TWO CASE STUDIES OF CONSOLIDATION SETTLEMENT ANALYSIS USING CONSTANT RATE OF STRAIN CONSOLIDATION TEST

KOJI SUZUKI\(^{a}\) and KAZUYA YASUHARA\(^{b}\)

ABSTRACT

Consolidation settlement of two Holocene marine clays improved by prefabricated vertical drain is analyzed in order to investigate the applicability of Constant Rate of Strain consolidation test by using the stress-strain relations determined by the test carried out at a strain rate of 0.02%/min. The clay deposits, found at Yokohama, Japan and Banjarmasin, Indonesia, are slightly to moderately overconsolidated. Since the final stresses inside the ground due to construction works are close to consolidation yielding stresses, settlement behavior of the clays is analyzed by considering their state of overconsolidation. Moreover, because the compressibility of clays significantly changes during the conversion from overconsolidated state to normally consolidated state, the analysis considers non-linearity in stress-strain relations. It is confirmed, from the comparison between the actual settlement and the result of the analysis, that Constant Rate of Strain consolidation test can be applied directly to detect the stress-strain relations mobilized in the field when clay deposit improved by vertical drain is concerned.

Key words: consolidation, constant rate of strain consolidation test, Holocene clay, settlement, stress-strain relation

IIC: D5/E2/H7

INTRODUCTION

Constant Rate of Strain consolidation test (CRST) has advantage to determine consolidation characteristics when it is compared with conventional Incremental Loading consolidation test (ILT). Determination of consolidation yielding stress \(\sigma'_y\) and compressibility around \(\sigma'_y\) from ILT is sometimes very difficult and uncertain for highly compressible clays because of huge spaces among measured points. On the other hand, CRST can provide virtually continuous measurement, which enables to make accurate determination of consolidation characteristics even for highly compressible clays.

Figure 1(a) shows \(e - \log \sigma'_y\) curves, where \(e\) is void ratio and \(\sigma'_y\) is consolidation stress, obtained from both CRST and ILT for Ariake clay, which is soft and compressible Holocene clay widely found in the western part of Kyushu Island, Japan. The advantage of CRST to determine stress-strain relation of clay specimen is well demonstrated in the figure. In order to determine the value of \(\sigma'_y\) from ILT, the points A and B in the figure have to be connected by a smooth line. However, even though complicated functions are used, the detection of \(\sigma'_y\) is not necessarily reliable for highly compressible clays as Kobayashi (1991) and Kobayashi and Mizukami (1992) pointed out. While in the curve from CRST, the yielding point is very clear, even though the curve is plotted in arithmetic scale as shown in Fig. 1(b).

CRST was proposed by Crawford (1964) in order to determine \(e - \log \sigma'_y\) relations of clays within a short time. Byrne and Aoki (1969), Smith and Wahls (1969) and Wissa et al. (1971) identified the theoretical background of this test. Since then a large number of studies concerning CRST were published. Umehara and Zen (1975 and 1979) used CRST to determine consolidation constants for very soft clay in which the effect of self-weight on consolidation behavior can not be ignored. Leroueil et al. (1983) studied in detail, strain rate dependence of \(\sigma'_y\) based on extensive field and laboratory tests including CRST. Leroueil et al. (1985) revealed, from the results of various types of consolidation tests including CRST, that one-dimensional behavior of clays is controlled by a unique stress-strain-rate relation.

However, in spite of the advantage indicated in Fig. 1, only a few attempts have been made to predict consolidation settlement based on the results of CRST. Umehara et al. (1983) applied CRST to the settlement analysis of a sea wall constructed on an intermediate soil in which silt size particle is predominant, and concluded that CRST is suitable to determine consolidation parameters for soils with relatively high permeability. Okumura and Suzuki (1991) analyzed the consolidation settlement of Holocene marine clay with using stress-strain relations obtained from CRST, and compared it with the actual behavior of

---

\(^{a}\) Graduate Student, Department of Urban and Civil Engineering, Ibaraki University, and Design Department, Civil Engineering Headquarters, Toa Corporation, Japan (ko_Suzuki@toa.const.co.jp).

\(^{b}\) Professor, Department of Urban and Civil Engineering, Ibaraki University, Japan.

Manuscript was received for review on March 5, 2004.

Written discussions on this paper should be submitted before July 1, 2005 to the Japanese Geotechnical Society, 4-38-2, Sengoku, Bunkyo-ku, Tokyo 112-0011, Japan. Upon request the closing date may be extended one month.
the ground. Miki et al. (1994) used CRST to determine the parameters of elasto-viscoplastic analysis for the deformation of a soft ground under a trial embankment.

In the present paper, the authors investigate the applicability of stress-strain relations determined from CRST at 0.02%/min for actual problems through two case studies of consolidation settlement of Holocene marine clays. The clay deposits, found in two different countries, were improved by prefabricated vertical drain PVD to accelerate the consolidation. The first case study was already reported by Okumura and Suzuki (1991), but the procedure of the calculation slightly differs from the original paper in order to analyze the consolidation settlement in two case studies in a consistent manner. The clays subjected to the analysis are slightly to moderately overconsolidated. Since the consolidation at the two sites took place at stresses near \( \sigma_{cr}^{s} \), where compressibility of clays significantly changes, non-linearity in stress-strain relations was taken into consideration. The results of the analysis showed good agreement with the measured settlements. Applicability of CRST to actual problem was verified through the case studies.

This paper presents the detail of the analysis and the comparison between the prediction and the actual settlement of the ground.

**OUTLINE OF THE CASE STUDIES**

Two case studies described in the present paper are 1) land reclamation in Isogo ward, Yokohama, Japan, 2) back filling of a quay wall in Banjarmasin, Kalimantan Island, Indonesia. The thickness of clay deposits exceeds 30 m for both sites, and PVDs were installed to accelerate the consolidation. The PVD installation in the two projects was carried out when the filling work progressed to some extent. Therefore, the consolidation started one-dimensionally at the beginning. Then, radius flow of pore water was added as PVDs were installed. In the case of Isogo, the entire depth of the clay deposit was improved, while in the case of Banjarmasin, only upper 19 m of the clay deposit was improved because the lower half was stiff. The drain spacing is 1.5 m square arrangement for Isogo site and 1.85 m square arrangement for Banjarmasin site.

Two clay deposits investigated in this study are in overconsolidated state as summarized in Table 1. According to the results of CRST, the magnitude of overconsolidation \( \sigma_{cr}^{s} - \sigma_{0}^{s} \), where \( \sigma_{0}^{s} \) is an initial value of effective overburden stress, ranges from 85 to 170 kPa for Isogo clay and from 45 to 270 kPa for Banjarmasin clay. The ratio of \( (\sigma_{cr}^{s} - \sigma_{0}^{s})/\Delta \sigma_{e} \), where \( \Delta \sigma_{e} \) is applied load due to construction works, varies from 0.7 to 1.4 for Isogo clay and from 0.8 to 4.9 for Banjarmasin clay. Maximum consolidation stress \( \sigma_{0}^{s} + \Delta \sigma_{e} \) (from 0.9 to 1.3 times for Isogo clay and from 0.5 to 1.1 times for Banjarmasin clay as much as \( \sigma_{cr}^{s} \)). Suzuki et al. (1995) demonstrated that the compressibility of clays significantly changes at the stresses around \( \sigma_{cr}^{s} \) and that constant value of compression index \( C_{c} \) never appears until consolidation stress exceeds at least three times as much as \( \sigma_{cr}^{s} \), as plotted in Fig. 2. This figure strongly suggests that the change of compressibility should be considered in consolidation analysis under the stress of \( \sigma_{cr}^{s}/\sigma_{cr}^{s} = 0.5 \) to 1.3.

Unfortunately, the settlement at the two sites was monitored only at the surface of the deposit. Therefore, the comparison between the result of the analysis and the actual behavior of the ground is limited to the total

---

**Table 1. Overconsolidation of the clays investigated by CRST**

<table>
<thead>
<tr>
<th>No.</th>
<th>Project</th>
<th>OCR</th>
<th>( \Delta \sigma_{e} ) (kPa)</th>
<th>( \sigma_{cr}^{s} - \sigma_{0}^{s} ) (kPa)</th>
<th>( (\sigma_{cr}^{s} - \sigma_{0}^{s})/\Delta \sigma_{e} )</th>
<th>( \sigma_{0}^{s}/\sigma_{cr}^{s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Land reclamation in Isogo ward, Yokohama, Japan</td>
<td>2.0-4.3</td>
<td>120</td>
<td>85-170</td>
<td>0.7-1.4</td>
<td>0.9-1.3</td>
</tr>
<tr>
<td>2</td>
<td>Back filling of a quay wall in Banjarmasin, Kalimantan Island, Indonesia</td>
<td>1.6-2.7</td>
<td>55</td>
<td>45-270</td>
<td>0.8-4.9</td>
<td>0.5-1.1</td>
</tr>
</tbody>
</table>

---

Fig. 1. Example of \( e - \log \sigma_{cr}^{s} \) and \( e - \sigma_{cr}^{s} \) curves obtained from CRST and ILT for Ariake clay.
amount of the settlement monitored at the surface of the deposits.

EQUATION USED FOR THE CONSOLIDATION ANALYSIS

Stress-strain relation of clay is generally not linear as demonstrated in Fig. 1. Mikasa (1963) derived one-dimensional consolidation equations that can use nonlinear stress-strain relation and pointed out that distinguishing the progress of consolidation in terms of stress and strain is very important. Equation (1) is the simplest version among equations he derived, in which $\varepsilon$ is compression strain; $z$ is depth; $t$ is elapsed time; and $c_v$ is coefficient of consolidation in vertical direction.

$$\frac{\partial \varepsilon}{\partial t} = c_v \frac{\partial^2 \varepsilon}{\partial z^2}$$

(1)

Because soft clay deposits are usually improved by vertical drain method, the use of Eq. (1) is very limited in actual construction works. Rendulic’s equation is very popular in the design of vertical drain, but it assumes linear stress-strain relation of clay. In order to analyze consolidation with vertical drain, and also analyze with considering non-linearity of stress-strain relation, Eq. (2) is used in this study. Suzuki et al. (2003) applied Eq. (2) to evaluate both settlement and strength increment of soft marine clay under a large-scale seawall constructed for the Kansai International Airport Phase II project, and reported that the actual consolidation behavior was successfully analyzed. Derivation of Eq. (2) is shown in the APPENDIX of this paper.

$$\frac{\partial \varepsilon}{\partial t} = c_v \frac{\partial^2 \varepsilon}{\partial z^2} + c_h \frac{\partial^2 \varepsilon}{\partial r^2} + c_h \frac{1}{r} \frac{\partial \varepsilon}{\partial r}$$

(2)

in which $\varepsilon$ is compression strain in vertical direction, $c_v$ and $c_h$ are coefficients of consolidation in vertical and horizontal direction, respectively, and $r$ is the distance from the center of drain well.

In order to analyze consolidation settlement by finite difference method, Eq. (2) is discretized by using the central differences with respect to the spatial variables and by using the forward difference with respect to the time variable. Figure 3 illustrates cylindrical mass of soil assumed for the analysis. The computer program used in this study always treats the consolidation of a cylindrical mass even before vertical drain is installed, when the consolidation is still one-dimensional. When the value of $t$ reaches the time of drain installation, the program introduces permeable boundary condition from A to B to C in Fig. 3. At the permeable boundary, excess pore water pressure is assumed to dissipate instantaneously. If unimproved zone remains in lower part of deposit, like in the Banjarmasin case, one-dimensional consolidation is assumed to take place along the line from C to D in Fig. 3.

When the subsoil is a multi-layer system, adjacent layers are considered sharing the same apparent speed of pore water flow in vertical direction and the same excess pore water pressure at the layer boundary. At the permeable boundary, compression strain can be given instantaneously from stress-strain relation because no excess pore water pressure generates. The computer program treats the penetration of compression strain into soil mass. Strain distribution in a layer obtained from the finite difference calculation is averaged to evaluate its settlement. The surface settlement is estimated by adding up those for all of the layers. At the same time, an average strain of a layer is translated into an average effective stress.

The calculation starts using $c_v$ and $c_h$ in overconsolidated state, $c_{\text{VOC}}$ and $c_{\text{HOC}}$. In the computer program, each layer has its own representative value of $\sigma_i'$. The program, at all times, compares the average effective stress of a layer with its $\sigma_i'$. When the average effective stress of a layer exceeds its $\sigma_i'$, $c_{\text{VOC}}$ and $c_{\text{HOC}}$ are converted to those in normally consolidated state, $c_{\text{VNC}}$ and $c_{\text{HNC}}$, as demonstrated in Fig. 4. In a real soil mass, since consolidation progresses from the place near permeable boundary, overconsolidated zone and normally consolidated zone exist simultaneously in a layer. However, the program cannot calculate this phenomenon, because $c_v$ and $c_h$ in Eq. (2) should be spatially constant in a layer as described in the APPENDIX. Instead, conversion from
overconsolidated to normally consolidated is considered is included
in the program to take place in a layer instantaneously and
and at a certain time when the average effective stress exceeds $\sigma'_c$. Continuity $c$, around $\sigma'_c$ is not con-
idered in the calculation.

Well resistance and smeared zone are well known fac-
tors that decrease the rate of consolidation. There are
some proposals to predict the process of consolidation with vertical drain with the consideration of both the
effects (Hansbo, 1981; Onoue, 1988). However, quantita-
tive evaluation of smeared zone effect is very difficult, be-
because it requires many factors to be identified such as the
extent of smeared zone, the effect of disturbance on the
consolidation characteristics inside smeared zone, and the
difference of permeability in vertical and horizontal
directions. In order to avoid this difficulty, apparent
value of $c_0$ was used in this study.

Effect of well resistance was also ignored. Figure 5
shows the coefficient of permeability $k$ obtained from
CRST for Isogo and Banjarmasin clays. The value of
Isogo clay is in the order of $10^{-7}$ cm/s. While the value of
$k$ for PVD is $1$ cm/s, according to the manufacturer’s
brochure. Therefore, the coefficient of well resistance $L$
(Yoshikuni, 1979) expressed by Eq. (3) becomes about
0.03 when $d_w = 5$ cm ($n = d_w/d_s = 33.9$) and $H = 15$ m
are used. This means that the delay in time for 90% degree
of consolidation is less than 1%. In the case of Banjarmasin
site, the clay has about one order less values of $k$ than
Isogo site as presented in Fig. 5. Then, the value of $L$ is
smaller when $H = 19$ m is used. The value of $n$ (= 41.8) is
larger than Isogo site. Consequently, the effect of well
resistance in Bajarmasin site is less than Isogo site.
Experimental data presented by Miura et al. (1993) also
support insignificance of well resistance when PVD is
concerned.

$$L = \frac{32}{\pi^2} \frac{k_c}{k_{w}} \left( \frac{H}{d_{w}} \right)^2$$

(3)

where, $k_c$ is the value of $k$ for clay, $k_w$ is the value of $k$ for
drain well, $H$ is the drainage distance in vertical direction,
and $d_w$ is the diameter of drain well.

**UNDISTURBED SAMPLING AND TEST
TECHNIQUE**

The Japanese standard type of fixed piston thin wall
sampler was used for undisturbed sampling for the two
projects. The quality of undisturbed samples retrieved by
this sampler has been proved by Tanaka (2000) and
Tanaka et al. (2001) to be the same as the Laval and
Sherbrooke samplers, which are reputable for their
ability to obtain high quality undisturbed samples.
Before the operation of sampling, borehole was drilled
with using cross bit and mud water circulation down to
the designated depths where undisturbed sampling
started.

CRST and ILT were performed for Isogo clay, while
only CRST was carried out for Banjarmasin clay.
Specimens for both consolidation tests, prepared from
undisturbed samples, were 6 cm in diameter and 2 cm in
height. In ILT, the load increment ratio was 1.0 and the
duration of each loading was 24 hours. In CRST, a back
pressure of 100 kPa was applied to the specimen. Pore
water pressure was measured at the bottom end of the
specimen, while drainage was allowed only at the top end
of the specimen. Consolidation in CRST was done with a
strain rate of 0.02%/min for all the specimens. Both JIS
A1227 and ASTM D4186 recommend to use different
strain rate according to plasticity index $I_p$. However,
when different strain rates were used according to $I_p$
of the specimens, stress-strain relations evaluated from
the test correspond to respective strain rates. It is not desir-
able, in the Authors’ opinion, to use such stress-strain
relations for the purpose of investigating the applicability
of CRST to consolidation analysis.

The same strain rate was adopted by Hanzawa et al.
(1990) to evaluate consolidation characteristics of Ariake
clay. They conducted a series of preliminary CRST with
different strain rates in order to determine the strain rate
which permits approximately full drainage condition,
and 0.02%/min was chosen. Because this rate can give
specimens nearly 30% of strain by 24 hours loading, it is
also convenient for daily laboratory works. Figure 6
presents pore water pressure $u_w$ measured at the bottom
of the specimen, together with $\sigma'_c$ and $u_w/\sigma'_c$ for Isogo
and Banjarmasin clays. The value of $u_w/\sigma'_c$ for Isogo clay
ranges from 0.02 to 0.03, while for Banjarmasin clay it
level of the reclaimed land exceeded the water level, which was almost +1.5 m with no significant change during the reclamation work based on the monitoring in the site. Equivalent diameter \(d_e\) of PVD and the diameter of effective circle \(d_{c1}\) used for the analysis are 5 cm and 150 \(\times 1.13 = 169\) cm, respectively.

**Profile of Isogo Clay**

Figure 8 presents the results of CRST and ILT conducted to investigate the consolidation characteristics of the clay together with its plasticity. Although consolidation tests were performed at the depth of 2 m, the value of \(\sigma_1'\) and OCR can not be determined due to the reason described later. Natural water content \(w_1\) and plasticity index \(I_p\) become small below the depth of 22 m. The deposit can be divided into upper and lower clays at the depth.

The values of \(\sigma_1'\) from CRST are larger than those from ILT by 30 to 60 kPa, except around the depth of 8 m and 16 m where the values of \(\sigma_1'\) from both CRST and ILT are almost the same. In average, \(\sigma_1'\) from CRST is 1.2 times as much as that from ILT. Leroueil (1996) reported that CRST gives higher value of \(\sigma_1'\) than ILT by 1.16 to 1.5 times. The difference of \(\sigma_1'\) due to test method in Fig. 8 agrees with his report. Until the depth of 18 m, the variations of \(\sigma_1'\) are parallel to \(\sigma_{so}^0\) as expressed by \(\sigma_1' = \sigma_{so}^0 + 100\) kPa (from CRST) and \(\sigma_1' = \sigma_{so}^0 + 60\) kPa (from ILT). Below that, the variations of \(\sigma_1'\) are proportional to \(\sigma_{so}^0\) as defined by \(\sigma_1' = 2.1 \times \sigma_{so}^0\) (from CRST) and \(\sigma_1' = 1.6 \times \sigma_{so}^0\) (from ILT). Since stress increment due to reclamation is \(\Delta \sigma_{so} = 120\) kPa, the clay moves to a normally consolidated state from overconsolidated state through the entire depth according to the results of ILT, while the results of CRST show that the lower clay remains overconsolidated even after the end of consolidation.

The value of consolidation obtained from both consolidation tests are plotted in Fig. 9. There is no difference in the values between the tests. The values of \(c_c\) in normally consolidated state \(c_{c(nc)}\) are 100 to 200 cm²/d for the upper clay and \(c_{c(nc)} = 500\) to 3000 cm²/d for the lower clay. The generation of pore water pressure during CRS compression is much smaller than that of
Banjarmasin clay as demonstrated in Fig. 6. However, the values of \( c_i \) form CRST are almost identical to those obtained from ILT as shown in Fig. 9. It can be said that the strain rate used in CRST can generate enough amount of pore water pressure, at least in normally consolidated state, to evaluate \( c_i \).

The thickness of the clay is changing as shown in Fig. 7. Sedimentation of soil particles generally progresses so as to make undulating ground surface flat. Because this process went at the site with the rise of the sea level, it is expected that SL4 to SL6 never appear at the area where the deposit is relatively thin. Instead, SL1 and SL2 are probable at such an area.

Hanzawa and Adachi (1983) discussed the causes of overconsolidation for Holocene clays, and concluded that clays become overconsolidated, without release of overburden stress, due to aging effect such as chemical bonding and secondary compression. According to them, distribution pattern of \( \sigma_i' \) is parallel to \( \sigma_{0i}' \) in the part subjected to chemical bonding, and is proportional to \( \sigma_{0i}' \) in the part subjected to secondary compression. They insisted that chemical bonded zone is found in the upper part of a deposit and secondary compressed zone in the lower half, because it is most probable that secondary compression progresses with destroying chemical bonded structure between clay particles.

The relation between \( \sigma_i' \) and \( \sigma_{0i}' \) of Isogo clay shown in Fig. 8 agrees with Hanzawa and Adachi (1983). However, the reclaimed area has the possibility of being dredged before, because the area had been used to moor ships. Therefore, the possibility of release of overburden stress cannot be ruled out as the cause of overconsolidation.

**Coefficient of Consolidation for the Analysis**

Since the clay in this case study remains overconsolidated during most period of consolidation, coefficient of consolidation in overconsolidated state \( c_{v(OC)} \) is a very important parameter for the analysis. The value of \( c_{v(OC)} \) is about 10 times as much as \( c_{v(OC)} \), as indicated in Fig. 9. However, both CRST and ILT can not provide the accurate value of \( c_{v(OC)} \). In CRST, pore water pressure generation at the bottom of the specimen is so small in overconsolidated state that the measurement of the pressure is not reliable. In ILT, the measurement of settlement started 6 seconds after the loading, which is likely to miss the initial part of the settlement and to give subsequent low estimation of \( c_{v(OC)} \).

Figure 10 indicates the results of specially conducted ILT, in which the start of the measurement is 2 seconds after the loading. Each loading step in this test lasted until primary consolidation was completed. The values of \( c_i \) in the figure are normalized by \( c_i \) at the maximum consolidation stress in the test (=640 kPa). As indicated in the figure, \( c_{v(OC)} \) is greater than \( c_{v(OC)} \) by 40 to 80 times. Since the loading operation and monitoring was made manually, uncertainty still exists in the values of \( c_{v(OC)} \) in the figure. Therefore, the analysis was conducted with using \( c_{v(OC)} \) being equal to 20, 40 and 80 times as much as \( c_{v(OC)} \), and compared with the actual behavior of the ground. This approach to determine \( c_{v(OC)} \) is effective, because \( c_{v(OC)} \) is not necessarily the major concern of soil investigation in routine basis and reliable value of it is not usually available.

Experiences in the past support the viability of the assumption that apparent value of \( c_i \) is equal to \( c_i \), when consolidation of soft marine clays improved by vertical drain is considered. The value of \( c_i \) for intact clay is generally higher than \( c_i \), as many researchers have report-
ed. On the other hand, the formation of smeared zone causes the reduction of the apparent value of $c_0$. Consequently, the assumption of $c_0 = c_0$ can be used for the analysis. Although slight modification was made on the value of $c_0$, this assumption worked well in the construction of the artificial island for the Kansai International Airport to predict the consolidation settlement of a Holocene clay in the site (Arai et al., 1991). Although it is unknown that this assumption is valuable for the consolidation at stresses near $\sigma'_c$, the same assumption expressed by Eq. (4) was first adopted to the analysis.

$$c_{0(OC)} = c_{0(OC)}, \quad c_{0(NC)} = c_{0(NC)} \quad (4)$$

Representative Stress-Strain Curves

According to the change in consolidation characteristics and plasticity index $I_p$, the clay deposit can be divided into six sub-layers, SL1 to SL6 as noted in Fig. 8. Representative stress-strain curves chosen from CRST and ILT for each sub-layer are shown in Fig. 11. Vertical axis in the figure is $\sigma'_c - \sigma'_0$, where $\sigma'_c$ is consolidation stress. Initial void ratio $e_0$ for each sub-layer, used to calculate compression strain $\varepsilon$, was defined as the void ratio at the stress where $\sigma'_c = \sigma'_0$ as illustrated in Fig. 12. The value of $e_0$ defined by Fig. 12 is always smaller than the void ratio of the specimen that is subjected to unavoidable swelling due to stress release. Until $\sigma'_c$ reaches $\sigma'_0$, the specimen is in reloading. Determination of $e_0$ in Fig. 12 expects this reloading to make the void ratio of the specimen close to that in intact state in the ground. In order to take the change in compressibility into calculation as precise as possible, a polynomial equation expressed by Eq. (5) was employed to approximate the stress-strain curves. Stress-strain curves obtained from the tests are well approximated by Eq. (5) as shown in Fig. 11.

$$\sigma'_c - \sigma'_0 = Ae_1^3 + Be_2^4 + Ce_3^3 + De_4^2 + Ee \quad (5)$$

Arrows in Fig. 11 show the values of $\sigma'_c$ for SL2 to SL6. Since stress-strain curves for SL1 do not show any bending point when they are drawn in arithmetic scale as demonstrated in Fig. 13, $\sigma'_c$ for SL1 cannot be identified from the consolidation test result. In order to determine $\sigma'_c$ for SL1, the relation between $\sigma'_c$ and $\sigma'_0$ in deeper part is employed as described later. Applicability of this kind of curves to the calculation was firstly investigated by estimating the settlement at the points A and B in Fig. 7, where the clay deposit is relatively thin and the settlement of SL1 is expected to be predominant.

Analysis at Points A and B

The thickness of the clay deposit at points A and B are 6.7 m and 9.1 m, respectively. As already discussed, $\sigma'_c$
of SL1 can not be determined from the stress-strain curve. The relation between $\sigma'_i$ and $\sigma''_i$ can be given by the equation, $\sigma'_i = \sigma''_i + (60 \text{ to } 100) \text{ kPa}$ for the upper part of the clay as presented in Fig. 8. Since the value of $\sigma''_i$ at the mid-layer of SL1 is 12 kPa, the value of $\sigma'_i$ for SL1 is extrapolated as 72 to 112 kPa. According to this extrapolation, $\sigma'_i = 100 \text{ kPa}$ was assumed for SL1. This value was used for both CRST-based and ILT-based analyses.

The value of $c_{v(NC)}$ used for the analysis is 170 cm$^2$/d, determined from the plots in Fig. 9. As already noted, the value of $c_{v(DOC)}$ used is 20, 40 and 80 times as much as $c_{v(NC)}$. The results of the analysis with single drainage condition from the top of the clay deposit are compared with the measured settlement in Fig. 14. Followings can be pointed out:

i) Stress-strain curves for SL1 shown in Fig. 11, from which $\sigma'_i$ can not be determined, were confirmed to be allowable for the analysis, and the assumed value of $\sigma'_i$ for SL1 was verified to be acceptable.

ii) The time-settlement curves obtained from the analysis with $c_{v(DOC)} = 40 \times c_{v(NC)}$ and $c_{v(DOC)} = 80 \times c_{v(NC)}$ have no marked difference.

**Analysis at Point C**

In the analysis for point C, where the clay is the thickest as shown in Fig. 7, $c_{v(NC)}$ used is 170 cm$^2$/d for the upper clay (SL1 to SL4) based on the plots in Fig. 9. Since the determination of $c_v$ for the lower clay (SL5 and SL6) is very difficult due to a large scattering, preliminary calculation was conducted with $c_{v(NC)} = 200, 400 \text{ and } 800 \text{ cm}^2/\text{d}$. It became evident from the calculation that $c_{v(NC)}$ for the lower clay does not affect the calculation when a greater value than 400 cm$^2$/d is used. According to this result, $c_{v(NC)} = 800 \text{ cm}^2/\text{d}$ is adopted for the lower clay. The value of $c_{v(DOC)}$ used was 20 and 40 times as much as $c_{v(NC)}$. Double drainage condition from both ends of the deposit was applied according to the description in the drilling log which mentions that there is thin sandy layer that covers mud stone stratum.

Results of the analysis with Eq. (4) are presented in Fig. 15, together with the actual settlement of the ground. Calculation based on ILT yields larger settlement than the observed one, in particular, after PVD installation. The final settlement obtained from the calculation is 1.25 time as much as the actual settlement. Calculation based on CRST with $c_{v(DOC)} = 20 \times c_{v(NC)}$ gives smaller evaluation of the actual settlement before the installation of PVD. When $c_{v(DOC)} = 40 \times c_{v(NC)}$ is assumed, CRST produces the prediction that well agrees with the measured settlement, excepting immediately after PVD installation.

As demonstrated in Fig. 15, CRST can well explain the actual settlement behavior in the site from the start of consolidation to the end. This result of the analysis
supports the applicability of CRST to evaluate stress-strain relations mobilized in the field as long as the clay deposit improved by vertical drain method is concerned.

The reason of ILT giving larger settlement than CRST is that compression strain at the same stress is larger in ILT than in CRST. Because stress-strain relation from ILT corresponds to slower strain rate than CRST, the larger compression strain can be attributed to the rate effect that makes $e = \log \sigma'_i$ (or $\epsilon - \alpha'$) curves move leftward with decreasing strain rate. The leftward movement gives smaller value of $\sigma'_i$ for ILT than CRST as already compared in Fig. 8. Therefore, larger settlement, obtained from the analysis at stresses around $\sigma'_i$, than the actual one suggests the possibility that $\sigma'_i$ of this clay obtained from ILT is smaller than that mobilized in the field.

The calculation provided very fast settlement after PVD installation, as shown in Fig. 15. This may be attributed to the following two reasons: 1) the formation of smeared zone reduces the apparent value of $\phi_{0}^{(OC)}$ even though the clay is in overconsolidated state, 2) because the conversion from overconsolidated to normally consolidated state progresses from the vicinity of drain well, the decrease in the apparent $\phi_{0}^{(OC)}$ takes place continuously, which is not considered in the calculation. In order to find out appropriate values of $\phi_{0}^{(OC)}$ for the analysis, Eq. (6), which assumes that $\phi_{0}^{(NC)}$ can be substituted as $\phi_{0}^{(OC)}$ even if clay is in overconsolidated state, was tried. Figure 15 also shows the results obtained from Eq. (6) combined with $\phi_{0}^{(OC)} = 40 \times \phi_{0}^{(NC)}$. Equation (6) yields better prediction than Eq. (4).

![Fig. 15. Result of the analysis at point C and measured settlement of Isogo clay](image)

![Fig. 16. Profile of Banjarmasin clay](image)

$$c_{0}^{(OC)} = c_{0}^{(NC)} = c_{0}^{(NC)}$$ (6)

**SETTLEMENT OF BACK FILLING OF A QUAY WALL**

The second case study is the consolidation settlement of the clay deposit induced by the back filling of a quay wall constructed in Banjarmasin, near the mouth of a river in Karimantan Island, Indonesia. The thickness of the clay deposit found at the site exceeds 30 m. A sand layer is resting below the clay deposit. PVDs were installed, with square arrangement of 1.85 m, after spreading of the filling material with 1 m thickness and before additional materials were filled. The values of $d_u$ and $d_l$ used for the analysis are 5 cm and $185 \times 1.13 = 209$ cm, respectively. Ground water level is almost the same as the existing ground surface. Stress increase induced by the filling was $\Delta \sigma_i = 55$ kPa in total.

**Profile of Banjarmasin Clay**

The results of CRST of Banjarmasin clay are summarized in Fig. 16 for neighboring three boreholes, together with index properties. Only CRST was performed as consolidation test for this clay. Decrease of $w_0$ is observed around the depth of 7 m and 18 m. The value of $\sigma'_i$ in the upper part of the clay deposit is equal to $\sigma'_i + 50$ kPa, while in the lower part it becomes proportional to $\sigma'_i$.
\[ \sigma'_i = 1.90 \times \sigma'_{i0} \] from the depth of 19 m to 24 m and \[ \sigma'_i = 2.65 \times \sigma'_{i0} \] beneath the depth of 26 m. Resembling to Iso-go clay, \( \sigma'_i \) is parallel to (in the upper part) and is proportional to (in the lower part) \( \sigma'_{i0} \). According to the change in \( w_o \) and \( \sigma'_i \), the clay deposit can be divided into six sub-layers as indicated in the figure. The values of \( c_{\sigma(OC)} \) are mostly from 15 to 30 cm²/d as indicated in Fig. 17. Four curves in the figure show higher values than others. These data were obtained from the samples taken from the depth of 4 to 8 m and around 18 m, where low values of \( w_o \) are observed in Fig. 16.

Since \( \Delta \sigma \) in this case is 55 kPa, the clay under the depth of 19 m remains overconsolidated through the entire period of consolidation. The installation of PVDs, therefore, were done down to the depth of 19 m.

**Stress-Strain Curves**

Stress-strain curves obtained from CRST are shown in Fig. 18 together with approximation by Eq. (5). The curves in the figure were made by cutting off the initial part, where \( \sigma'_i \) is less than \( \sigma'_{i0} \), from the test results as demonstrated in Fig. 12. For SL2 and SL4 to SL6, average relation was approximated. While for SL3, one curve was subjected to approximation to represent SL3. The curves from SL1 are the same type as those from SL1 in Iso-go clay, in which bending point is not observed when arithmetic scale is used. Since two curves out of three have almost the same relation as the curves from SL2 in the range of \( \varepsilon > 0.1 \), it can be expected that the compressibility of SL1 is almost the same as that of SL2. Plasticity index of SL1 presented in Fig. 16 is also almost the same as that of SL2. From this regard, stress-strain relation in the analysis for SL1 is substituted by that for SL2.

**Result of the Analysis**

Not only the upper part of the clay, but also the lower part where PVDs were not installed, was subjected to the settlement calculation. The value of \( c_{\sigma(OC)} \) for SL1 to SL6 used for the calculation is 20 cm²/d, which is an average value determined according to Fig. 17. Since the combination of Eq. (6) with \( c_{\sigma(OC)} \) being 40 times as much as \( c_{\sigma(NC)} \) well explained the actual settlement of the ground as demonstrated in the first case study, the same combination was applied for the second case study. The result of the analysis with double drainage condition at the top and the bottom of the deposit is plotted in Fig. 19 together with the actual measurement of the settlement. Calculation with Eq. (6), which assumes that \( c_{\sigma(NC)} \) can be used as \( c_o \) for the entire period of consolidation with vertical drain, is presented in dotted line in the figure. This equation gave successful result for the first case study, however, for the second case study, actual settlement is much faster than the dotted line after the installation of PVD. Rigid line in the figure represents the result of the calculation with Eq. (4), in which \( c_{\sigma(OC)} \) is equal to \( c_{\sigma(OC)} \). The rigid line better explains the actual behavior of the ground as shown in the figure.

As shown in Fig. 19, sudden decrease is observed in the speed of the settlement obtained from the calculation immediately after the third stage of loading. This phenomenon is caused by the conversion of SL1 and SL2 from overconsolidated state to normally consolidated state. Corresponding tendency is observed in the measured settlement at elapsed days of 280 in Fig. 19. Although, the method of determining the apparent value
of $c_{\text{v(OCC)}}$ is rather different from the first case study, stress-strain relation obtained from CRST is also confirmed to be applicable for the actual prediction of consolidation settlement through the second case study.

The analysis focused on the upper part of the deposit (SL1 and SL2) was also performed because the lower part (SL3 to SL6) has the value of $\sigma'_f - \sigma_0$ that varies from 80 kPa to more than 200 kPa, and is still overconsolidated under the load increment of $\Delta \sigma_0 = 55$ kPa. The result well explains the actual settlement as presented by broken line in Fig. 19.

**DETERMINATION OF $c_{\text{v(OCC)}}$**

In the first case study, the assumption of $c_{\text{v(OCC)}} = c_{\text{v(NO)}}$ gave preferable prediction as presented in Fig. 15. While in the second case study, the same assumption did not provide acceptable result, but the assumption of $c_{\text{v(OCC)}} = c_{\text{v(NO)}}$ did as demonstrated in Fig. 19. When the values of $\sigma'_f$ are relatively high, the value of $c_{\text{v(OCC)}}$ becomes important in the design of vertical drain method. Here in this section, the difference in how to determine $c_{\text{v(OCC)}}$ is studied.

Vertical component of pore water flow is usually so small when $c_s = c_v$ is assumed that only horizontal flow is considered in the design of vertical drain. In the case study of Banjarmasin site, $c_s = c_v$ was assumed to the entire period of consolidation with PVD. On the other hand, because the clay was still in overconsolidated state when PVDs were installed, consolidation of Isogo site was analyzed with $c_0$ and $c_v$ that was greater than $c_s$ by 40 times, until the clay became normally consolidated. Therefore, the effect of vertical flow is expected to be larger in Isogo site than in Banjarmasin site.

Figure 20 presents results of calculation that compares the consolidation with radius and vertical flow (designated R & V in the figure) and that with only radius flow (designated R in the figure) for Isogo site. The calculation assumed that PVDs were installed at $t = 0$. Other conditions such as loading pattern and stress-strain relations for the calculation are the same as those for Fig. 15. In the calculation with R & V drainage, top and bottom of the deposit are considered as drainage boundary. Figure 20 clearly shows that the radius flow plays predominant role in the consolidation with vertical drain even though $c_s$ is higher than $c_v$.

In order to study the difference in how to determine $c_{\text{v(OCC)}}$, the value of $c_{\text{v(NO)}} / d_e^2$ from Eq. (7), by which the time factor of consolidation with vertical drain $T_h$ is defined, are compared for each case study. The values of $c_{\text{v(NO)}}$, $d_e$ and $c_{\text{v(NO)}} / d_e^2$ are listed in Table 2. The value of $c_{\text{v(NO)}} / d_e^2$ for the first case study is in the order of $10^{-5}$. The clay in the second case study has far smaller value of $c_{\text{v(NO)}}$ and larger value of $d_e$ than in the first. Consequently, $c_{\text{v(NO)}} / d_e^2$ is about one order less than the first. Although further experience and investigation are required to conclude how to determine $c_{\text{v(OCC)}}$, it can be said tentatively that $c_{\text{v(NO)}}$ can be substituted by $c_{\text{v(NO)}}$ when $c_{\text{v(NO)}} / d_e^2$ is larger than $1 \times 10^{-3} d^{-2}$.

$$T_h = \frac{c_{\text{v(NO)}}}{d_e^2} \cdot t$$

(7)

**CONCLUSIONS**

Consolidation settlement that occurred in two different marine clays improved by PVD was analyzed. Stress-strain relations for the analysis were determined from CRST and ILT for the first case study, and only from CRST for the second. All of CRST were conducted at a strain rate of 0.02%/min. The clay deposits subjected to the analysis are slightly to moderately overconsolidated, and the analysis was conducted with considering their state of overconsolidation. In order to take the change in compressibility into consideration as precise as possible, a polynomial expression was introduced to approximate the stress-strain relation obtained from the consolidation
The results of the analysis based on CRST showed good agreement with the actual settlement behavior of the ground for the two case studies, even though the generation of pore water pressure during CRST is very much different in two clays as presented in Fig. 6. CRST conducted at a strain rate of 0.02%/min can appropriately evaluate stress-strain relation in the field, when clay deposit is improved by vertical drain.

2) The result of the analysis based on ILT was also compared in the first case study. The analysis yielded larger settlement than the actual behavior, suggesting that ILT for Isogo clay underestimates the value of $\sigma'_z$ mobilized in the field.

3) The values of $c_{(OC)}$ and $c_{(NCS)}$ are important to analyze the consolidation in overconsolidated state. Both CRST and ILT underestimate $c_{(OC)}$. The value of $c_{(OC)}$ being about 40 times as much as $c_{(NCS)}$ is found to be suitable from the comparison of the results of the analysis with the actual settlement behavior.

4) The assumption of $c_{(OC)}=c_{(NCS)}$ gave preferable results for the first case study, while $c_{(OC)}$ was likely for the second. It can tentatively be concluded that $c_{(OC)}$ can be substituted by $c_{(NCS)}$ when the value of $c_{(NCS)}/d_z^2$ exceeds $10^{-3} d_z^{-1}$.

REFERENCES


APPENDIX

Equation to Analyze the Consolidation with Vertical Drain

Equilibrium equations with effective stresses in axisymmetric condition are written by Eq. (A1), in which $u$ is excess pore water pressure. The effect of self-weight of soil on consolidation is assumed to be ignorable.

$$\frac{\partial \sigma_{zz}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{zz} - \sigma_{uu}}{r} = \frac{\partial u}{\partial r} = 0$$

$$\frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{zz}}{r} + \frac{\sigma_{rr} - \sigma_{uu}}{r} = \frac{\partial u}{\partial z} = 0$$

(A1)

Stress-strain relation usually obtained from consolidation test routinely carried out in laboratory is in the form of Eq. (A2), which defines stress-strain relation in only vertical direction.

$$\frac{d\varepsilon_{zz}}{d\sigma_{zz}} = f(\sigma_{zz}) = m_u$$

(A2)
Rendulic’s equation has been widely accepted in the design of vertical drain. As many researchers pointed out, it assumes that Poisson’s ratio $\nu$ is equal to 0.5. Therefore, $\nu = 0.5$ can be expected as a practical assumption for consolidation with vertical drain. Under the condition of $\nu = 0.5$, $\sigma_{zz}^t$, $\sigma_{z\theta}^t$ and $\sigma_{zz}^d$ are identical and shear stress is 0, when soil skeleton is considered as elastic. Then, Eq. (A1) can be reduced to Eq. (A3).

$$\frac{\partial \sigma_{zz}^t}{\partial r} + \frac{\partial u}{\partial r} = 0$$

$$\frac{\partial \sigma_{zz}^d}{\partial z} + \frac{\partial u}{\partial r} = 0$$

(A3)

When a clay deposit is widely loaded like reclamation work, it can be assumed that deformation in horizontal direction during consolidation is ignorable. Law of conservation of mass in such a case can be expressed as Eq. (A4), where $e_{zz}$ is compression strain in vertical direction and $v_z$ and $v_h$ are apparent speed of water flow in vertical and horizontal direction, respectively.

$$\frac{\partial e_{zz}}{\partial t} = \frac{\partial v_z}{\partial z} + \frac{\partial v_h}{\partial r} + \frac{v_h}{r}$$

(A4)

According to Darcy’s law, $v_z$ and $v_h$ are written as Eq. (A5).

$$v_z = - \frac{k_v}{\gamma_w} \frac{\partial u}{\partial z}$$

$$v_h = - \frac{k_h}{\gamma_w} \frac{\partial u}{\partial r}$$

(A5)

Substituting Eq. (A5) into Eq. (A3), we get Eq. (A6) as Darcy’s law in terms of effective stress.

$$v_z = - \frac{k_v}{\gamma_w} \frac{\partial \sigma_{zz}^t}{\partial z}$$

$$v_h = - \frac{k_h}{\gamma_w} \frac{\partial \sigma_{zz}^d}{\partial r}$$

(A6)

Substitution of Eq. (A6) into Eq. (A4) leads to Eq. (A7)

$$\frac{\partial e_{zz}}{\partial t} = \frac{\partial}{\partial z} \left( \frac{k_v}{\gamma_w} \frac{\partial \sigma_{zz}^t}{\partial z} \right) + \frac{\partial}{\partial r} \left( \frac{k_h}{\gamma_w} \frac{\partial \sigma_{zz}^d}{\partial r} \right)$$

(A7)

From Eq. (A7), we get

$$\frac{\partial e_{zz}}{\partial t} = \frac{\partial}{\partial z} \left( c_v \frac{\partial e_{zz}}{\partial z} \right) + \frac{\partial}{\partial r} \left( c_h \frac{\partial e_{zz}}{\partial r} \right)$$

(A8)

where

$$c_v = \frac{k_v}{\gamma_w} \frac{\partial \sigma_{zz}^t}{\partial z} = \frac{k_v}{\gamma_w m_v}$$

$$c_h = \frac{k_h}{\gamma_w} \frac{\partial \sigma_{zz}^d}{\partial r} = \frac{k_h}{\gamma_w m_v}$$

(A9)

Assuming that $c_v$ and $c_h$ are spatially constant, we can simplify Eq. (A8) as Eq. (A10). In Eq. (2) in this paper, subscript $zz$ is omitted to write.

$$\frac{\partial e_{zz}}{\partial t} = c_v \frac{\partial^2 e_{zz}}{\partial z^2} + c_h \frac{\partial^2 e_{zz}}{\partial r^2} + \frac{c_h}{r} \frac{\partial e_{zz}}{\partial r}$$

(A10)

Because strain-deformation relations are not considered, strain distribution evaluated from Eq. (A10) is expected to differ from actual one in the ground. However, since this equation has the same form as Rendulic’s equation that is usually used in the design of vertical drain, Eq. (A10) gives the same time-settlement curve as Rendulic’s equation when average strain is concerned. In addition to that, although the ratios of $k_v/m_v$ and $k_h/m_v$ should be constant, Eq. (A10) allows using non-linear stress-strain relation.

Multi-dimensional consolidation is, in general, impossible to be expressed by a single equation. However, as long as consolidation with vertical drain is concerned, a practical approximation can be made by Eq. (A10).