ANALYSIS OF CRUSHING OF GRANULAR MATERIAL UNDER ISOTROPIC AND BIAXIAL STRESS CONDITIONS

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ABSTRACT

Granular materials forming part of natural slopes, embankments, subgrades of foundations and pavement structures are subjected to both static and dynamic loads during their engineering lives. As a result of these loads, particle crushing occurs. The present study focuses on the evolution of crushing in a simulated granular material subjected to different combinations of biaxial stresses. It was found that the Discrete Element Method (DEM) can be used to visualize and understand the evolution of crushing experienced by a granular material under these conditions. Even though DEM does not normally consider particle breakage, it is possible to simulate crushing by replacing one particle that has failed in tension with a combination of many particles of different sizes. The results from the developed simulations indicated that the samples tended to achieve a fractal distribution of particle sizes, although at the end of the simulations these distributions were still being dominated by the size of the original particles. Changes in other properties of the simulated granular material such as the void ratio and the shear strength were also recorded and analyzed. It was found that the internal friction angle decreased as a result of particle crushing.

Key words: biaxial compression, discrete element method, micro mechanics, particle crushing, strength (IGC: D5/D6)

INTRODUCTION

Granular materials forming part of natural slopes, embankments, subgrades of foundations and pavement structures are subjected to both static and dynamic loads during their engineering lives. As a result of these loads, particle breakage in the form of abrasion and total fragmentation occurs. Particle breakage causes settlements and a reduction of the hydraulic conductivity of the granular material (Vallejo, 2003). Moreover, other engineering properties such as the internal friction angle will change from their original values as crushing evolves. Previous researchers have found that granular materials undergoing crushing exhibit a non-linear failure envelope (Bishop, 1966; Miura and O-hara, 1979; Feda, 2002; Vallejo, 2003). However, recent ring shear tests on carbonate sand presented by Coop et al. (2004) showed that the mobilized angle of shearing resistance does not considerably change regardless of the severe degradation of the tested material. Changes in the original properties of a granular material affect the stability of engineering structures supported on it. Consequently, granular crushing is a potentially detrimental phenomenon that deserves to be carefully examined.

Thus, computer simulations using the Discrete Element Method (DEM) are presented. A simulated uniform material was subjected to isotropic compression in order to study the crushable behavior of granular materials located at great depths. Also, deviator stresses were applied to samples subjected to constant values of confining stresses in order to simulate the crushable behavior of granular materials located near the application of the loads (such as granular materials underneath of footings and piles, or forming part of granular bases in pavement structures).

PREVIOUS WORK

Based on laboratory tests, some general factors have been associated with the occurrence of crushing in granular materials:
—Crushing is directly related to particle strength (Feda, 2002; Hardin, 1985; Lade et al., 1996; McDowell and Bolton, 1998).
—A uniform granular material composed of large particles crushes more readily than one composed of smaller particles of the same material (Lee and Farhoomand, 1967; Hagerty et al., 1993; Hardin, 1985; Lade et al., 1996; McDowell and Bolton, 1998).
—Angular particles crush more easily than rounded particles (Lee and Farhoomand, 1967; Hagerty et al., 1993).
- Uniform soils exhibit more crushing than well graded soils having the same maximum size (Lee and Farhoomand, 1967; Lade et al., 1996).
- Crushing starts under a smaller value of vertical stress in a uniaxial compression test if the granular material is in a loose state rather than in a dense state (Hagerty et al., 1993; McDowell and Bolton, 1998, Nakata et al., 2001b).
- Crushing of a granular material continues with time in the form of creep (Terzaghi and Peck, 1948; Lee and Farhoomand, 1967; Feda, 2002; Lade et al., 1996; Takei et al., 2001; Leung et al., 1996; McDowell and Khan, 2003).
- More crushing is produced during the shearing stage than during the consolidation stage in triaxial tests at high pressures (Bishop, 1966).
- Crushing is accelerated by the addition of water to the crushable material (Lee and Farhoomand, 1967; Miura and Yamanouchi, 1975).

Most of the reported results were obtained using triaxial compression and uniaxial compression machines, mainly on glass beads and sands.

Numerical simulations in the form of the Discrete Element Method (DEM) can be used for the visualization of crushing. Since the original DEM developed by Cundall and Strack (1979) did not consider particle breakage, different solutions have been proposed in overcome this constrain. One solution to this problem is to treat each granular particle as a porous agglomerate built by bonding uniform smaller particles (Jensen et al., 2001; McDowell and Harireche, 2002; Cheng et al., 2003). Another solution is to replace the particles that are fulfilling a predefined failure criterion with an equivalent group of smaller particles (Tsoungui et al., 1999; Lang, 2002). This study uses the second solution and a new simplified tensile failure criterion that can be easily implemented on DEM.

BASES OF THE COMPUTER SIMULATION

The PFC2D program produced by Itasca was used to study the evolution of crushing in granular materials subjected to different configurations of biaxial stresses. In the PFC2D program (based on DEM), particles are idealized as disks that interact with each other at their contacts. This interaction is mainly governed by three models: the stiffness model, the slip model, and the bonding model (Itasca, Theory and Background, 2002). Only the first two models were used in the simulation presented in this paper. The PFC2D program does not consider particle breakage.

Particle Breakage Criterion

Since the PFC2D program does not allow particle breakage, a new subroutine using the FISH language (Itasca, FISH in PFC, 2002) was programmed in order to allow this to happen. The simplified failure criterion adopted during the simulation considers:

- Only particles with a coordination number equal to or smaller than 3 are able to be broken. Particles break more easily when their coordination number is low (Tsoungui et al., 1999; Lade et al., 1996; Nakata et al., 2001a).
- For those particles having a coordination number smaller than or equal to 3, the real loading configuration such as the one presented in Fig. 1(a) is assumed to be equivalent to the one obtained in a diametrical compression test such as the Brazilian test, as shown in Fig. 1(b). In this way, the induced tensile stress, \( \sigma_t \), can be approximated with the expression presented on Fig. 1(b), where \( P_1 \) is the value of the highest contact force acting on the particle, \( L \) is the thickness of the disk (unit thickness for the simulated case), and \( D \) is the diameter of the disk.

- The tensile strength of a particle having a radius of 1 mm is predefined as \( \sigma_{\text{max}1 \text{mm}} = 3 \times 10^6 \text{Pa} \), and it is assumed that the tensile strength of a particle with a radius \( r \), \( \sigma_{\text{max}}(r) \), is related to \( \sigma_{\text{max}1 \text{mm}} \) according to the following relationship (where \( r \) is expressed in mm):

\[
\sigma_{\text{max}}(r) = \sigma_{\text{max}1 \text{mm}} \left( \frac{1}{r} \right)^{-1}
\]

In this way, particles with a radius greater than 1 mm have a tensile strength smaller than \( \sigma_{\text{max}1 \text{mm}} \), and particles with a radius smaller than 1 mm have a tensile strength greater than \( \sigma_{\text{max}1 \text{mm}} \). Particle size is important since many researchers have found that the tensile strength of an individual particle is a function of its size (Billam, 1971; Nakata et al., 2001b).
- Every particle with a coordination number smaller than or equal to 3 is allowed to break if \( \sigma_t > \sigma_{\text{max}}(r) \).
It has been found that the distribution of fragments produced by a tensile failure depends on the nature of the tested material (Nakata et al., 1999; Takei et al., 2001). For the purpose of this simulation it was assumed that if a particle is fulfilling the previously established failure criterion, it is allowed to break into a group of 8 particles, as shown in Fig. 1(c). This distribution of sizes is some how similar to the distribution presented in Fig. 2, which is a picture of a real aggregate broken under diametrical compression (point load test).

In order to implement the failure criterion, a new subroutine was programmed using the FISH language. This subroutine automatically checks if a given particle is fulfilling the failure criterion. If it does, the particle that broke is deleted and replaced by the set of particles shown in Fig. 1(c). The subroutine does not restrict smaller particles from continuing to break.

### Configuration of the Samples and Sequence of the Simulated Biaxial Tests

The biaxial box had originally a width of 5 cm and a height of 13 cm. The horizontal walls had a total length of 7 cm, so the biaxial box could expand horizontally. The shear and normal stiffnesses of the walls forming the box were set to $1 \times 10^9$ N/m. These walls were assumed to be frictionless. The vertical walls were allowed to move horizontally in opposite directions in order to control the applied horizontal stress. The upper horizontal wall was allowed to move vertically in order to control the applied vertical stress. In this way, the applied principal stresses were controlled using a FISH servo-mechanism programmed by the authors. Both vertical and horizontal stresses were calculated as the average between the applied stresses on the two opposite walls. It should be noted that in the PFC$^{2D}$ software, walls forming a box...
can be designed so they do not interact (Itasca, Theory and Background, 2002). For example in the biaxial test the movement of the upper horizontal wall is not influenced by the movement of the two vertical walls. Thus, they can overlap with no interaction.

After the construction of the box, 165 circular particles having a uniform radius of 3 mm were generated inside. Their positions were randomly chosen by the program, having the limitation of no overlaps between particles. The density of the particles was set to 2500 kg/m³, their normal and shear stiffnesses were set to $1 \times 10^8$ N/m, and their friction coefficient was set to 0.7. A gravity field of 9.8 m/s² was used during the simulations. The generated particles were allowed to fall and accommodate inside the box, thus the initial height of the samples was less than 13 cm.

The simulation started subjecting the sample to an isotropic compression up to a confining pressure equal to $18 \times 10^5$ Pa. Three copies of this original sample were saved at different values of confining pressure ($\sigma_3 = 1 \times 10^5$ Pa, $5 \times 10^5$ Pa, and $10 \times 10^5$ Pa). Then, deviator stresses were applied to the three copies while the correspondent $\sigma_3$ in each case was kept constant. The deviator stresses were induced by moving downward the upper wall of the box with a constant velocity of $1 \times 10^{-7}$ m/step. Granular crushing developed as a result of the isotropic compression and the application of the deviator stresses.
RESULTS OF THE SIMULATIONS

Isotropic Compression

Figure 3 shows the void ratio of the upper and lower regions of the sample at different values of applied confining pressure. Also, some snap shots of the sample at different confining pressures are presented in this figure. These snap shots show that crushing started at a confining pressure between $5 \times 10^5$ Pa and $6 \times 10^5$ Pa. It can be seen how a sudden change in the trend of the void ratio curves also took place at these stresses, and the sample exhibited a well defined normal compression line after this. At low values of isotropic confining pressure, the upper and lower parts of the sample had different values of void ratio, but after crushing this difference was considerably reduced.

The snap shots presented in Fig. 3 show that most of the crushing took place at the boundaries of the sample. This was expected since these particles were transmitting large forces and their coordination numbers were small due to the imposed geometrical constrains. The color of the particles in Fig. 3 corresponds with the generation of crushing that they represent. It was found that no more than a third generation of crushing was produced. This is due to the fact that the tensile strength of the particles increased with a reduction on their size.

Shearing at $\sigma_3 = 1 \times 10^5$ Pa

A copy of the original sample subjected to the isotropic compression was recorded at a confining pressure of $1 \times 10^5$ Pa. It can be noted from Fig. 3 that the sample at this stage was far from experiencing crushing. After this, a deviator stress was induced on the sample while $\sigma_3$ remained constant. The values of the applied deviator stress and the induced vertical and volumetrical strains were recorded. The test was ended at a vertical strain of 0.3. It was observed that the volume of the sample did not significantly change during the test. Thus, the value of volumetric strain was $-0.0116$ at the end of the simulation (negative sign represent a decrement in volume). No crushing was produced during this simulation. The detailed results of this test are not presented in this paper since they are not relevant for the analysis.

Shearing at $\sigma_3 = 5 \times 10^5$ Pa

A copy of the original sample subjected to the isotropic compression was recorded at a confining pressure of $5 \times 10^5$ Pa. As it was mentioned before, this confining pressure was very close to the yield stress that caused the beginning of crushing. A deviator stress was induced while $\sigma_3$ remained constant. Figure 4 shows the obtained values of deviator stress and the correspondent vertical strains. Also, some snap shots of the sample at different values of vertical strain are presented with their correspondent values of volumetric strain. The deviator stress vs. vertical strain curves present spike drops as a consequence of particle crushing and particle rearrangement. It is interesting that when crushing started, the sample momentarily experienced a small negative value of deviator stress since $\sigma_1$ was smaller than $\sigma_3$. Figure 4 shows that crushing started at a small value of vertical strain, and continued until the end of the test. Even though some of the initial crushing took place at the boundaries of the sample, it did not concentrate in these regions. Moreover, by comparing Figs. 3 and 4 it can be observed that the regions where most of the crushing took place were not exactly the same. No particles representing more than a third generation of crushing were generated during this simulation.

Analyzing the values of volumetric strain in Fig. 4, it can be inferred that the sample presented a contractive behavior due to the generated crushing. Figure 5 shows some details of the sample close to the peak deviator stress and at the end of the test. It can be seen in these
snap shots that the fragments generated as the product of particle crushing reorganize in an efficient way, so it is logical that the volume of the sample was significantly reduced as a consequence of crushing. On the other hand, the last three snap shots on Fig. 4 show that after a vertical strain of 0.28, the sample experienced a very small amount of dilation. This was a consequence of the densification of the simulated material.

The snap shots presented in Figs. 4 and 5 show how the simulated group of particles tended to evolve from a uniform granular material into a well graded mixture of sizes. As presented by Turcotte (1986), the distribution of sizes generated by fragmentation is fractal in nature, and can be described with the following power law:

$$N(R > r) = k(r)^{-D_f}$$

Where $N(R > r)$ is the number of particles with radius $R$ greater than a given value $r$, $k$ is a proportionality constant and $D_f$ is the fragmentation fractal dimension. $D_f$ can be used as a measure of particle size distribution. Well graded mixtures containing different sizes have higher values of $D_f$, while uniform size mixtures dominated by particles having the original size exhibit lower values. A maximum value of 1.5 (two dimensional case) is
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reported by Turcotte as the maximum $D_f$ empirically found in materials subjected to impulse loads.

Figure 6 shows the particle size distribution of the sample as a function of the induced vertical strain. It also includes the power regression line for the distribution obtained at the end of the simulation. The higher value obtained for the square of the correlation coefficient, $R^2$, in the power regression shows that the distribution obtained as a result of crushing is fractal in nature, as suggested by other authors (Turcotte, 1986; Bolton and McDowell, 1997; McDowell and Bolton, 1998; Coop et al., 2004). However, it can be noted that the fragmentation fractal dimension obtained from this power regression is equal to 1. This means that even though the sample is evolving into a well distributed mixture of sizes, it is still being dominated by the original particles, as can be seen in the last of the snapshots.

Figure 6 also shows that the granular crushing took place between a vertical strain of 0.28 and 0.4. As it was pointed out before from the volumetric strain results, the sample experienced a small amount of dilation during these values of vertical strain. Thus, the dilation due to the produced densification of the sample suppressed the contractive behavior due to the particle breakage.

Shearing at $\sigma_3 = 10 \times 10^5$ Pa

A copy of the original sample subjected to the initial isotropic compression was recorded at a confining pressure of $10 \times 10^5$ Pa. As it can be seen from Fig. 3, the sample presented some crushing as a result of this isotropic compression. A deviator stress was induced on the sample while $\sigma_3$ remained constant. The values of the applied deviator stress and the induced vertical and volumetrical strains are presented in Fig. 7. Snap shots of the sample at different values of vertical strain are also presented in this figure. Additionally, some details of the sample at vertical strains of 0.20 and 0.42 are provided in Fig. 8.

It can be observed that a significant amount of particle breakage was produced due to the induced deviator stress. Particles representing a fourth generation of crushing were produced due to the severe crushing. Opposite to the sample sheared at $\sigma_3 = 5 \times 10^5$ Pa, this sample exhibited a contractive behavior during all the simulation. This was expected since the confining pressure was significantly higher than the one previously used.

The grain size distribution of the sample as a function of the induced vertical strain is presented in Fig. 9. A power regression line was fitted to the grain size distribution of the sample at the end of the test. The higher value obtained for the square of the correlation, $R^2$, shows that the obtained distribution was fractal in nature. The obtained fragmentation fractal dimension at the end of the test was equal to 1.169. This value is higher than the one obtained after shearing the sample subjected to $\sigma_3 = 5 \times 10^5$ Pa. This shows that more crushing was produced during the test with $\sigma_3 = 10 \times 10^5$ Pa than during the test with $\sigma_3 = 5 \times 10^5$ Pa. Nevertheless, as it can be observed in Figs. 7 and 8 the sample was still being dominated by the original large particles. Thus it was expected that the obtained fragmentation fractal dimension was smaller compared to the maximum value of 1.5 reported in the literature.

Change in the Internal Friction Angle

The Mohr circles were plotted taking the peak values of deviator stress at each confining pressure. After this, a Mohr-Coulomb failure envelope was calculated independently for each circle. Figure 10 shows the obtained three circles, failure envelopes, and values for the internal friction angle $\phi$. It can be seen that the internal friction angle was considerable reduced due to the generated particle crushing (from $27^\circ$ to $20.5^\circ$). Joining the failure
These will be the first to fail affecting the overall deformation of the granular structure and its progressive failure.

The simulations do not consider the influence of weak particles. Nevertheless, the authors consider that the assumption associated with the simulations provides an adequate representation of the evolution of crushing in granular materials.

CONCLUSIONS

The evolution of crushing in a granular material subjected to different combinations of biaxial stresses was simulated using DEM. In order to allow particle breakage, a simplified failure criterion was implemented in a FISH subroutine that interacted with the course of the simulations. This failure criterion was based on the magnitude of the loads applied to the particle, the particle coordination number, and the particle size. Particles fulfilling the failure criterion were deleted and replaced by a group of new particles following certain rules that agree with experimental observations.

The results of the isotropic compression showed that the simulated material resembled well the behavior of a real crushable material, presenting a yield stress and a well-defined normal compression line. The tests with shearing stages at constant values of confining pressure showed that crushing evolved differently in the isotropic confining test and these tests. Not only more crushing was produced during the tests with deviator stresses, but also the regions where particle breakage concentrated were different. It was found that the samples at the end of both biaxial tests at $\sigma_3 = 5 \times 10^5$ Pa and $\sigma_3 = 10 \times 10^5$ Pa had a fractal grain size distribution dominated by large original particles. More particle crushing was found at the end of the test at $\sigma_3 = 10 \times 10^5$ Pa than at the end of the test at $\sigma_3 = 5 \times 10^5$. Finally, it was found that the internal friction angle of the simulated material was reduced as a consequence of granular crushing.

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REFERENCES


DISCUSSION OF THE RESULTS OF THE SIMULATIONS

The simulations presented in the study only consider circular particles. As it was mentioned before, angular particles break more easily than rounded particles because of stress concentrations at their angularities. Therefore, real samples containing angular particles could experience more crushing than the samples analyzed by the simulations. However, the authors believe that adopted simplifications are justified since the results presented in this study were developed in order to understand and visualize the evolution of crushing in a simplified way.

The proposed failure criterion used in the simulations considers that the strength variations in the idealized granular material depend only on the size of the particles. This is established by Eq. (1). The simulations do not consider variations in strength of the material making the particles even though they could be of the same size. If the assembly has a limited number of weak particles, these will be the first to fail affecting the overall deformation of the granular structure and its progressive failure.