PLASTICITY MODELING OF THE EFFECT OF SAMPLE PREPARATION METHOD ON SAND RESPONSE

ACHILLEAS G. PAPADIMITRIOU1), YANNIS F. DAFALIAS2) and MITSUTOSHI YOSHIMINE3)

ABSTRACT

Experimental evidence shows that different preparation methods produce sand samples with distinctly different stress-strain response. This paper explores and draws conclusions from the measured differences on the monotonic undrained triaxial response of Toyoura sand, prepared by different methods at the same values of void ratio and initial effective stresses. This is achieved by comparing data and simulations performed with a recently developed plasticity constitutive model, which accounts for the effect of inherent fabric anisotropy on the mechanical response. The inherent fabric anisotropy is represented by a second order symmetric fabric tensor, and its effect on the response at different loading directions is expressed by an appropriate dependence of certain constitutive ingredients on a joint isotropic invariant of the loading direction and the fabric tensor. Use of this constitutive scheme to simulate the aforementioned data on Toyoura sand exploits the fact that the preparation method affects both the dilatancy and the hardening response in a systematic manner. Under the premise that these effects are due to the different inherent fabric created by the preparation method, it follows that the foregoing simulations do not require changes in constitutive equations or entirely different sets of model constants. On the contrary, only model constants related to this inherent fabric anisotropy scheme need readjustment, providing insight to how the sample preparation method affects the response and what can be done to model it.

Key words: anisotropy, constitutive equation of soil, dilatancy plasticity, sand, soil structure, test procedure (IGC: D3/D6/E13)

INTRODUCTION

Different researchers around the world have proposed throughout the years a great number of preparation methods of sand samples for laboratory testing. Details on the various methods are beyond the purpose of this paper. What is of interest are comments on the various techniques, and if possible some insight on their relation to in-situ conditions. For example, the water sedimentation method is one of the most popular methods for sample preparation, and possibly because it is thought of well representing the sand fabric of natural fluvial or hydraulic fill deposits (Kuerbis and Vaid, 1988; Vaid et al., 1999). From another point of view, moist placement (or tamping) method is usually adopted for the purpose of creating very loose samples or the purpose of preventing segregation of well-graded material. Common are also methods using dry material, like the dry deposition method, which has the benefit of forming uniform samples by adjusting the height of pluviation, while some variations of the method include additional compaction (e.g. Yoshimine et al., 1998). If compaction of the sample is performed by use of a thin rod, then the gravitational (transverse isotropic) fabric of the soil is presumably destroyed leading to an isotropic fabric (Oda, 1981). On the other hand, tapping on the mold is thought to destroy less the gravitational fabric leading to intermediate levels of anisotropy (Miura and Toki, 1982).

It becomes obvious that there is a breadth of techniques (and variations thereof) for preparing a sand sample in the laboratory, and also no standardisation in the terminology (i.e. similar techniques appear with different names). Unfortunately, the decision on which technique is to be used depends on the available equipment and personnel expertise, rather than on the kind of fabric intended to be reproduced. Moreover, there is no established link between different techniques and expected results. Aiming at establishing such a link, this paper first classifies the preparation methods into various groups for better understanding of their differences, independently of their specific name in various papers. Hence, Tables 1 and 2, categorize the methods in terms of the moisture content of the sand during deposition, the depositional method, and the densification (compaction) procedure used.

Top on of this, of most interest to this paper is how
different methods (i.e. different initial fabrics) affect the stress strain response of sand. As far as undrained cyclic loading is concerned, this effort has started since the 1970s (e.g. Ladd, 1974, 1977; Mulilis et al., 1977; Tatsuoka et al., 1985). On the contrary, related research for undrained monotonic loading is less extensive, mainly because it has been performed on the basis of the concept of critical void ratio and steady state theory (e.g. Casagrande, 1936; Castro and Poulos, 1977; Poulos et al., 1985), which implies uniqueness of the steady state strength, independent of initial fabric, drainage conditions, stress or strain controlled testing, and loading manner and direction.

Interestingly, Been et al. (1991) shows data verifying this overall uniqueness of the steady state (or of the critical state, equivalently in this paper). Despite the fact that this is truly an attractive simplifying concept, especially for constitutive modelling, thenceforth data from a series of researchers have started to relax this over-simplification. For example, Mooney et al. (1997, 1998) show dependence of the critical state line in the void ratio e-mean effective stress p space on the loading direction and manner, while Yoshimine and Ishihara (1998) show the same for the ultimate steady state line. Regarding the effect of initial fabric, which is of primary interest here, Ishihara (1993) and Verdugo and Ishihara (1996) showed results that the critical state of Toyoura sand for triaxial compression was not influenced by the sample preparation method. Zlatovic and Ishihara (1997) showed the same for Nevada sand. On the contrary, Alarcon and Leonards (1988) and Dennis (1988) suggested that the critical state of sandy soils could be affected by the sample preparation method, while DeGregorio (1990) showed test results on Ottawa sand that clearly indicate that the positions of the critical state lines in the void ratio e-minor principal effective stress o-3 space for triaxial compression depend on the sample preparation method (see Fig. 1).

While the dependence of the critical (or steady) state line in the e-σ3 (or the e-p) space on initial fabric is still in debate in the literature, the respective dependence of the quasi steady state line (or equivalently of the phase transformation line) is undeniable (e.g. Ishihara, 1993; Zlatovic and Ishihara, 1997). In other words, the initial fabric seriously affects the undrained behavior of sand at least before reaching critical state. Nevertheless, no paper has been yet found in the literature with general conclusions on the effects of initial fabric that can prove useful in practical applications. Aiming at this goal, a series of triaxial tests were performed on Toyoura sand, prepared with four (4) distinctly different methods. Each set of four (4) differently prepared samples was consolidated at the same initial conditions of p and e, in order to underline the effect of initial fabric on the response. Moreover,

<table>
<thead>
<tr>
<th>Water content of the soil</th>
<th>Deposition method</th>
<th>Following densification procedure denoted in Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1] Saturated with water</td>
<td>Sedimentation (pluviation) through water</td>
<td>[a], [c]</td>
</tr>
<tr>
<td>[2] Moist (typically 5% for clean sands)</td>
<td>Placement (by spoon, etc.)</td>
<td>[b], [c-i], [d]</td>
</tr>
<tr>
<td>[3] Dry</td>
<td>Placement (by spoon, funnel, etc.)</td>
<td>[a], [b], [c], [d]</td>
</tr>
<tr>
<td></td>
<td>Pluviation through air (from nozzle, sieve, etc.)</td>
<td>[a]</td>
</tr>
</tbody>
</table>

Table 2. Compaction and density adjustment methods for sand samples

<table>
<thead>
<tr>
<th>[a] No densification (as deposited)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[b] Vertical tamping on the top of the soil</td>
</tr>
<tr>
<td>[i] Using smaller tamper (typically about a half of the sample diameter)</td>
</tr>
<tr>
<td>[ii] Using larger tamper (nearly the same as the sample diameter)</td>
</tr>
<tr>
<td>• Lower frequency by hand (typically 1 Hz to 5 Hz)</td>
</tr>
<tr>
<td>• Higher frequency by electric vibrator (typically 10 Hz to 100 Hz)</td>
</tr>
<tr>
<td>[c] Horizontal hitting of the mold</td>
</tr>
<tr>
<td>[i] With surcharge on the soil surface</td>
</tr>
<tr>
<td>[ii] Without surcharge on the soil surface</td>
</tr>
<tr>
<td>[d] Rod plunging</td>
</tr>
</tbody>
</table>
by running sets of four (4) differently prepared samples at widely different void ratios, it became possible to investigate whether there is a coupling of initial conditions and fabric in determining the sand response. Finally, running both triaxial compression and extension tests enables the investigation of how the initial fabric manifests in two (2) distinctly different loading directions and manners.

The data alone show enough for a first approximation of the effect of initial fabric on sand response. In parallel, a recently developed anisotropy scheme embedded in a sand plasticity constitutive platform is used for the simulation of the test data. This scheme was proposed by Dafalias et al. (2004) and has been shown to offer an accurate simulation of the effect of different loading directions and manners on the response of sand samples, which is a clear indication of anisotropy. Here, this scheme is used for simulating the response under the same loading directions and manners (in triaxial compression and extension), but on samples with de facto different initial fabrics. This simulation adds to the interpretation of the data, since it:

a) Pinpoints which elements of response, e.g. the dilatancy, the elastic or plastic moduli, are systematically affected by the preparation method, and which are not,

b) Quantifies the deduced systematic effects, in terms of the model constants, and thus establishes calibration processes for differently prepared sand samples.

In this way, this paper establishes a link between the preparation methods and differences in the mechanical response and practically provides a numerical tool for taking into account these effects in practical applications. For this purpose, TRIAXIAL TESTS ON SAND PREPARED WITH DIFFERENT METHODS outlines details on the executed triaxial tests and on the preparation methods used. ANISOTROPIC STATE PARAMETER presents the anisotropic state parameter A proposed by Dafalias et al. (2004), which takes into account the initial fabric and the loading direction, while INTRODUCTION OF ANISOTROPY IN THE CONSTITUTIVE MODEL presents how the parameter A is used in constitutive modelling. EFFECTS OF SAMPLE PREPARATION METHOD ON SAND RESPONSE compares simulations and data, while INITIAL FABRIC AND MODEL CONSTANTS comments on model calibration and the relation of model constants to initial fabric. ‘UNIQUENESS’ OF THE CSL? explores the importance of having the CSL in the e-p space a function of anisotropy, while the paper ends with conclusions on the effects of sample preparation methods on sand response, as well as on the use of constitutive modelling for their simulation.

TRIAXIAL TESTS ON SAND PREPARED WITH DIFFERENT METHODS

The samples used in this study had a height of about 100 mm and a diameter of about 50 mm. In all tests, Toyoura sand was used with a mean particle diameter of $D_{50} = 0.17$ mm, a minimum void ratio of $e_{\text{min}} = 0.597$ and a maximum void ratio of $e_{\text{max}} = 0.977$. The aforementioned four different preparation methods used are: dry deposition, wet tamping, air pluviation and dry rodding. The particulars of each method are described below:

1) **Dry deposition**: Dry sand was deposited in five layers in the mold. In each layer, dry sand was deposited in the loosest state using a paper funnel (Ishihara, 1993), followed by tapping of the mold to achieve target density. This method is the same as that adopted by Yoshimine and Ishihara (1998).

According to the categorization of Tables 1 and 2, this method is outlined as 3-c-ii-low frequency.

2) **Wet tamping**: Moist sand with 5% water content was placed in five layers in the mold. In each layer, vertical tamping was applied to the deposit using a flat tamper that had a diameter almost equal to that of the sample. The applied tamping energy was higher in the upper layers resulting in samples of relatively increased homogeneity, as detected by their uniform deformation pattern during shear. Given the diameter of the tamper, the sand was practically compacted one-dimensionally in the vertical direction, and thus the resultant initial (particles') fabric must be much different from that in samples prepared by the usual wet tamping (or moist placement) method that uses a smaller diameter tamper (e.g. Verdugo and Ishihara, 1996; Ishihara, 1993). According to the categorization of Tables 1 and 2, this method is outlined as 2-b-ii-low frequency.

3) **Air pluviation**: Dry sand was pluviated through air from a slit of 1 mm width and 10 mm length. The density of the sample was adjusted by changing the height of pluviation fall (0 cm to 60 cm). According to the categorization of Tables 1 and 2, this method is outlined as 3-a.

4) **Dry rodding**: Dry sand was deposited in five layers at the loosest state in the same manner as the dry deposition method, and each layer was densified by kneading with a metal rod with a 5 mm diameter. The end of the rod was rounded in a half-spherical...
shape. The number of roddings for one layer was changed to achieve target density. According to the categorization of Tables 1 and 2, this method is outlined as 3-d.

After saturation at a Skempton B value above 0.96, the samples were isotropically consolidated to a mean effective stress $p = 100$ kPa. After approximately one hour, the consolidated samples were sheared undrained under monotonic loading conditions (axial strain rate, 1%/min.). Both triaxial compression and extension tests were performed. The void ratio of the samples was measured to an accuracy of 0.001 from the value of water content and dry weight of sand measured after testing, using the same method described by Verdugo and Ishihara (1996). A total of 34 tests were used for the purpose of this paper, all initially consolidated at $p = 100$ kPa, but had void ratios ranging from $e = 0.725$ to $e = 0.944$, which corresponds to a huge variation of relative density from 8.7% up to 66.3% . Table 3 presents the void ratios of the 34 samples, denoting the sample preparation method used and the shearing direction applied.

The test results are not shown here, since they will be shown in EFFECTS OF SAMPLE PREPARATION METHOD ON SAND RESPONSE in comparison to the simulation runs.

### ANISOTROPIC STATE PARAMETER

Illustration of the anisotropy of a sand sample presupposes the definition of a fabric tensor, which portrays the orientation of either the particle contact normals, or of the particles themselves. According to micromechanical investigations by Tobita (1989), Oda et al. (1985), and Oda and Nakayama (1988), the initial orientation of particles undergoes limited change at very large macroscopic shear deformation reaching critical state, while the orientation of contact normals changes appreciably. Hence, in order to describe this aspect of fabric, which remains relatively unchanged by subsequent deformation, a constant-valued second-order tensor $F$ is defined portraying the orientation distribution of the particles themselves, as proposed by Oda (1999). For the special, but very common, case of transverse isotropy with axis $z$ the vertical direction (gravity deposition direction), $F$ can be written in reference to its principal directions $(z, r, \theta)$ as (Oda and Nakayama, 1988):

\[
[F] = \begin{bmatrix}
  a & 0 & 0 \\
  0 & \frac{1}{2}(1-a) & 0 \\
  0 & 0 & \frac{1}{2}(1-a)
\end{bmatrix}
\]  

in terms of a scalar-valued quantity $a$, which is an indirect measure of particle orientation and varies from 0 to 1. The case of $a=0$ corresponds to a fabric formation where particles “lie” entirely on the $r$-$\theta$ (horizontal) bedding plane, while $a=1$ implies a fabric formation where particles are oriented parallel to the vertical $z$ direction and tightly packed along $r$ and $\theta$ (unlikely case). Finally, $a=1/3$, corresponds to a statistically isotropic orientation of particles. It is expected that the most common cases will be in the range of $0 < a < 1/3$, i.e. with a preference towards horizontal orientations.

In order to introduce the relative effect of the orientation of particles on the mechanical response of a sand sample, it is important to take into account the relative orientation of the loading direction with respect to the fabric. The loading direction and the fabric are both expressed by second-order tensors, and their relative orientation can be measured by their joint invariants. Li and Dafalias (2002) and Dafalias et al. (2004) proposed different expressions for quantifying this orientational difference, both of which define a scalar-valued anisotropic state parameter $A$. Being the simplest of the two, the definition in Dafalias et al. (2004) is adopted here, which employs the first joint invariant of $F$ and $n$, the latter being a normalized loading direction. In particular, $n$ is the deviatoric unit tensor (i.e. $trn = 0$, $trn^2 = 1$) that lies along the normal to the yield surface trace in the $\pi$-plane of the stress ratio $r$ ($=s/p$) space, where $s$ is the stress deviator ($s = a - pI$). To account for the effect of the third stress ratio invariant, i.e. of the Lode angle $\theta$ associated with it, given by $\cos 3\theta = \sqrt{6}trn^2$, the anisotropic state parameter $A$, is defined as (Dafalias et al., 2004):

\[
A = g(\theta, c)F:n
\]  

where function $g$ of $\theta$ becomes $g = 1$ for triaxial compression and $g = c < 1$, $c$ being the ratio of the absolute values of $A$ in triaxial extension and compression.

For radial monotonic loading (emanating from $r = 0$ at a constant Lode angle $\theta$), like the triaxial paths of interest in this paper, Dafalias et al. (2004) presents an analytical

<table>
<thead>
<tr>
<th>Preparation method</th>
<th>Triaxial compression (TC)</th>
<th>Triaxial extension (TE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry deposition (DD)</td>
<td>0.738</td>
<td>0.741</td>
</tr>
<tr>
<td></td>
<td>0.823</td>
<td>0.745</td>
</tr>
<tr>
<td></td>
<td>0.929</td>
<td>0.766</td>
</tr>
<tr>
<td></td>
<td>0.944</td>
<td>0.815</td>
</tr>
<tr>
<td>Wet tamping (WT)</td>
<td>0.758</td>
<td>0.749</td>
</tr>
<tr>
<td></td>
<td>0.781</td>
<td>0.779</td>
</tr>
<tr>
<td></td>
<td>0.857</td>
<td>0.828</td>
</tr>
<tr>
<td></td>
<td>0.914</td>
<td>0.844</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.884</td>
</tr>
<tr>
<td>Air pluviation (AP)</td>
<td>0.725</td>
<td>0.741</td>
</tr>
<tr>
<td></td>
<td>0.780</td>
<td>0.762</td>
</tr>
<tr>
<td></td>
<td>0.803</td>
<td>0.771</td>
</tr>
<tr>
<td></td>
<td>0.859</td>
<td>0.837</td>
</tr>
<tr>
<td>Dry rodding (DR)</td>
<td>0.762</td>
<td>0.734</td>
</tr>
<tr>
<td></td>
<td>0.782</td>
<td>0.774</td>
</tr>
<tr>
<td></td>
<td>0.808</td>
<td>0.807</td>
</tr>
<tr>
<td></td>
<td>0.826</td>
<td>0.814</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.827</td>
</tr>
</tbody>
</table>
expression for $A$ as a function of parameter $a$. Lode angle $\theta$, ratio $c$, the intermediate principal stress ratio $b = (c_2 - c_1)/(c_1 - c_3)$ and angle $\alpha$ of the major principal stress direction $c_1$ with respect to the $r$-$\theta$ bedding plane (no confusion should arise between the Lode angle $\theta$ and the cylindrical coordinate $\theta$). In figure format, and for $c = 0.75$, this expression for $A$ takes the form presented in Fig. 2, i.e. triaxial compression and extension performed for the traditional ($\alpha = 0$, $b = 0$) and ($\alpha = 90^\circ$, $b = 1$) pairs, correspond to the minimum and maximum values of $A$, denoted by $A_i$ and $A_r$ respectively. Based on the general expression, these values are interrelated by:

$$A_i = -\frac{1}{c}A_r = \sqrt{\frac{3}{2}} \left( a - \frac{1}{3} \right)$$  \hspace{1cm} (3)

From Eq. (3) and Fig. 2, for transverse isotropy (which is the usual case in-situ), the traditional triaxial loading paths pose as the two extreme cases of the anisotropic state parameter $A$, with all other loading paths lying in between.

It has to be underlined that although tensor $F$ remains constant for a given sand and a given inherent fabric, the anisotropic state parameter $A$ changes during loading, in general, due to changes in the effective stress tensor that affect the normalized loading direction $n$ and the Lode angle $\theta$ entering Eq. (2). The state parameter $A$ remains constant throughout loading only in special cases, namely load paths with constant values of angle $\alpha$ and ratio $b$, like the triaxial compression and extension paths mentioned above (note Eq. (3)). Besides these special cases, the state variable $A$ generally changes during loading until it reaches critical state, where the effective stress tensor remains constant and $A$ retains a constant value.

**INTRODUCTION OF ANISOTROPY IN THE CONSTITUTIVE MODEL**

**Sand Response and Anisotropic State Parameter $A$**

There is plenty of evidence in the literature (e.g. Yoshimine et al., 1998; Nakata et al., 1998) that as the angle $\alpha$ increases from $0^\circ$ to $90^\circ$ and the ratio $b$ increases from 0 (triaxial compression) to 1 (triaxial extension), the sand response becomes significantly more contractive and soft. Based on Fig. 2 and for $a < 1/3$ (usual values), as the angle $\alpha$ increases from $0^\circ$ to $90^\circ$ and the ratio $b$ increases from 0 to 1, the anisotropic state parameter $A$ increases as well, from negative values for triaxial compression to positive for triaxial extension. Hence, it is reasonable to assume that as $A$ increases, the sand response becomes more contractive and soft.

The foregoing change in response may be simulated by various constitutive schemes. One of the most efficient methods is that of Li and Dafalias (2002) and Dafalias et al. (2004), i.e. to make the constitutive ingredients that control contractiveness and softening, namely the dilatancy $D$ and the plastic modulus $K_p$ of a constitutive model, functions of $A$. Although the two papers propose different expressions for the same purpose, their common characteristic is simplicity, in the sense that parameter $A$ is set to affect merely the scalar-valued $D$ and $K_p$, but no other scalar-valued or tensor-valued quantity. The important point is that the parameter $A$ does not enter the analytical expression of the yield function, hence, the change in $A$ due to changes in the effective stress tensor (see ANISOTROPIC STATE PARAMETER) does not enter Prager’s compatibility law, otherwise known as the consistency condition, which is simply the equation setting the differential of the yield function expression equal to zero. Had $A$ been included in the analytical expression of the yield expression, the derivative of $A$ with respect to time would have been considered in the consistency equation, leading to needless constitutive complexity. In fact one may say that a great simplifying advantage of the proposed constitutive framework is exactly the inclusion of the effect of anisotropy only at the level of dilatancy and plastic modulus dependence on it via $A$, avoiding unnecessary and complicated dependences of the yield surface expression on such anisotropy. This became possible because it was shown by proper interpretation of experimental data on Toyoura sand (Dafalias et al., 2004) that the anisotropy to mainly affect the aforementioned two entities (dilatancy and plastic modulus).

Based on its simplicity, the foregoing proposal is generic in concept and can be applied as an add-in to any critical state constitutive model. For this paper the proposition of Dafalias et al. (2004) is adopted in conjunction with the constitutive model platform outlined in APPENDIX A.

**The Dependence of $D$ on $A$**

As mentioned above, it is reasonable to assume that as $A$ increases, the sand response becomes more contractive. In order to simulate this effect, the dilatancy $D$ needs to become an increasing function of $A$. Based on Eq. (A5), this may be materialized by either of two obvious alternatives: $a)$ to make the coefficient $A_d$ an increasing function of $A$, or $b)$ to appropriately alter the value of ‘distance’
(\(M^d-\eta\)) by making \(M^d\) such a function (see also Eq. (A7)). Experimental evidence shows that due to anisotropy, different loading directions and manners will definitely lead to different locations of the quasi-steady state line in the \(e-p\) space (Ishihara, 1993; Yoshimine et al., 1998), and probably to different locations of the respective steady (or critical) state line (e.g. Riemer and Seed, 1997; Mooney et al., 1997, 1998), while the critical stress ratio \(\eta=M\) remains relatively unchanged (Mooney et al., 1997, 1998). Changing \(A\) complies only with the first and the last of the above experimental observations. To comply with all three observations, Li and Dafalias (2002) and Dafalias et al. (2004) proposed to change \(M^d\) indirectly, by appropriately relocating the critical state line in the \(e-p\) space from which the \(\psi=e-e_s\) is measured. Recalling Eq. (A8), this proposition is realized by rendering \(e_0\) a decreasing function of \(A\), according to (Dafalias et al., 2004):

\[
(4) \quad e_0 = e_A \exp(-A)
\]

which in conjunction with Eq. (A8) relocates the CSL in the \(e-p\) space parallel to itself as a function of \(A\).

**The Dependence of \(K_p\) on \(A\)**

As mentioned above, it is reasonable to assume that as \(A\) increases the sand response becomes softer. In order to simulate this effect, the plastic modulus \(K_p\) needs to become a decreasing function of \(A\). Qualitatively, this is achieved by merely the downward relocation of the critical state line with increasing \(A\), already introduced in Eq. (4). This because an increase in \(A\) reduces \(e_0\), Eq. (4), which decreases, in turn, the absolute value of the negative \(\psi\), entering Eq. (A4) for \(K_p\). Nevertheless, to obtain sufficiently softer response it was found necessary to adopt an additional direct dependence of \(K_p\) on \(A\), via the dependence of \(h\), Eq. (A9). Dafalias et al. (2004) proposed the following relation:

\[
(5) \quad h_0 = h_A [1 + k_0 - k_0 (A^\alpha / (A^\alpha - A))] 
\]

which for \(A=A_s\) (triaxial compression) yields \(h_0 = h_A\), and for \(A=A_e\) (triaxial extension) yields \(h_0 = k_0 h_A\), with the \(k_0<1\) (at least for the usual cases, when \(a<1/3\)). Since parameter \(A\) changes during loading, in general, so does the value of \(h_0\) according to Eq. (5). Hence, \(h_0\) is a state variable whose value is affected by the model constant \(e_A\).

EFFECTS OF SAMPLE PREPARATION METHOD ON SAND RESPONSE

**Accuracy of Model Simulations**

Dafalias et al. (2004) showed that the proposed anisotropy scheme via \(A\) simulates successfully the effects of loading direction and manner for similarly prepared samples of Toyoura sand (the tests of Yoshimine et al., 1998 and Nakata et al., 1998). As shown in Table 3, in this paper the same Toyoura sand was tested extensively (34 tests) in the triaxial cell, after being prepared with 4 different sample preparation methods. Given the model constants presented in Table 4 (detailed discussion will be presented in INITIAL FABRIC AND MODEL CONSTANTS), simulations were run for all 34 triaxial tests and their comparisons to the measurements are presented in Fig. 3 through 6. Each figure presents the effective stress paths and the stress strain relations of similarly prepared triaxial tests, in turn by dry deposition (DD), wet tamping (WT), air pluviation (AP) and dry rodding (DR), respectively. Observe that the model simulates the data very well, irrespective of void ratio, loading direction and manner, and sample preparation method. Furthermore, note that irrespective of sample preparation method, the triaxial extension (TE) data are consistently more contractive and soft than the triaxial compression (TC) data initiated by the same initial conditions of \((p, e)\), a conclusion in agreement with previous observations in the literature (e.g. Yoshimine et al., 1998; Nakata et al., 1998; Riemer and Seed, 1997).

**Sample Preparation Method and Stress Strain Response**

In order to gain insight on the effects of sample preparation method on sand response, most of the data and the simulations of Fig. 3 through 6 are re-plotted in Figs. 7, 8 and 9. Specifically, Fig. 7 compares data and simulations for all tests with void ratio \(e=0.78\), irrespective of sample preparation method. Similarly, Fig. 8 compares data and simulations for all tests with void ratio \(e=0.82\), irrespective of sample preparation method. Fig. 9 compares data and simulations for all TC tests with \(e=0.86\) and all TE tests with \(e=0.74\). Observe that the WT method (the only of the methods at hand that prepares the samples with wet sand) seems to give more dilative and stiff response than the three remaining methods, for both TC.
and TE, irrespective of void ratio. For the latter 3 (dry) methods, the response in TC is differently affected than that in TE. Namely, AP gives the most contractive and soft response in TE, while the same appears to hold for DR in TC. This general conclusion underlines the increasing importance of the water content during preparation, relative to the deposition method used during the preparation of the sample. Careful examination of data in the literature verifies that samples prepared wet usually give consistently more dilative and stiff response than dry samples, and this is probably best depicted by the comparatively larger cyclic resistance to liquefaction that such samples exhibit (e.g. in the data of Ladd, 1974; Mulilis et al., 1977; Ladd, 1977; Miura and Toki, 1982; Tatsuoka et al., 1986).

Sample Preparation Method and the Quasi Steady State

More distinct conclusions can be drawn by comparing...
the measured values of $p$ and $e$ at the quasi steady state (or at phase transformation) with the quasi-steady line procuring from parametric runs of the model. This is performed in Figs. 10(a), (b), (c) and (d) for the results pertaining to samples prepared by DD, WT, AP and DR, respectively. Observe that the simulated quasi-steady state lines (solid lines for TC and dashed lines for TE) generally agree well with the data, for all preparation methods. Furthermore, note that the model predicts an asymptotic approach of the quasi-state lines to the $p = $ initial $p$ line ($p = 100$ kPa here), in accordance to the literature (e.g. Yoshimine and Ishihara, 1998). Finally, observe the quasi-steady state lines (and the data) for TC fall consistently above the respective for TE, showing that for such preparation methods (and inherent fabrics) the response in TC is consistently more dilative and stiff than the response in TE.

Again the differences between the methods are better
depicted by direct comparison of the quasi steady state lines for the various methods. This is performed in Fig. 11. Observe that the range of the (black) lines for TC falls generally above the range of (gray) lines for TE, but the two ranges practically border one another, showing that the effect of sample preparation on the sand response may be as important as the effect of loading direction and manner. This figure also shows that the range of the (gray) lines for TE is larger than the range of (black) lines for TC, denoting that the response in TE seems to be more affected by the sample preparation method than the response in TC, in accordance to observations by Miura and Toki (1982). Finally, observe the quantitative verification of the observation that the 3 dry methods give distinctly more contractive and soft response than the wet (tamping) method, for both TC and TE, since the lines
Sample Preparation Method and the Critical State

Unfortunately, all 34 triaxial tests were terminated far before critical state. Hence, no conclusions can be drawn related to the uniqueness (or not) of the critical state line with respect to the sample preparation method on the basis of the available data. As a result, Fig. 12 presents the critical state lines resulting from the calibration of the proposed anisotropy scheme, without any data for comparison. Note that all lines are parallel, with the TC (black) lines overlying the respective TE (gray) lines, because $a < 1/3$ for all preparation methods (see Table 4). To verify the reliability of the adopted calibration, observe in Fig. 12 that the critical state line that Verdugo and Ishihara (1996) proposed for Toyoura sand under TC lies within, and near the middle of, the (shaded) range of the critical state lines for TC in this paper. As proposed, the critical state line of Verdugo and Ishihara (1996) is considered characteristic of two sample preparation methods, that of dry deposition (practically the same with DD in this paper) and that of moist placement (similar to WT in this paper, but using a small diameter tamper). Based on Fig. 12, notice that the line of Verdugo and Ishihara (1996) is quite near the line for DD in this paper (within experimental error margin), but definitely lies below the line for WT. This is an indication that the size of the tamper plays a significant role in the formation of initial fabric. Namely, large size diameter
EFFECT OF SAMPLE PREPARATION METHOD

119

leads to far more dilative and stiff response.

INITIAL FABRIC AND MODEL CONSTANTS

As shown in Table 4, different sets of parameters were used for each set of differently prepared Toyoura sand samples, which were estimated by separate calibration procedures, on the basis of the guidelines of Dafalias et al. (2004). Observe in Table 4 that out of the fourteen (14) model constants, eleven (11) are common for all sets of data, and only three (3) of the constants need to be changed to account successfully for the effects of sample preparation method on sand response, namely $a$, $e_A$, and $h_A$. Notably, these three (3) constants, along with $k_h$, are the constants related to the proposed anisotropy scheme. In the following, details are presented on the physical meaning of these four (4) constants.

Constants $k_h$ and $a$

Constant $k_h$ expresses the relative difference in stiffness (i.e. the value of the plastic modulus $K_p$) to be expected for the two extreme cases of loading direction and manner for transverse isotropy, i.e. for TC and TE (see Eq. (5)). The value of $k_h$ proves independent of the sample preparation method, and takes the constant value $k_h = 0.2$ (see Table 4). In parallel, constant $a$ expresses macroscopically the relative difference of the critical state line locations in the $e$-$p$ space for TC and TE, by means of its use in determining parameter $A$ via $F$, Eqs. (1) and (2). In terms of equations, this is a product of the conditions $trF=1$ and $trn=0$, which make the value of $A$ normalizable to the value of $a$, or more accurately the value of $(a-1/3)$ as shown in Fig. 2. Based on Table 4, the value of constant $a$ proves to have little variation with the preparation method, since $a = 0.277 - 0.310$ for the four (4) methods.

The fact that the two above constants show weak dependence on the preparation method implies that these two constants are more affected by the nature of the grains (mineralogy, shape and size distributions), rather than the manner that the grains are deposited in the sample. This becomes more evident if one studies the use of constant $a$ in determining $A$. This constant takes values usually between 0 and $1/3$, but values near 0 can only be achieved for absolutely horizontal packing of extremely longitudinal (montmorillonite-like) grains. Such grain shapes are unlikely for sands, hence the values for sands are usually closer to $1/3$ (e.g. the values for Toyoura sand in this paper). Furthermore, the more spherical the grain shape, the closer the value of $a$ to $1/3$, which is an extreme value that can only be achieved by an ideal case of spherical grains of the same size. Despite the fact that for the same sand the values of constant $a$ for different preparation methods fall within a narrow range, their actual values do provide some insight to the preparation method at hand. For example, of the four methods, it is DR that has the value of $a$ closer to $1/3$, a result that agrees well with the literature attributing to this preparation method relatively isotropic initial fabric (Oda, 1981).

Constants $e_A$ and $h_A$

The test data presented in this paper have indicated that whenever dilativeness increases, so does stiffness, a trend commonly found in sand data testing (e.g. Yoshimine et al., 1998; Nakaia et al., 1998). In order to study how this trend is related to the preparation methods, one may practically study how the preparation method affects the values of the model constants related to the proposed anisotropy scheme. As mentioned above, constants $a$ and $k_h$ express merely relative differences in response for different loading directions and manners. The overall dilativeness and stiffness of the response is governed by constants $e_A$ and $h_A$ in Eqs. (4) and (5), respectively.

In order to study how the values of $e_A$ and $h_A$ are affected by the sample preparation method, Fig. 13 is presented. This figure shows that an increase of $e_A$ (due to a different sample preparation method) is usually
accompanied by an increase of the respective value of \( h_A \) or vice versa. It should be underlined that this does not mean that the two constants are interrelated, despite the Toyoura sand-specific exponential-type trend between \( e_A \) and \( h_A \) of Fig. 13. In addition, observe in Fig. 13 that AP, DR and DD that prepare the samples using dry sand (‘dry’ methods) are well simulated by relatively small values of \( e_A \) and \( h_A \), which are close to one another as compared to the much larger values of \( e_A \) and \( h_A \) for the WT method, the only method that uses wet sand (‘wet’ method). In quantitative terms, the \( h_A \) for WT is more than double the values for the ‘dry’ methods, practically leading to less than half the strains for WT as compared to the ‘dry’ methods.

Hence, unlike \( k_b \) and \( a \), model constants \( h_A \) and \( e_A \) show a strong dependence on the preparation method. This dependence does not imply that constants \( h_A \) and \( e_A \) are state variables of some sort. Rather, it underlines the fact that sand materials should be differentiated in terms of both the nature and the inherent fabric of the grains, where the term ‘nature’ reflects mineralogy, grain shape and size distributions, while the term ‘inherent fabric’ reflects particle orientation (which is a function of the preparation method). Yet, of the various model constants for sands, some are more affected by the preparation method (\( e_A \) and \( h_A \)) and others less so (\( k_b \) and \( a \)). Overall, all four (4) constants along with their use in the proposed anisotropy scheme comprise an integrated framework for defining initial fabric and its effect on sand response.

### ‘UNIQUENESS’ OF THE CSL?

Given that the relocation of the CSL in the \( e-p \) space as a function of loading direction and manner and/or sample preparation method is a debatable issue in the literature, it is acknowledged that the relocation of the CSL as a function of \( A \) (Eq. (4)) could be considered non-desirable by engineers that proclaim uniqueness. In that case, one could model the undeniable strong effect of anisotropy on the dilatancy \( D \), by making the coefficient \( A_d \) an increasing function of \( A \), i.e.

\[
A_d = A_A \exp (A)
\]

In such a case, \( A_d \) would become a state variable depending on the value of the model constant \( A_A \). As an example, Fig. 14 compares the data from two tests after DR preparation at the same void ratio, with the simulations performed with the proposed model and constants (black lines) and with another set of simulations (gray lines) that use Eq. (6) instead of Eq. (4) without otherwise altering the values of model constants. The new version of the anisotropy scheme uses a unique CSL in the \( e-p \) space with \( e_0 = e_A = 0.884 \) in this case, instead of a variable CSL as a function of \( A \), that for TC and TE has CSL lines with \( e_0 := 0.91 \) and 0.865, respectively (see Fig. 12). As deduced from Fig. 14, the effect of this constitutive option becomes important after the quasi-steady state and all the way to critical failure, while for the initial parts of loading (up until the quasi-steady state) the differences are unimportant.

Although the data in this figure tend to vote for the option of a variable CSL, the truth is that this issue cannot be resolved on the basis of the present data set, since the tests were stopped far before the critical failure. Nevertheless, we believe that for practical purposes, i.e. boundary value problems, having or not the CSL a function of \( A \) is relatively unimportant, given that critical state failure is rarely encountered with possible exception in an event of a shear band formation. What is important is to ensure that the dilatancy \( D \) is a function of \( A \), with either Eq. (4) or Eq. (6). This is what leads to satisfactory simulations up until the quasi-steady state, which is, without doubt, a better index of the undrained collapsibility of sandy soils compared to the critical state (Ishihara, 1993; Yoshimine and Ishihara, 1998; Yoshimine et al., 1999).
CONCLUSIONS

This paper studies the effect of sample preparation method on the monotonic undrained triaxial sand response, by comparing simulations and data from 34 new triaxial compression and extension tests. The tests were performed on Toyoura sand samples prepared with four (4) different methods: the dry deposition (DD), the wet tamping (WT), the air pluviation (AP) and the dry rodding (DR). The simulations were performed with an appropriate constitutive model, that of Dafalias et al. (2004). This combinatorial study led to the following conclusions:

a) There are three basic criteria for differentiating preparation methods: 1) the water content of the sample (dry, moist, saturated), 2) the deposition method (placement or pluviation) and 3) the compaction method used to obtain the desired density (vertical tamping, horizontal tapping or nothing).

b) Of the three criteria, the first seems most critical in formulating the initial fabric. For example, the ‘wet’ sand used in the WT method leads to far more dilative and stiff response than the ‘dry’ sample methods (DD, AP and DR) that lead to small relative differences. This difference on the basis of the water content is also backed by cyclic data, which show larger cyclic resistance to liquefaction for ‘wet’ samples (e.g. Mulilis et al., 1977; Ladd, 1977; Tatsuoka et al., 1986). Second in importance is probably the third criterion, i.e. the compaction method. For example, DR seems to lead to the most isotropic fabric, in accordance to the literature (Oda, 1981), while there are important effects of the size of the tamper in WT (compare our data to the respective of Verdugo and Ishihara, 1996).

c) Practically, the effect of sample preparation on sand response may be as important as the effect of loading direction and manner. For example, the differences in response between TC and TE may be counter-balanced by different preparation methods (observe similarities between TC after DR and TE after WT). Anyway, sand response in TE seems to be more affected by the sample preparation method than the response in TC, in accordance to observations by Miura and Toki (1982).

d) The sample preparation method affects the quasi-steady state of the sample, but its effect on the critical state is in debate in the literature. The presented data do not offer any insight to the latter issue, since testing was terminated far before critical state. In other words, the sample preparation method definitely affects the overall dilativeness and stiffness of the sample, at least up until the quasi-steady state. Hence, the proposed anisotropy scheme is set to affect the dilatancy coefficient $D$ and the plastic modulus $K_p$ of the constitutive model. With regard to the former, it proposed to be affected indirectly by relocating the critical state line in the $e$-$p$ space as a function of anisotropy. If the model is not a desirable constitutive aspect, satisfactory simulations (at least up until the quasi-steady state) may also be obtained by directly scaling the dilatancy coefficient as function of anisotropy.

e) The proposed anisotropy scheme is generic in format and may be incorporated in any critical state constitutive model. It is governed by four (4) model constants, and merely changing the value of three (3) of them ensures good simulations of the effects of sample preparation method. Of the four constants, two (constants $k_0$ and $a$) are weakly related to the preparation method, while the other two (constants $e_A$ and $h_A$) increase or decrease significantly for different preparation methods.

Overall, this paper establishes a link between the sample preparation method and the mechanical response of sand, and furthermore presents an appropriate inherent fabric anisotropy scheme that may be used to predict the sample preparation method effects, by merely altering the values of model constants in a systematic manner. Nevertheless, for design purposes, the issue of reconstituting sand samples in the laboratory for accurately predicting the in-situ response still remains problematic. The reason is that the link between different sample preparation methods in the lab and the in-situ sedimentation and stress history has not yet been established. We believe that research along this path will help bridge the gap between expensive testing of frozen sand samples and the cheaper (but of ambiguous reliability) alternative of using empirical design methods based on in-situ testing, and the findings as well as the methodology of this paper will help in this effort.

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REFERENCES


APPENDIX A: THE CONSTITUTIVE MODEL PLATFORM

The emphasis in this paper is on triaxial loading applied along the (z, r, θ) directions, with z being the vertical direction, which imposes equality of stresses and strains on the r and θ (horizontal) principal directions. Hence, the constitutive model platform on which the anisotropy scheme is introduced is presented with the use of the usual (p, q) stress and (εp, εq) strain quantities, where  

\[ p = (σ_r + 2σ_θ)/3, \quad q = σ_r - σ_θ, \quad ε_p = (ε_r + 2ε_θ)/3 \]  

The modeling platform is based on the works by Manzari and Dafalias (1997) and its modification by Li and Dafalias (2000) that both did not account for inherent fabric anisotropy, and the subsequent extensions by Dafalias and Manzari (2004) and Dafalias et al. (2004) that introduced the fabric constitutive elements. The model is briefly outlined here in the triaxial space formulation, merely for better explanation of the introduced anisotropy scheme.

It is a critical state model with kinematic hardening that induces plastic strains only when there is change in the triaxial stress ratio \( η = q/p \). Hence, the yield surface has the form of an open wedge with apex at the origin of axes, and its description in triaxial space is given by:

\[ f = |\eta - α| - m = 0 \]  

where \( m \) is a constant scalar related to the opening of the yield surface, while scalar \( α \) is the kinematic hardening variable that locates the bisector of the yield surface.

The total (elastic+plastic) strain rates (denoted by a superposed dot) are given by:

\[ \dot{\varepsilon}_p = \frac{q}{3G} + \frac{2}{3} \frac{pη}{K_p}; \quad \dot{\varepsilon}_q = \frac{p}{K} + \frac{\sqrt{2}}{3} D \frac{pη}{K_p} \]  

where \( G \) and \( K \) are hypoelastic elastic moduli given by:

\[ G = G_p \left[ (2.97 - ε) \left( \frac{p}{p_a} \right)^{1/2} \right] K = \frac{2(1 + v)}{3(1 - 2v)} G \]  

with \( p_a \) being the atmospheric pressure and \( v \) the Poisson’s ratio. Parameters \( K \) and \( D \) in Eq. (A2) are the plastic modulus and the dilatancy parameter, respectively, given by:

\[ K_p = \frac{2}{3} \sqrt{\frac{2}{3}} \frac{p}{p_a} \text{ph}(M^p - η) \]  

\[ D = \frac{2}{3} A_{\delta} \left[ 2 - \sqrt{\frac{\langle M^\delta \rangle - η}{A_{\delta}}} \right] (M^\delta - η) \]  

where the \( \langle \cdot \rangle \) are the Macauley brackets, and the peak \( M^p \) and the dilatancy (phase transformation) \( M^\delta \) triaxial stress ratios are considered variable with the state.
parameter $\psi = e - e_c$ (Been and Jefferies, 1985), according to (Li and Dafalias, 2000):

\begin{align}
M^c &= M \exp (- n^c \psi) \\
M^d &= M \exp (n^d \psi)
\end{align}

The $M$ is the critical state stress ratio, which takes the values $M_c$ and $M_e$ for triaxial compression and extension, and $n^c$ and $n^d$ are model constants. Note that for loading in triaxial extension, $-\eta$ substitutes for $\eta$ in Eqs. (A4) and (A5). The CSL in the $e$-$p$ space on which $e_c$ lies is given by (Li and Wang, 1998):

\begin{equation}
e_c = e_o - \lambda_c (p/p_u)^\xi
\end{equation}

where $e_o$, $\lambda_c$ and $\xi$ are constants.

As deduced from Eqs. (A4) and (A5), the signs of $K_p$ and $D$ depend on the stress-ratio ‘distances’ $(M^c - \eta)$ and $(M^d - \eta)$ respectively, which take positive as well as negative values. In particular, positive and negative $K_p$ means hardening and softening response (with respect to stress ratio), while positive and negative $D$ corresponds to contractive and dilative response.

It has to be underlined that the most important feature of the above equations is the dependence of the dilatancy and the peak stress ratios on $\psi$, Eqs. (A6) and (A7), which was first introduced by Manzari and Dafalias (1997) in a linear form, and subsequently modified by Li and Dafalias (2000) into the exponential dependence of Eqs. (A6) and (A7). This is because it assures gradual transition to critical state, where $\psi = 0$ and $\eta = M = M^c = M^d$ according to Eqs. (A6) and (A7), which yields infinite shear straining ($K_p = 0$) and zero dilatancy ($D = 0$) according to Eqs. (A4) and (A5), respectively.

Finally, note that while $A_d$ in Eq. (A5) is a positive factor and $d_{ref}$ is a reference distance with respect to $(M^d - \eta)$, accurate simulations require $h$ in Eq. (A4) to be a function of the current state variables according to:

\begin{equation}
h = G_{d}h_{io} \frac{1 - c_i e}{(p/p_u)^{-1/2}} \frac{1}{|\eta - \eta_{io}|}
\end{equation}

which introduces, instead of $h$, two model constants $h_{io}$ and $c_i$. The $\eta_{io}$ is the initial value of $\eta$ updated at any new loading initiation, which occurs whenever $\eta$ changes sign (multiaxial definition, by Dafalias, 1986). Hence, $\eta = \eta_{io}$ holds whenever $\eta$ changes sign, providing for a zero plastic strain increment at the first step of any new loading initiation, via Eqs. (A9), (A4) and (A2). Practically, each time the stress point crosses the yield surface from within a new load path initiates and the zero plastic strain increment via Eq. (A9) ensures a smooth elastoplastic transition. The obvious instability of a zero value in the denominator of $h$ at this instance is addressed numerically by adding a ‘small’ quantity to the denominator of Eq. (A9). This quantity is ‘small’ enough (in the order of $10^{-10}$) to disallow the instability and not alter practically the end results.

Full definition of the model requires the kinematic hardening law, which in this case takes the simple form of $\dot{\varepsilon} = \dot{\eta}$, whenever plastic loading occurs.