Collision and Rebound of Spherical Particles in Liquid

Motivation and Summary. Our group is interested in understanding and predicting “bedload” transport of sediment by a turbulent flow, whereby particles slide, roll, or jump along a sediment bed, a process which shapes rivers and the seacoast, and strongly influences flooding phenomena. Despite its importance, bedload transport has resisted detailed understanding because of its extreme complexity, which includes turbulent boundary flow, particle-fluid interaction, and interparticle tribology. The present work considers the latter aspect. In particular, we focus on the coefficient of restitution between liquid-immersed particles, which is observed to vanish when the Stokes number rises above a certain threshold (see Fig. 2 below). This coefficient influences dissipation of mean-flow energy, and thus figures prominently in theories of granular flow and of bedload sediment transport.

Why this threshold behavior? Assuming perfectly smooth, rigid spheres in incompressible, constant property fluid, classical lubrication theory for normal approach yields a viscous resistance force $F = -6\pi \mu R^2 v/h$, where $R$ is the reduced radius of the two spheres, $v$ is relative velocity, and $h$ is the minimal gap between particle surfaces. This yields the following trajectories in the phase plane (solid arrows in Fig. 1):

$$-St = \ln(x/R) + St^I$$

where $St$ is the instantaneous Stokes number $mv/6\pi \mu R^2$, and $St^I$ is a constant of integration that Wells (1993) refers to as the “impact Stokes number”. By these assumptions, the spheres always stop; increasing $St^I$ only makes the final gap smaller. Barnocky and Davis (1988; hereinafter “BD”) modelled the “blocking” effect of surface roughness as follows; once particles touch at asperity tops, the dissipative action of the fluid is neglected until the particles separate again, after which equation (1) again holds but with the sign of $v$ reversed. Evaluating the trajectory from an initial gap $x_o$ through impact and back to $x_o$, the total coefficient of restitution, which accounts for both viscous and solid-solid contact dissipation, is found to be:

$$e_{tot} = e_{dry} + \frac{1 + e_{dry}}{St^I} \ln \frac{x}{x_o}$$

in which $e_{dry}$ is the coefficient of restitution for dry impact. Thus surface roughness, by blocking particle approach and promoting conversion of kinetic energy to elastic internal energy, is predicted to strengthen rebound. From Fig. 1, it is apparent that higher incoming velocity will, above a certain threshold, yield stronger rebound.

This paper presents our experiments on particle rebound from a plate in water, which is analogous to collision of two particles. We extend the domain of validation of BD’s result to finer roughness, and investigate additionally the practically important effect of energy dissipation at asperities. The conclusion considers implications for Particle Dynamics Simulations by “fictitious domain” techniques.

Experiment. Steel or glass plates used as target walls were placed on the bottom of a water tank. Glass beads with various diameters $d$ were dropped on the target walls; almost no rotation observed when the particles were falling. A Pulnix CCD camera was synchronized with a strobe lamp to record 648x484 pixel$^2$ images at 120 Hertz.

The diameter of the glass beads were from 1.05mm to 2.35mm and the density indicated by the manufacturer is 2500kg/m$^3$. The glass beads were selected for sphericity so that the difference between the largest and smallest diameter of a given particle was less than 5% of average diameter. The surfaces of the beads were also checked with a microscope to ensure that no crack existed.

Target glass plates were of three types: smooth, sandpapered, and rough with molded hemisphere-like asperities. Corresponding steel target plates were polished, sandblasted, and grooved with a scraper. Values of their surface roughness, measured with the Surftest 211 (Mitutoyo, São Paulo, Brazil) equipment, are presented in Table 1. The surface roughness of the beads was seen from micrographs to be comparable to that of the smooth glass plate.

![Fig. 1. Phase trajectories from lubrication solution (solid lines), and BD extension thereto (dashed). Vertical grey lines indicate two hypothetical heights of asperity tops; ratio of incoming to outgoing velocity determined by $e_{dry}$ (shallower slope of green curve corresponds to lower $e_{dry}$). Experiments at resolution $x_{m}$ would miss weak rebound from lower roughness.](image-url)
To examine the effects of surface roughness on energy dissipation during solid-solid contact, impact trials were conducted in air (open symbols in Fig. 2). For the glass target, there is no significant difference between the average coefficients of restitution for the three surfaces, although more variance is observed for the rough surfaces; it appears that surface roughness does not affect the amount of energy dissipation in the glass during impact. By contrast, the corresponding data from the stainless steel plates yielded significantly weaker rebounds from the roughened surfaces; we believe this to be due to plastic deformation of the asperities. In fact, the grooved surface, which was nominally rougher than the sandblasted surface, gave somewhat stronger rebound than the sandblasted, presumably because the summit heights were less variable.

The effect of roughness on rebound strength in water were different for glass and steel. For glass, rougher target plates yielded stronger rebound, and the amount of increase agrees rather well with the prediction of equation (2). For steel targets, the rougher surfaces have a lower value of $e_{\text{dry}}$, and this outweighs the blocking effect expressed by the second term of equation (2), so that the rebound from the rough surfaces are now weaker. Again, the differences predicted to occur by equation (2) are in reasonable agreement with the data.

**Conclusion.** The BD model appears to satisfactorily predict the global effect of roughness on coefficient of restitution for rebound from an elastic material (glass) and an elastic-plastic material (stainless-steel). Future work should consider the variance of rebound velocity. In addition, effort is still necessary to bridge the rather small scales considered in the BD and related theories with those observed in typical experiments. In the near future, we will tackle these issues in the context of contact modeling for Particle Dynamics Simulations by “fictitious domain” techniques, for which we will require a force model that is able to recover the observed coefficients of restitution.

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**References**


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