Large Eddy Simulation of Some Benchmark Turbulent Flows Using “FrontFlow/Blue”

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1. INTRODUCTION

With the development of the technique of numerical simulation, Large Eddy Simulation (LES) has begun to be applied to industrial flows, such as flow in a turbomachine\(^1\). However, before a LES code can be used in industrial flows, a set of benchmark tests should be performed to evaluate the performance of the code. Homogeneous isotropic turbulence and fully developed plane channel flow are two typical cases of the benchmark flows. Direct Numerical Simulation (DNS) data\(^2,3\) of the two flows are available, which makes it convenient to evaluate the results of LES. The objective of the current computation is to validate the performance and the accuracy of our developing LES code “FrontFlow/Blue”. “FrontFlow/Blue” is a general-purpose finite element program that calculates incompressible unsteady flows in arbitrarily-shaped geometries. It was primarily designed for predicting unsteady flows in turbomachinery and simulating sound pressure spectrum. The main target of this study is to check the accuracy of the energy spectrum and the turbulence statistics for the benchmark turbulent flows using this code.

2. GOVERNING EQUATION

The governing equations of LES for incompressible flow are spatially filtered continuity equation and Navier-Stokes equations:

\[
\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial \overline{\rho} \overline{u}_i}{\partial x_i} = 0 \tag{1}
\]

\[
\frac{\partial \overline{\rho} \overline{u}_i}{\partial t} + \frac{\partial \overline{\rho} \overline{u}_i \overline{u}_j}{\partial x_j} = -\frac{1}{\overline{\rho}} \frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \frac{\partial \overline{\rho} \overline{u}_i}{\partial x_j} + \frac{\partial \overline{\tau}_{ij}}{\partial x_j} \right] \tag{2}
\]

where \(\overline{\rho}\) is the grid scale velocity component, \(\overline{\rho}\) is the density and \(\nu\) is the kinematic viscosity.

In Eq. (2), the subgrid-scale (SGS) Reynolds stress tensor \(\overline{\tau}_{ij}\) is defined by:

\[
\overline{\tau}_{ij} = \overline{\rho} \overline{u}_{i} \overline{u}_{j} - \overline{\rho} \overline{u}_{i} \overline{u}_{j} \tag{3}
\]

The Smagorinsky closure\(^4\) is applied to the SGS stress \(\overline{\tau}_{ij}\):

\[
\overline{\tau}_{ij} = \frac{1}{3} \overline{\rho} \overline{\rho} \overline{u}_{i} \overline{u}_{j} \overline{S}_{ij} \tag{4}
\]

where

\[
\overline{\rho} \overline{\rho} \overline{u}_{i} \overline{u}_{j} \overline{S}_{ij} = \frac{1}{2} \left[ \frac{\partial \overline{\rho} \overline{u}_{i}}{\partial x_j} + \frac{\partial \overline{\rho} \overline{u}_{j}}{\partial x_i} \right] \tag{5}
\]

The quantity \(\overline{C_s}\) is the Smagorinsky coefficient and \(\Delta\) is the size of the grid filter. In the study, Dynamic Smagorinsky model (DSM)\(^5\) with modification by Lilly\(^6\) is used, where \(\overline{C_s}\) is determined locally in time and space. It should be noted that we use DSM to compute the SGS stress in all the following computations of LES.

3. NUMERICAL METHODS

Both an explicit time-accurate streamline upwind scheme\(^7\) and an implicit Crank-Nicolson (CN) scheme have been implemented in “FrontFlow/Blue” for integrating the momentum equations with respect to time. For both schemes, the spatial discretization is based on the Finite Element Method (FEM). The implemented algorithms have second order accuracy both in time and space and we believe that the second order scheme is most appropriate for use in engineering applications of LES. However, the accuracy of the second order scheme should be checked in benchmark problems before it can be applied to engineering problems. Both Arbitrary Boundary Marker and Cell (ABMAC)\(^8\) method and Fractional Step (FS) method are employed to solve the pressure equation. Bi-CGSTAB\(^9\) method is used as the matrix solver of the global linear equations that result from the FS method as well as the CN scheme.

4. RESULTS OF BENCHMARK TURBULENT FLOWS

4.1 Results of Homogenous Isotropic Turbulence

At first homogenous isotropic turbulence of \(Re = 121.1\) (based on r.m.s. velocity and the length of the computational domain) was computed. The computational domain is a \((2\pi)^3\) box and the num-
The number of grid nodes is 64^3. The dimensionless time increment is set to \( \Delta t = 0.00316 \). Uddin et al.\(^{10}\) have computed the flow. Their conclusions are that the agreement between LES results and DNS database of Tanahashi et al.\(^2\) is good and the results using DSM are the best among all the LES results. In this paper, we continue the work by confining our attention to the effects of the numerical conditions including the pressure algorithm and the time increment.

Figure 1 shows the effect of two pressure algorithms: ABMAC method and FS method. Here the temporal algorithm is fixed as the explicit time-accurate streamline upwind scheme. The maximum divergence of the velocity using FS method is much smaller than that of ABMAC method because of the higher convergence rate of FS method. However, the pressure algorithm has little effect on the result of the energy spectrum at \( t = 3.792 \) as shown in Fig. 2. The reason is that this flow is free turbulence, thus the pressure does not play an explicit role on the energy spectrum.

![Fig. 1 Maximum divergence of the velocity field in homogenous isotropic turbulence of Re = 121.1.](image1)

![Fig. 2 The effect of the pressure algorithm on the energy spectrum in homogenous isotropic turbulence of Re = 121.1.](image2)

We also calculated the higher Reynolds number case of with different grids: 128^3, 64^3 and 32^3, where \( \Delta t \) is set to 0.0015. The same conclusion can be drawn in this case from the energy spectrum at \( t = 2.55 \), which is shown in Fig. 3.

To investigate the effect of the time increment, we calculated homogenous isotropic turbulence of Re = 121.1 using \( \Delta t = 0.00316, \Delta t = 0.00632, \Delta t = 0.0158, \Delta t = 0.0632, \Delta t = 0.158 \) and \( \Delta t = 0.316 \). The corresponding CFL numbers are 0.2, 0.4, 1.0, 4.0, 10, and 20, where the CFL number is defined as \( \text{CFL} = \max \left( \frac{|u|}{\Delta x}, \frac{|v|}{\Delta y}, \frac{|w|}{\Delta z} \right) \). The pressure algorithm is fixed as FS method and we use the implicit CN scheme as the temporal algorithm. Figure 4 presents the energy spectrum of different time increment. It can be concluded that the agreement of LES and DNS data\(^2\) is quite good with CFL less than 10. From CFL = 10, the energy at the low wave number range decreases and the energy at the high wave number range increases. Figure 5

![Fig. 3 The effect of the pressure algorithm on the energy spectrum in homogenous isotropic turbulence of Re = 304.9.](image3)

![Fig. 4 The effect of the time increment on the energy spectrum in homogenous isotropic turbulence of Re = 121.1.](image4)
shows the effect of the time increment on the vortical structure in homogenous isotropic turbulence. The vortical structure is visual-
ized by iso-surfaces of the second invariant $Q$ of the velocity
gradient, which is defined as $Q = -\frac{1}{2}(s_i s_j - w_i w_j)$, where $s_i$, $w_j$ are the symmetric and antisymmetric parts of the velocity gradient
tensor. From Fig. 5, the difference of vortical structures between
CFL = 0.2 and CFL = 4.0 is small and both of them are similar to
that of DNS$^2$. For CFL = 10, some large vortices disappear and
change into small vortices. The phenomenon is consistent with the
energy spectrum of the vortices. From DNS data$^3$, we can know
that the physical Kolmogorov time scale is $\tau_k = \left(\frac{\nu}{\varepsilon}\right)^{\frac{1}{2}} \approx 0.1$, where $\varepsilon$ is the dissipation rate of the turbulence energy. In Fig. 4
and Fig. 5 it is shown that if the time increment is less than the
Kolmogorov time scale, the result of LES is reasonable. This con-
clusion is similar to that of Choi and Moin$^{11}$ in fully developed
channel flow. It is implied that we can select the time increment of
LES according to the Kolmogorov time scale, so that the CFL num-
cr can be larger than 1, which results in the reduction of the total
computational cost.

4.2 Results of Fully Developed Plane Channel Flow
Fully developed plane channel flow at $Re = 180$ (based on fric-
tion velocity $U_r$ and channel half-width $h$) was computed using
“FrontFlow/Blue”. The computational domain is $2\pi h \times 2h \times \pi h$
and the dimensionless time increment is set to $\Delta t = 5 \times 10^{-4}$.

Figure 6 shows the mean velocity profile using different grid
resolution and pressure algorithm. The LES results are compared
with DNS data of Kim et al.$^3$. The effect of the pressure algorithm
is apparent in the channel flow. When ABMAC method is used, the
mean velocity is overpredicted in the log-law region as compared
with the result of FS method using the same $128^3$ grid.

Furthermore, we fixed the pressure algorithm as FS method and
compared the results of three different grids: one is $128^3$ grid with
minimum wall distance $y^+_\text{min} = 1$, the second is $64^3$ grid with $y^+_\text{min} = 1$ and the third is $128^3$ grid with minimum wall distance $y^+_\text{min} = 0.05$. The implicit CN scheme is used as the temporal algorithm in
the third grid while the explicit time-accurate streamline upwind
scheme is used in all the other grids. Results show that only with
$128^3$ grid and fine resolution in near-wall region can we obtain a
A quite good result that is very close to DNS data. The reason is that if the grid resolution is not enough or the convergence rate of the pressure equation is slow, the wall reflection effect of the pressure cannot be predicted accurately. As a result, the turbulence intensity in the streamwise direction is overpredicted and the turbulence intensities in the other two directions are underpredicted (see Figs. 7–9). The underpredicted wall-normal turbulence intensity results in the overprediction of the mean velocity in the log-law region.

Figure 10 shows iso-surfaces of the second invariant $Q$ in the channel flow (LES, FS, 128$^3$ grid, $y^+ = 0.05$).
5. CONCLUSIONS

Large eddy simulations of homogeneous isotropic turbulence and fully developed plane channel flow were performed to validate the performance of our LES code “FrontFlow/Blue”. Results show that the agreement of LES results and DNS data is quite good if appropriate numerical parameters are selected. The effect of the pressure algorithm is very small in the homogeneous isotropic turbulence while its effect is apparent in the channel flow. The results of the homogeneous isotropic turbulence show that the time increment should be set below some critical values that seem to be determined by the physical time scale rather than the CFL condition. The results of the channel flow demonstrate that both the pressure algorithm and the grid resolution play important roles in the computation.

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