the parameters used in three-dimensional analysis reflect well the characteristics of alumina balls. Figure 7 shows the observed results of 2D sectional excavation in aluminum rod mass (Shahin et al., 2004). It is seen that the results of the mass of alumina balls under the plane strain condition are almost the same as those of aluminum rod mass. This justifies the choice of the parameters of alumina balls that are the same as the parameters of aluminum rods.

Figures 8 and 9 show the distributions of the earth pressures of the model tests and corresponding numerical
analyses. Figure 10 is the observed 2D earth pressure distribution of aluminum rod mass, which is reproduced from the paper by Shahin et al. (2004) to compare the results of this series. The earth pressures for different ground depths are plotted with the same ratio of the actual earth pressure. The left vertical axis represents earth pressures normalized by the initial vertical stress $\sigma_{0v} (= \gamma D)$ on the base level, where $\gamma$ is the unit of ground materials ($\gamma = 22.3$ kN/m$^3$) and $D$ is the depth of soil.
cover. The right vertical axis represents the actual value of earth pressure in Pascal. The dotted vertical line in these figures represents the center of the lowering block. As observed for 2D model tests, very shallow ground \((D/B = 0.5)\) produces little changes in earth pressure distributions, despite some pressure decrease in the edges of lowering blocks, and for \(D/B = 1.0, 2.0\) and \(3.0\) the earth pressure on the lowering block decreases due to arching effect. It is also noticed that the earth pressure on the lowering block decreases suddenly after applying only 0.05 mm displacement in this block. The change of earth pressure is observed up to \(d = 1\) mm, after which the earth pressure becomes almost constant. Earth pressure of the numerical analyses in Fig. 8 quantitatively shows very good agreement with the results of the model tests. These results also reveal that 3D simultaneous excavation can be simulated as two-dimensional plane strain condition in the finite element analysis.

**Series II (3D Block by Block Excavation)**

Figure 11 shows the observed surface settlements of the model tests of Series II. The values represent the surface settlement for applied displacement of \(d = 4\) mm at blocks A, D, E, F and J. Here, the measurement section of surface settlement is along the line between the blocks E and F. Figure 12 is the computed surface settlements of Series II. The computed values are taken from the section shown in Fig. 3(b). When lowering blocks A to D almost no settlement occurs at the measuring section, and surface at the measuring section only starts to settle significantly when block E is lowered. Surface settlement at the measuring section reaches almost its maximum value when block F is lowered. There is a good agreement between the observed and computed profiles of settlements in shape and quantity.

Figure 13 shows the history of the observed earth pressure on block F of Series II, where the left vertical axis represents normalized earth pressures and right vertical axis represents actual value of earth pressure. The horizontal axis in each column indicates the displacement of the corresponding lowering block. The letter in each column denotes the position of the lowering block. Legend represents the positions of earth pressures at the tunnel cross-section along the tunnel axis. Since one small load cell is 2 cm in width, the distance of center of gravity of the right most load cell in the transverse direction is 3 cm from the tunnel axis. It is noticed in these figures that earth pressure at block F starts to increase when block C is lowered, except for \(D/B = 0.5\) where it occurs for lowering block E. For \(D/B\) equal to 0.5 and 1.0,
maximum earth pressure at block F is observed when lowering block E. On the other hand, for \( D/B \) equal to 2.0 and 3.0, earth pressure at block F becomes maximum value when block D is descended. For the soil covers of \( D/B = 1.0, 2.0 \) and 3.0, earth pressure decreases sharply at block F when this block is lowered. However, earth pressure at block F increases up to a certain value when block G and the next block H are lowered. This is due to the formation of arching in the direction of excavation. Figure 14 illustrates the history of the computed earth
pressure on block F of Series II. Since finite element analyses were carried out for one half the model ground, earth pressures of two points of block F are plotted. Numerical analyses of this series slightly underestimate the arching effect in the longitudinal direction of the tunnel (especially increase of the earth pressure on block F when applying downward displacement at block D). Although there is a little difference between observed and computed results before lowering block F, computed results explain well the characteristics of earth pressure of model tests at every step of excavation.

Figures 15 and 16 illustrate the same distributions of the earth pressures of the model tests and corresponding numerical analyses of series II in a different arrangement. The results represent the values of earth pressure at the transverse reading section at the middle of the tunnel, for different positions of the excavation front as indicated in the legend. Measured results span only over the width of the load cell block, while computed results are shown for the whole section. These figures clearly show the reduction of earth pressure at the measuring block for lowering this block, and increase of earth pressure when excavation front goes away from the measuring block. A perfect match between computed and measured results can be observed.

**Series III (3D Sequential Excavations)**

Figure 17 represents the observed surface settlements of sequential excavations for $D/B = 0.5, 1.0, 2.0$ and $3.0$. Figure 18 shows the computed surface settlements
corresponding to the observed ones of this series. Legends show the position of the excavation fronts, where value zero indicates the excavation front is in the position of the observation section. In the legends, negative sign indicates the excavation front is behind the measuring section and positive sign is used when the excavation front has passed the measuring section. Definition of the excavation front is shown in Fig. 2(b). Figures 19(a) and 19(b) show observed and computed surface settlements at the center of middle cross section of tunnel, respectively. Abscissas represent the position of excavation front from the middle cross section of the tunnel. Legends illustrate the depth to width ratio of tunnel. For $D/B = 0.5$, surface settlement occurs when the excavation front is very close to the measuring section, and the maximum surface settlement is noticed when excavation front passes a certain distance beyond the measuring section. For further excavations, there is almost no influence at the measuring section. But, for $D/B = 1.0, 2.0$ and 3.0, surface settlement is observed at the measuring section even when the excavation front is 4 cm before this section. Thus, for deeper tunnel ground, the influence of excavation at the measuring section appears before the excavation front reaches that section. Similar response is obtained in the numerical analyses. Comparing the surface settlements of simultaneous excavation (Figs. 5 and 6), block by block excavation (Figs. 11 and 12) and sequential excavation (Figs. 17 and 18), it is revealed that the magnitudes of surface settlements of Series III are slightly smaller than those for Series I and II in every case, but the shape of surface settlement troughs are similar in these series. In Series III, the shear zone develops continuously in the direction of excavation as well as in the direction of cross section, so that total amount of volume expansion in the ground in Series III is a little larger than other Series—the ground material shows positive dilatancy during shear. This is the reason why the surface settlements in Series III are slightly
smaller than the others. It is described in the previous paper (Nakai et al., 1997) that soil dilatancy influences the surface settlements in tunneling.

Figure 21 shows the history of earth pressure at a point that is 6.5 cm from the center of transverse cross section for the model tests and 6.75 cm for the numerical analyses of series III as shown in Fig. 20. The point is located outside the tunnel block. The results of the numerical analyses were measured keeping similarity to the results of the model tests. Horizontal axis represents the longitudinal distance from the middle cross section of the tunnel. Left vertical axis represents earth pressure normalized by the product of the unit weight and depth of the ground for model tests and by the initial stress for numerical analyses. From these figures, it is revealed that earth pressure at the point increases before the excavation front reaches the same section. Since the region of arching is small for $D/B = 0.5$, significant reduction of earth pressure is not seen at the point. However, for $D/B = 1.0, 2.0$ and $3.0$, earth pressure of the point increases before the excavation front arrives at the measuring section and then decreases suddenly when this section is reached. For these depths of soil ground, arching effect extended laterally up to the point where the load cell was located. Therefore, the region of arching increases with ground depth. Numerical analyses produce the same results as the model tests, both in shape and quantity.

Figure 22 shows the computed normalized earth pressure distributions along the transverse direction of middle section (along shaded element in Fig. 3(a)). Legend indicates the position of excavation fronts after imposing 4 mm displacements at each section. Figure 23 shows the histories of earth pressure at the five small elements on the middle cross-section of the tunnel. Abscissas represent position of excavation fronts. Legend of this figure illustrates the position of each element along the tunnel centerline towards the transverse section. In these figures, it is seen that earth pressure increases when excavation front progresses to the measuring section, and it becomes maximum value when the excavation front is 2 cm before the measurement section for all the values of soil cover. Earth pressure is nearly zero at the tunnel cross-section when the excavation front is at the measuring section, and earth pressure increases gradually with the face goes away from this section. This is because, in 3D condition, arching takes place at the measuring section in both transverse and longitudinal directions, even in the case of very shallow tunnel ($D/B = 0.5$), and
the advance of excavation front from the measuring section breaks the formation of arching in the longitudinal direction. Thus, earth pressure depends on the distance of the face of excavation. Although arching diminishes at the measuring section when the excavation front goes away in a certain distance, earth pressure of that position does not reach to its original (initial) value because of the redistribution of earth pressure in the measuring section. Figure 23 reveals that the numerical analyses using subloading $t_0$ model can simulate earth pressure of tunnel excavation precisely. Since earth pressure becomes zero at the place of excavation, excavation is possible by NATM method.

Comparing computed earth pressure distributions of Series III (Fig. 22) and Series I (Fig. 9) it is found that final earth pressures distributions are different in these two series. The reason is that in 2D condition arching exhibits only in one direction (transverse direction) and it is formed with a semi-circular shape. However, in 3D condition, it can be imagined that arching is formed with a semi-spherical shape, since arching occurs in both transverse and longitudinal directions. The above discussions indicate the necessity of the three-dimensional simulation for tunnel excavation.

**CONCLUSIONS**

Model tests and elastoplastic finite element analyses were carried out to simulate the excavation of tunnels in three-dimensional conditions. The effect of 3D excavation on surface settlement and earth pressure was discussed. The influences of soil cover and construction process on surface settlement and earth pressure of the ground were also investigated. From the present study, the following conclusions can be drawn:

1. For very shallow tunnel ($D/B = 0.5$), surface settlement occurs when the excavation front is very close to the measuring section, but for relatively deeper ground surface settlement is observed at the measuring section even when the excavation front reaches at a certain distance before this section.

2. Surface settlement of 3D sequential excavation is slightly smaller than those of 3D block by block and 3D simultaneous excavation. Earlier works by Nakai et al. (1997) showed the same results for tunnel excavation in sandy ground. It is then necessary to take into consideration the proper three-dimensional construction process for a precise prediction.

3. Earth pressure decreases at the position of excavation, and increases at the surrounding of tunnel excavation because of arching effect. The acting earth pressure at the place of excavation becomes almost constant and independent of soil cover, when tunnel depth is greater or equal to the width of tunnel.

4. Arching is formed not only in the direction of cross section but also in the excavation direction. Since arching is formed in both transverse and longitudinal direction of excavation, even in very shallow tunnel, earth pressure should be predicted as three-dimensional problem.

5. In 3D sequential excavation, earth pressure is almost
zero at the excavation front irrespective of the soil cover. This was neither observed in 3D simultaneous excavation nor in 3D block by block excavation. Apparently a soil arch is perfectly formed during the more continuous sequential excavation, while the arch is partly disturbed during the stepwise block by block excavation, reduces the efficiency of the load transfer process.

Three-dimensional finite element analysis in which the elastoplastic stress-strain behavior of soil and the construction process are properly taken into consideration, is a powerful tool for the prediction of ground movements and earth pressure in tunneling.

ACKNOWLEDGEMENTS

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REFERENCES


APPENDIX: SUBLOADING $t_N$ MODEL

The constitutive model (subloading $t_N$ model) used in numerical analyses, despite of using a small number of parameters, can well describe properly the following typical features of soils (Nakai and Hinokio, 2004):

I. Influence of intermediate principal stress on the deformation and strength of soils.

II. Dependence of stress paths on the direction of plastic flow.

III. Influence of density and/or confining pressure on the deformation and strength of soils.

A brief description of the features of this model is as follows:

Plastic strain increment is divided into component $\delta \epsilon^{(PA)}_p$ which satisfies associate flow rule in the space of modified stress $t_N$ and isotropic compression component $\delta \epsilon^{(IC)}_p$ as Eq. (A1).

$$\delta \epsilon^{(PA)}_p = \delta \epsilon^{(AF)}_p + \delta \epsilon^{(IC)}_p$$  \hspace{1cm} (A1)

These two strain increment components are expressed by the Eqs. (A2) and (A3) as,

$$\delta \epsilon^{(AF)}_p = A \frac{\partial f}{\partial t_N}$$  \hspace{1cm} (A2)

$$\delta \epsilon^{(IC)}_p = K \langle dt_N \rangle \delta \epsilon^{(i)}_p$$  \hspace{1cm} (A3)

Here, $f$ is the yield function using modified stress $t_N$, $A$ is the proportionality constant, $\delta$ is Kronecker’s delta and $\langle \cdot \rangle$ denotes Macaulay bracket. As Eq. (A2), by defining yield function $f$ with modified stress $t_N$ and considering associate flow rule at $t_N$-space instead of $\sigma$-space, feature (i) is obtained. Modified stress $t_N$ is defined as

$$t_N = a_k \sigma_{kk}$$  \hspace{1cm} (A4)

where, $a_k$ is the symmetric tensor whose principal values are assigned as the direction cosines of Specially Mobilized Plane (SMP). Dividing plastic strain increment into two components as Eqs. (A1) to (A3), for same yield function, this model can take into consideration feature (ii). Referring to the subloading surface concept by Hashiguchi (1980) and revising it, i.e., adding the term $G(p)$ in the denominator of proportionality constant $A$ of normal consolidated condition, feature (iii) is considered.

Finally, $A$ and $K$ can be expressed as Eq. (A5)

$$A = \frac{\partial f}{\partial \sigma_{kk}} \frac{1}{t_N} \langle dt_N \rangle \left[ 1 + \frac{\partial f}{\partial t_N} + G(p) \right]$$

$$K = \frac{1}{1 + \frac{\partial f}{\partial t_N} \left( 1 + \frac{G(p)}{a_k} \right) \frac{1}{t_N} \langle dt_N \rangle}$$  \hspace{1cm} (A5)

Solid line of Fig. A1 shows the yield surface passing through the present state of stress at P. Point A shows the void ratio corresponding to that state of stress. Variable $p$ is the difference between the void ratios of present state of stress (point A) and normal consolidation condition (point B) at the same stress state. In the definition of $A$ as in Eq. (A5), $p$ decreases with plastic deformation and eventually it becomes zero. To satisfy this condition, $G(p)$ should be a increasing function of $p$ which satisfies $G(0) = 0$, such as

$$G(p) = a \cdot p^2 \quad \text{(a: material parameter)}$$  \hspace{1cm} (A6)

From Fig. A1, by using the ratio of $t_N$ and $t_{N_{ic}}$, $p$ can be expressed as Eq. (A7)

$$p = (\lambda - \kappa) \ln \left( \frac{t_{N_{ic}}}{t_N} \right)$$  \hspace{1cm} (A7)

Here, $t_{N_{ic}}$ is related to the plastic volumetric strain $\epsilon^{pl}_v$ as
ERRATA

"A simple elastoplastic model for normally and over-consolidated soils with unified material parameters" by Teruo Nakai and Masaya Hinokio, Vol. 44, No. 2, April 2004, pp. 53–70.

The following corrections should be made to the original paper:

Page 56, Eqs. (12) and (19): \( p, q, t_s \) and \( t'_s \) in the original equations should be changed to \( dp, dq, dt_s \) and \( dt'_s \), respectively as follows:

\[
\begin{align*}
dp \cdot d\varepsilon_p + dq \cdot d\varepsilon_q &= 0 \quad (12) \\
 dt_s \cdot d\varepsilon_c + dt'_s \cdot d\varepsilon_{mp} &= 0 \quad (19)
\end{align*}
\]

Page 58, Eq. (28): \( \varepsilon = 0 \) should be added at the end of the original equation as follows:

\[
f = \ln \left( \frac{t_N}{t_{NO}} + \frac{1}{\beta} \left( \varepsilon_B - \frac{\rho}{1 + e_0} \right) \right) = 0 \quad (28)
\]