COMPACTION BANDS AND OEDOMETRIC TESTING IN CEMENTED SOILS

MARCOS ARROYO(i), RICCARDO CASTELLANZA(ii) and ROBERTO NOVA(iii)

ABSTRACT

This paper presents a summary of recent work on cemented soils at Milan University of Technology (Politecnico). Oedometric and triaxial tests have been performed on lightly bonded soils of medium to very high porosity. Soils tested vary from a rather conventional silica sand-lime mixture to more unusual materials, including expanded clay aggregates, fragmented marine shells or stabilized metallurgical residues. A simple but powerful elasto-plastic bonded soil model is employed to select testing procedures and interpret the results obtained. Both experimental results and model simulations are employed here to illustrate and explore the onset of compaction bands, a new form of localization previously observed in rocks but whose appearance is here first signaled for bonded soils.

Key words: cemented soils, coefficient of earth pressure at rest, compaction bands, elasto-plasticity, mathematical modelling, oedometric tests (IGC: D5)

INTRODUCTION

During the last two decades cemented soils have been the subject of major research efforts by the geotechnical community. This interest springs at least from two differing sources. The first one is a clearer understanding that many behavioural differences between natural soils and their remoulded counterparts may be traced to the loss of bonding that remolding implies. This has been proved extensively working mostly with two broad categories of soils: natural clays (e.g. Cotecchia and Chandler, 2000) and calcareous sands (e.g. Carter et al., 1999). Similar observations have been also made with stronger granular bonded materials like calcarenites (Lagioia and Nova, 1995) or sandstones (Cuccovillo and Coop, 1999). When reasonably intact and homogeneous samples may be retrieved (like in clays or sandstones) the approach is based on a systematic comparison of reconstituted and intact samples; for more delicate and/or variable deposits, like calcareous sands, testing of reconstituted artificially cemented samples is commonplace.

The other motivation for cemented soil study is an ever-expanding industrial practice (Porbaha, 1998) where soils and other materials like wastes are engineered to improve their physical or chemical properties using a variety of binding agents (lime, cement, fly ash etc.). Although fine grained soils are usually harder to engineer, attention to improved granular soils (Dupas and Pecker, 1977; Zhu et al., 1995; Schnaid et al., 2001) has been perhaps even greater than that given to improved fine grained soils (Uddin et al., 1997; Miura et al., 2001; Uddin and Buensuceso, 2002). Most work in this area has focused on mix design; testing several laboratory mixes to ascertain the optimum improvement. There has been less testing on field samples of improved soils.

All this experimental work has inspired many efforts explicitly oriented to model bonded soil behaviour (Gens and Nova, 1993; Rouainia and Wood, 2000; Kavvadas and Amorosi, 2000; Baudet and Stallebrass, 2004; Cudny and Vermeer, 2004 to name a few). All these are continuum elasto-plastic models, aiming to reproduce homogeneous element tests such as those supposedly recorded in research programmes like the ones quoted before. With rare exceptions (Lagioia and Potts, 1997) this work has generally sidetracked the question of how non-homogeneities or localized deformations may affect the recorded results and, consequently, the models developed to summarize and extrapolate those results. This question has extra interest when it is noted that bonded soils may be prone to more non-homogeneities than previously realized, as compaction bands, a new type of discontinuity, enters the scene.

Compaction bands were first noticed in sandstone outcrops as thin tabular zones of pure compressional deformation aligned orthogonal to the maximum compressive stress (Mollema and Antonellini, 1996). They were later observed in the laboratory when relatively high porosity sandstones were subjected to triaxial compres-
The phenomenon is not restricted to rocks, but appears also in a variety of stiff but porous materials (metal foams, for instance). Hence, there is a strong expectation for it to be important in cemented soils, which have both high initial porosity and stiffness as generally acknowledged characteristics.

Research at the Politecnico di Milano on the subject of bonded soil is long established (Nova, 1992; Lagioia, 1994; Castellanza, 2002). This paper summarizes recent work done at the Politecnico di Milano, with particular emphasis on oedometric behaviour and the observation and prediction of compaction bands. First a description of the testing apparatus employed and of the variety of tested materials is included. It follows a brief description of the main features of the elasto-plastic model used to guide and interpret the experimental work. Experimental evidence of compaction bands in cemented soils is then described. This evidence is best appreciated after the theoretical conditions for compaction banding in an oedometer are recalled. Finally, we particularize these theoretical conditions for the basic elasto-plastic bonded soil model previously introduced, thus establishing the critical parameters controlling this new instability.

### MATERIALS AND TESTING METHODS

A variety of bonded soils have been tested, whose main properties are summarized in Table 1. The two with a longest tradition are natural calcarenites (such as in Lagioia, 1994) and an artificial mixture of silica sand and lime (such as in Castellanza, 2002). This paper summarizes recent work done at the Politecnico di Milano, with particular emphasis on oedometric behaviour and the observation and prediction of compaction bands. First a description of the testing apparatus employed and of the variety of tested materials is included. It follows a brief description of the main features of the elasto-plastic model used to guide and interpret the experimental work. Experimental evidence of compaction bands in cemented soils is then described. This evidence is best appreciated after the theoretical conditions for compaction banding in an oedometer are recalled. Finally, we particularize these theoretical conditions for the basic elasto-plastic bonded soil model previously introduced, thus establishing the critical parameters controlling this new instability.

### Table 1. Bonded soils and their characteristics

<table>
<thead>
<tr>
<th>Label</th>
<th>Name</th>
<th>Description</th>
<th>Initial void ratio</th>
<th>Specific gravity $G_s$</th>
<th>Unit dry weight $\gamma_d$ [kN/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC</td>
<td>Natural Calcarenite</td>
<td>Natural soft rock: calcareous grains cemented with Calcite (CaCO$_3$ = 98%)</td>
<td>1.15</td>
<td>2.71</td>
<td>12.6</td>
</tr>
<tr>
<td>LCS</td>
<td>Lime Cemented Sand</td>
<td>Artificial soft rock: silica sand ($D_{50}$ = 0.75 mm) cemented with lime (Lime = 15%)</td>
<td>0.75</td>
<td>2.62</td>
<td>15.0</td>
</tr>
<tr>
<td>LCL</td>
<td>Lime Cemented LECA</td>
<td>Artificial soft rock: LECA grains (Light Expanded Clay Aggregate; grain specific weight = 6.14 kN/m$^3$, $D_{50}$ = 1.5 mm) cemented with lime (Lime = 15%)</td>
<td>1.35</td>
<td>2.31</td>
<td>5.4</td>
</tr>
<tr>
<td>AC</td>
<td>Artificial Conchiliades</td>
<td>Artificial soft rock: shells ($D_{50}$ = 7 mm) and silica sand ($D_{50}$ = 0.75 mm) cemented with 15% of lime.</td>
<td>1.54</td>
<td>2.60</td>
<td>10.2</td>
</tr>
<tr>
<td>JA</td>
<td>Jarofix</td>
<td>Artificial bonded soil: metallurgical residue of fine silt size ($D_{50}$ = 9 mm) (see Arroyo et al., 2005)</td>
<td>2.25</td>
<td>3.15</td>
<td>9.7</td>
</tr>
</tbody>
</table>

To investigate the mechanical behaviour of these materials two kinds of tests have been employed to date: oedometric and triaxial. The oedometric tests here referred were performed using a specially designed flexible ring (WTD) (Castellanza and Nova, 2004), mounted on a triaxial frame (Fig. 2). This disposition allows for continuous vertical displacement control of the test. Axial load is measured using a load cell. The ring is fairly flexible and its average circumferential stretch is measured by three strain gauges resolving about 1.0 µin/m. The induced radial strains are very small and the kinematic conditions are therefore very close to the oedometric ideal. From these minimal radial strains an average radial stress on the sample is readily obtained (see Castellanza and Nova, 2004). For isotropic materials, assuming axial symmetry and frictionless bases and ring, a complete stress strain relationship is hence derived. Further data on specimen preparation and calibration procedures for the WTD can be found in Dell'Orto and Torre (2003).

Triaxial tests were conducted using a Bishop-Henkel cell modified by Lagioia (1994) to sustain up to 7 MPa cell pressures (Fig. 3(a)). Vertical displacement was externally and internally measured. The internal transducers were strain-gauged pendulum inclinometers (Ackerley et al., 1987, Fig. 3(b)). Saturation was achieved by CO$_2$ flushing and back-pressure application. Results on cemented sand samples were corrected for membrane penetration effects; this and other details are fully described in Chiamone and Scotti (2003). More details of the Jarofix testing procedures are presented in Arroyo et al. (2005).

### CONSTITUTIVE MODEL

The models here employed to describe and explore bonded soil behaviour belong to a modeling framework
conceived by Nova (1992) and then extended by Nova and co-workers (di Prisco et al., 1992; Gens and Nova, 1993; Lagioia and Nova, 1995). That framework employs the formalism of elasto-plasticity allowing for a non-associative flow rule and several hardening variables. The last aspect is particularly important, as will be shown below.

The current model formulation is fully detailed elsewhere (Nova et al., 2003). Here we just report (in Table 2) the parameter sets and initial conditions for all the numerical tests and simulations later presented. Shapes for both yield surface and plastic potential are described using the expression given by Lagioia et al. (1996). Elastic behaviour is described by a four-parameter hyperelastic formulation (Borja et al., 1997). Note that both formulations, i.e. that of the plastic surfaces and that of the elastic behaviour, include as particular cases some classical approaches. With a suitable set of parameters, for instance, the Cam-clay ellipse may be recovered for both plastic surfaces. The same happens with the hyperelastic formulation, which includes as special cases isotropic constant elasticity or a bulk modulus variable with the logarithm of mean stress (Butterfield, 1979).

Apart from stress (or elastic strain) in this framework any material state is described using just two independent scalar variables with stress dimensions, $p_s$ and $p_m$ (a third independent scalar variable, $p_t$, was initially proposed, but for practical applications $p_t = k p_m$ is assumed, with $k$ a constant). Of these two internal variables, the first is a stress memory variable, akin to the classical preconsolidation pressure, whereas the second summarizes the effects of bonding and/or structure (here treated as synonymous). Both variables control the size of the current yield surface and of the inner reference surface corresponding to a fully destructured material, as shown in Figure 2. Displacement controlled soft oedometer (WTD).

Fig. 2. Displacement controlled soft oedometer (WTD)
in Fig. 4. In our current model formulation their evolution or hardening laws are given by:

\[
\begin{align*}
\dot{\rho}_s &= \rho_s \rho_s (\dot{\varepsilon}^\rho + \dot{\varepsilon}^\rho) \\
\dot{\rho}_m &= -\rho_m \rho_m (|\dot{\varepsilon}^\rho| + \dot{\varepsilon}^\rho)
\end{align*}
\]

Here \(\varepsilon^\rho\) and \(\varepsilon^\rho\) represent, respectively, volumetric and deviatoric plastic strains, whereas \(\rho_s\), \(\zeta_s\), \(\rho_m\) and \(\zeta_m\) are material parameters.

The evolution rules of the two internal variables are qualitatively different. Whereas the stress memory related, \(\rho_s\), can either increase or decrease, depending on the state of stress and history of the material, the bond strength linked \(\rho_m\) can only decrease with plastic strains. Consequently, either softening or hardening can take place, depending on the relative amount of the rate of change of the internal variables. In a further development (Nova et al., 2003) \(\rho_m\) has been made also dependent on a weathering variable that may represent, for instance, solute concentration, therefore establishing a chemomechanical coupling.

The inclusion of an independent hardening/softening variable to account for bonding was the key conceptual
element brought about by Nova (1992) and Gens and Nova (1993). This idea has been successful and was later incorporated almost without change into other, more complex, elasto-plastic models of bonded soil behaviour. The added complexities are necessary, for instance, to obtain a better match of anisotropic behaviour (Rouainia and Wood, 2000; Cudny and Vermeer, 2004) or stiffness nonlinearity (Baudet and Stallebrass, 2004). However, they are neither necessary nor convenient when the purpose is to explore the consequences of this shared constitutive choice.

COMMENTS ON MODEL CALIBRATION

Ease of calibration is sometimes a more compelling demand on a soil model than a precise performance. Since the overall performance of this model has been already illustrated (see Lagioia and Nova, 1995; Nova et al., 2003), we will focus now on calibration issues. When an ample database of good quality test results is available, as in Lagioia (1994), the calibration task is relatively easy. Several optimization strategies, either formal (e.g. Rouainia and Wood, 2000) or informal, may be employed to obtain a good match to a series of selected results, but ample test databases are not always available and it is therefore desirable to have calibration procedures as straightforward as possible.

Of the plastic functions that are employed to build the model (plastic potential, yield and hardening functions) some are easier to calibrate than others. The plastic potential proposed by Lagioia et al. (1996) is obtained through integration of the stress-dilatancy rule of Eq. (2):

$$d = m_2 (M_3 - \eta) \left( \frac{a_2 M_3}{\eta} \right)$$

where $m_2$, $a_2$ and $M_3$ are model parameters, controlling the shape of the plastic potential.

This may be easily calibrated with a few drained triaxial tests, since each test provides numerous data about the evolution of dilatancy ($d = \Delta \varepsilon / \Delta \varepsilon_n$) with stress ratio ($\eta = q/p'$). As an example, in Fig. 5 we present results for a lime-cemented sand and for Jarofix. The slope $m_2$ is closer to the classical Cam-clay value of 1 for the finer Jarofix than for the coarser cemented sand.

Calibration of the yield function is rather more difficult. Firstly, if several yield points belong to a single yield surface it is essential that the samples from which they are obtained share the same initial state ($p_s$ and $p_m$ in this model). This requirement makes the task very difficult in obtaining yield surfaces for natural materials, whose initial bonding and damage degree at the time of testing may be highly variable. Even for laboratory prepared materials, identification of yield points is sometimes delicate and yield data are notably disperse. Since the usual approach provides only a single yield point from every test, a solution is to do many tests (Fig. 6), but this is a rather onerous task. A possible alternative is to obtain more data from every test. Tatsuoka and Ishihara (1974) employed an elaborate stress path to obtain various yield points for virgin sand during triaxial compression. For lime cemented sand, simulations (Fig. 7) with the above model indicated the possibility of following a stress path quasi-tangent to the yield surface if yielding was induced through a cycle of increased pore pressure at constant deviatoric strain. Figure 7 shows an example of the experimental results obtained when this idea was applied; there is a marked yield and after that a stretch of the stress path that would
give a substantial amount of information on the shape of the yield surface. The hardening function for $p_s$ belongs also to an unbonded material. It may then be obtained by testing reconstituted samples. For the common case where critical states are attained, its evolution depends only on plastic volumetric strain ($\zeta_v = 0$ in Eq. (1)) and an oedometric test will be enough to obtain the remaining parameter. In fact, it can be shown that, when the elastic part is chosen to coincide with the classical Butterfield law, this parameter $p_s$ is equal to $1/(\lambda^* - \kappa^*)$, where $\lambda^*$ is the slope of the oedometric virgin compression curve in the $e_v - \log p'$ plane.

The evolution or hardening function for $p_m$ is more difficult to calibrate. In principle the two parameters that enter its formulation ($p_m$ and $\zeta_m$) may be directly obtained comparing the destructuration process (i.e. intact vs. remoulded samples) in oedometric and isotropic compression paths ($\zeta_m$ plays a role in the first case but not in the second). Two problems may arise. The first has to do with the necessary length of the stress path to achieve full destructuration. The second is subtler and has to do with the possibility that the tests may not be able to follow the destructuration process if this results in a non-uniform element response. As the results in the following sections indicate, this is a possibility not to be easily disregarded.

**EXPERIMENTAL OBSERVATIONS OF COMPACTION BANDS**

The observation of compaction bands on porous sandstone formations prompted the search for the same phenomena in the laboratory. Triaxial compression of porous ($e = 0.30$) sandstones sometimes occurs in conjunction with an essentially flat deviator-axial strain curve, which may harden again after the compaction takes place (Fig. 8(a)). At the flat section of the curve the tangent modulus is nil. Olsson (1999) argued that localized compaction might happen at that stage, since
the nil tangent modulus may be interpreted as a bifurcation condition extending the well-known Rudnicki-Rice framework of shear localization (Rudnicki and Rice, 1975). Postmortem visual sample examination produced already some evidence of compaction banding.

Curves with a very similar brittle-plastic shape to that of porous sandstones had been previously observed in softer geological materials, like natural calcarenites (Lagioia and Nova, 1995, ε ≈ 1.15) (Fig. 8(b)). Using a similar calcarenite such behaviour was easily reproduced, and, again like in sandstones, postmortem examination of the samples does suggest an uneven tabular compaction (Fig. 8(c)), not unlike that simulated by Lagioia and Potts (1997). These materials had void ratios well over those of sandstones, and, at least for the case of the natural calcarenite studied by Lagioia, the same flat-curve phenomenon was also noted in isotropic and $K_0$ compression tests. This last observation made also probable the appearance of compaction bands during oedometric tests. Indeed, oedometric tests do facilitate the observation of purely compressive inhomogeneities, since samples are more constrained than triaxial and simultaneous shear banding is impossible.

Results from a strain controlled oedometric test on a sample of artificial conchilides are shown in Fig. 9, alongside their simulation. Soon after the deviator peak has been reached, the strain driven calculated stress path curves backwards and experiences a sort of 'curl', and only afterwards the stress deviator increases again. This curl means that the vertical stress reaches a peak, decreases and then increases again. Here it is clear that the usual load-controlled oedometric test would have been unstable, since it would jump over the curl. But, as we will show below, this peak in axial stress is also indicative of a possible non-homogeneous behaviour even during a strain-controlled test. In fact, when the specimen was examined after the end of the test, it showed some evidence of increased compaction in bands located near its top and bottom.

To confirm that such banding occurred when the vertical stress reached the peak, another test was conducted on a specimen made of intact shells cemented with lime and water. Sand was not employed, in order to make the internal structure more fragile and the instability more evident. The test was conducted in a special oedometer with a transparent wall. The achievement of the local maximum of vertical stress was accompanied by shell breakage in the top part of the specimen. The aspect of this compaction band is documented by Fig. 10.

The location of the experimental compaction band near the top part of the specimen may be related to stress concentrations near the loading platen. It is interesting that similar band locations have been reported by Olsson (1999) testing sandstones and by Bolton and Marketos (2005) using a DEM simulation of the problem.
Material instabilities for elasto-plastic behaviour may be studied within a comprehensive framework developed by Nova and co-workers (Nova, 1989; Imposimato and Nova, 1998; Nova, 2003). An axial stress peak in a confined compression test in which radial strains are constrained to be zero, means that the stiffness coefficient linking together axial stresses and strains is zero. This may be seen as follows. For an isotropic material, in axisymmetric conditions such as those imposed by the test, the stress-strain relationship can be written as:

\[
\begin{bmatrix}
\sigma_a \\
\sigma_t
\end{bmatrix}
= 
\begin{bmatrix}
D_{aa} & D_{at} \\
D_{ta} & D_{tt}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_a \\
\varepsilon_t
\end{bmatrix}
\tag{3}
\]

From the kinematic condition \( \varepsilon_t = 0 \) and the peak axial stress condition \( \sigma_a = 0 \), it immediately follows that \( D_{aa} = 0 \). This corresponds to the compaction band condition obtained by Rudnicki (2002) following a different strand of reasoning.

Nova and Imposimato (1998) show that the condition \( D_{aa} = 0 \) is equivalent to \( C_{rr} = 0 \), where \( C_{rr} \) is the compliance term linking the incremental radial strain and incremental radial stress. This form is more convenient, since for an elasto-plastic model we have in general:

\[
\dot{\varepsilon}_{ij} = C_{ijhk} \sigma_{hk} + \frac{1}{H} \frac{\partial f}{\partial \sigma_{ij}} \frac{\partial f}{\partial \sigma_{hk}} \sigma_{hk}
\]

\[H = -\frac{\partial f}{\partial p_t} \frac{\partial \phi}{\partial \sigma_{tt}} \frac{\partial \phi}{\partial \sigma_{kk}} \tag{4}\]

where \( C^e \) represents the elastic compliance tensor, \( f \) the yield function, \( g \) the plastic potential, \( H \) the hardening modulus and \( p_t \) the plastic state variables. Softening is identified as the situation where \( H \) is negative. From the relations above, a general expression of the condition \( C_{rr} = 0 \) may be obtained as follows:

\[C_{rr} = C_{rr} + \frac{1}{H} \frac{\partial g}{\partial \sigma_{rr}} \frac{\partial f}{\partial \sigma_{tt}} = 0
\]

\[HC_{rr} = -\frac{\partial g}{\partial \sigma_{tt}} \frac{\partial f}{\partial \sigma_{rr}} \tag{5}\]

APPLICATION TO THE BONDED SOIL MODEL

The usefulness of the previous compaction band criterion lies in its application to a particular constitutive model. In fact, when applied to the model previously described, it allows some extra insight on the consequences of the introduction of a debonding variable.

As Eq. (4) indicates, the plastic modulus \( H \) depends
directly on the evolution rules of the hardening variables. For bonded soil models we have introduced these rules in Eq. (1), giving the evolution of $p_s$ and $p_m$. These may be related to a different pair of variables, $p_c$ and $p_r$, through:

$$p_c = p_s + p_m + p_r$$

$$p_r = kp_m$$  (6)

Choosing $p_s$ and $p_c$ as the pair of hardening variables simplifies the formulation of the yield surface and plastic potential. It does also simplify the expression for the plastic modulus $H$. It should be noted that $k$, which is directly related to traction resistance, is small in soils. As a result the variation of $p_r$ is much smaller than that of $p_c$ and may be disregarded in compaction-dominated processes.

A further assumption at this stage is that the elastic strains are much smaller than the plastic strains and its contribution to the total strain may therefore be neglected. This assumption is fair for an oedometric test in a porous bonded material. Using these assumptions, the evolution of $p_c$ during an oedometric test may be approximated as:

$$p_c = \frac{\partial p_c}{\partial \varepsilon_c} \dot{\varepsilon}_c^p = \left[ \left( 1 + \frac{2}{3} \frac{\zeta_m}{\zeta} \right) \rho_c, p_c - (1 + k) \left( 1 + \frac{2}{3} \frac{\zeta_m}{\zeta} \right) p_m p_m \right]$$  (7)

The condition $C_{rr} = 0$ would now read as:

$$\frac{\partial f}{\partial p_c} \frac{\partial p_c}{\partial \varepsilon_c} \frac{\partial g}{\partial \sigma_s} \frac{C_{rr}}{\partial \sigma_s} = \frac{\partial g}{\partial \sigma_t} \frac{\partial f}{\partial \sigma_t}$$  (8)

It is now necessary to introduce the expressions for plastic potential $(g)$ and yield function $(f)$. As we are here focusing on the consequences of the mixed hardening-softening behaviour characteristic of bonded soils, we will choose an associated model where $f$ and $g$ have the shape given by the same ellipse. We will also assume isotropic elasticity. With these assumptions, we will have in fact recovered Cam-clay but for the modified hardening rule accounting for destructuration and the simpler elastic model. As shown in the APPENDIX the resulting expression can be written in the following form:

$$1.75p_mp_m - p_s p_s = \frac{2E}{1-\nu} C(M,\eta)$$  (9)

where $E$ is the elastic Young’s modulus, $\nu$ is the Poisson’s ratio and $C(M,\eta)$ is a function, given in the APPENDIX, of the stress ratio and critical state line slope $M$.

In Fig. 11, function $C(M,\eta)$ is evaluated for a range of $M$ values and for stress ratios below the critical i.e. those ensuring plastic compaction. The function $C$ is always non-negative, and this means that the right hand side of Eq. (9) should also be positive for a compaction band to occur. As the experimental evidence suggests, strong bonds (high $p_m$) and high void ratios (low $p_s$) favour this phenomenon.

In fact, Eq. (9) suggests also that compaction banding during oedometric loading would also be favoured when the ratio between the destruction rate (measured by $p_m$) to compaction rate (measured by $p_s$) is high. We can explore this issue further with a sensitivity analysis of the model behaviour in oedometric conditions. In Figs. 12 and 13 we examine the influence of varying the initial value of $p_m$ for small initial $p_s$. In the first case the parameter $p_m$ is smaller than $p_s$, while the opposite happens in the second case. The peak in axial stress,
which is the telltale of a discontinuous compaction possibility, increases always with a higher initial $p_m$ but much more so when $p_m/p_s$ is high. In a last numerical example, Fig. 14 shows that the apparition of peaks on the axial stress-strain curve is not really controlled by the destructuration rate $p_m$ or the initial bonding $p_m$ alone, but rather by the ratio $p_m/p_s$. This is exactly what would be suggested by inspection of Eq. (9). This result also enhances the general credibility of Eq. (9), since the simulations in these figures had been run with a somewhat more complex, (but more realistic), non-associated, hyperelastic, variant of the bonded soil model.

In the previous figures the oedometric test was also presented in terms of a $K_0$ vs axial stress curve. The generally sigmoidal shape of these curves fits well with experimental results, like those collected in Fig. 15 or those reported by Zhu et al. (1995). The axial load peak manifestation in this diagram is that of a backwards loop around the level of the upper $K_0$ limit. This phenomenon will not be recorded at all in a load-controlled test (like those in Zhu et al. or in Fig. 16(a)) that instead would just jump over the peak. It is not even clear, if compaction banding follows the peak, what will be recorded in a strain-controlled test with external measurements. It is remarkable, therefore, that the experimental curves for calcarenite in Fig. 15(b)) and for conchiliades in Fig. 9(c)) do suggest an incipient backward loop near the upper $K_0$ limit.

For a given material (i.e. for a set of parameters and initial state variables) the compaction band condition expressed by Eq. (9) defines a surface in a state space spanned by $(p_m, p_s, \eta/M)$. On the other hand, the material evolution during an oedometric test will also define a trajectory in this space. It is then possible to visualize the achievement of a compaction band condition as the intersections of both. Figure 16 explores this procedure. Two intersections appear, at points B and C, points that mark local extremes of axial stress, and, in the case of point C a global minimum of the hardening variable $p_c$. The onset of compaction bands would be triggered by the first extreme at B. Note that this point does not correspond to a peak in deviatoric stress; peak that is located at point A.
DISCUSSION

We have shown above how it is possible to obtain, in softer geomaterials, the same laboratory evidence for compaction banding that was initially obtained for sandstones. Later work on porous sandstones, however, has produced clearer identification of compaction banding during standard triaxial compression, mainly through acoustic emission records (Olsson and Holcomb, 2000) and quantitative microstructural analysis (Baud et al., 2004). The observed characteristics of these discontinuities have been variable, sometimes they travel along the sample as a single compaction front and then disappear, and sometimes they appear in groups and remain frozen.

The gathering of experimental evidence for compaction banding in soil-like materials is, arguably, still in its infancy. Discontinuities developing orthogonal to the sample axis are ill-suited for detection with current instruments (e.g. on sample measurements, like the inclinometers shown in Fig. 3(b)). Post-testing microscopic examination of samples in search for discontinuities may not be as straightforward in finer materials as it is in sandstones. It is not even clear what may be the relevant microstructural unit that is being crushed. Acoustic emission or other laboratory geophysical measurements are also generally more difficult in soils than in rocks.

When planning further experimental work, it should be noticed that test or sample imperfections may control the patterning of the compaction bands (i.e. presence of one or many bands, band(s) location(s) within the sample, coalescence and/or disappearance of the bands; Lagioia and Potts, 1997; Vajdova and Wong, 2003). However the onset and possibility of compaction band inception is something that, as shown above, may be explored at the constitutive level with a relatively simple model.

CONCLUSIONS

The study of bonded soil behavior has made important progress in the last 15 years. A number of models have been proposed where a previous elasto-plastic model is enhanced introducing new hardening variables to account for bonding. The evolution rules proposed for these variables are such that bonding always decreases with mechanical loading, even when the soil is being densified.

From a general point on view of elasto-plastic formulations, this inclusion introduces a new possibility, that of softening during compaction or on the “wet” side, to use a classical expression. It is well known that a softening behavior may lead to material bifurcations and discontinuities; the classical example is shear banding on dilating soils. A different type of discontinuity is associated with this newer kind of softening: compaction bands.

The conditions for the onset of compaction bands are known. Here they were applied to a rather simple bonded soil model showing that, in oedometers, compaction bands may indeed happen during compression of porous bonded materials. This result is satisfactory since it corroborates laboratory observations of the same phenomena. A notable consequence of this result is that for this kind of materials oedometer testing should always take place under strain control. This will avoid the instabilities associated with the peak in the axial stress-strain curve. Note however, that even using strain control would not be enough to avoid the loss of test homogeneity associated with compaction bands. More research is needed to explore the consequences of this feature for the measured...
ACKNOWLEDGMENT

This work has been supported by an EU Grant within the project Degradation and Instabilities in Geomaterials (DIGA Project—Improving Human Potential Program—Research Training Network HPRN-CT-2002–00220).

REFERENCES

COMPACCTION BANDS IN CEMENTED SOILS


APPENDIX

Here, we will here study an associated case; where both yield and plastic surfaces for the bonded soil model are equal, therefore the condition for compaction band onset, Eq. (8), is now given by;

\[
\frac{df}{\sigma_x} = \frac{df}{\sigma_y} = \frac{df}{\sigma_z} = \frac{2E}{1-\nu} \frac{df}{\sigma_x} \quad (A1)
\]

where \( E \) is the isotropic elastic Young modulus. On the other hand

\[
\frac{df}{\sigma_x} = \frac{df}{\sigma_y} = \frac{df}{\sigma_z} = \frac{1}{3} \frac{df}{\sigma_\theta} + \frac{df}{\sigma_\phi} \quad (A2)
\]
The yield surface for the bonded soil may now be written and using the Cam-clay elliptical shape;

\[ f = \rho^* \left[ 1 + \hat{h}^{*2} \right] - \sigma_c = 0 \]
\[ \rho^* = \rho + \rho_c \]
\[ \rho_c = \rho + \rho_n + \rho_s \]
\[ \hat{h}^* = \frac{q}{M \rho^*} \] \hspace{1cm} (A3)

The derivatives needed are;

\[ \frac{\partial f}{\partial \rho_c} = -1 \quad \frac{\partial f}{\partial \rho^*} = 1 - \hat{h}^{*2} \quad \frac{\partial f}{\partial q} = \frac{2}{M} \hat{h}^* \] \hspace{1cm} (A4)

Substitution and simplification lead to the following expression;

\[ - \frac{\partial \rho_c}{\partial \varepsilon_n^p} = \frac{2E}{1-\nu} \left( \frac{1}{3} (1 - \hat{h}^{*2}) - \frac{\hat{h}^*}{M} \right)^2 \]
\[ \frac{1}{3} (1 - \hat{h}^{*2}) + \frac{2\hat{h}^*}{M} \]

At this point it should be noted that;

\[ \eta^* = \frac{q}{\rho + \rho_c} = \frac{q}{\rho} = \eta \] \hspace{1cm} (A6)

With the extra hypothesis of \( \xi^* = 0 \) (critical states for the remoulded material) and \( \zeta_m = 1 \), (equal weight of deviatoric and volumetric plastic strain on destructuration) Eq. (6) is obtained.