1. Introduction

Sag sections are the major basic segment bottlenecks on Japanese expressways. A sag section is a road segment on an expressway in which the vertical slope increases at a small but constant rate. Because the rate of slope increase is slight and constant, vehicle speeds are significantly reduced before drivers become aware of the change in slope. According to a research report by the Japanese National Institute for Land and Infrastructure Management, 60.5% of congestion incidents on Japanese expressways in 2011 occurred on uphill slope and sag sections and caused congestion amount of 105,649 km/h. Traffic managers find an urgent need to monitor and predict the occurrence of congestion, which requires systematic observation on the bottleneck phenomena at sag sections and analysis of its mechanism. The first systematic observation was made by Koshi et al. Major features of traffic flow condition at sag section bottleneck were concluded. The bottleneck capacity would drop to around 2,200~2,700 veh/h/2lane, much lower than the theoretical capacity when the traffic demand reached to around 3,000 veh/h/2lane. Rapid speed reduction was observed at the start of bottleneck activation. On the other hand, as observed at many expressway bottlenecks, traffic capacity is a time-dependent feature and its definition remains not fully solved. Congestion also occurs at a lower flow rate after a constant high flow passing the bottleneck. In other words, congestion occurs at same expressway bottlenecks under different flow rate.

After observing traffic flow condition at sag section for several years, researchers realize that the congestion is formed with an aggregation of individual car-following behavior. Xing and Koshi studied the car-following behavior of drivers at the start of congestion with aerial image data. They modeled car-following behavior for every individual driver with trajectory data. They found that 75% of drivers in a platoon decelerate intensely while their spacing with the leading vehicle decreases slightly. This finding suggests certain kind of car-following behavior maybe a possible explanation for congestion occurrence.
at sag sections. It also suggests that variability in the behavior of drivers may be the main cause of the probabilistic occurrence of congestion and the lack of a certain observable bottleneck capacity.

A simple understanding of congestion formation because of car-following behavior at sag section is: Cars first decelerate because of the vertical grade. When drivers noted the speed drop, they try to restore the speed by acceleration. A series of small adjustment follows after the unnoticed deceleration and intended acceleration. Drivers naturally tend to keep a longer spacing with leading car in order to have enough spacing for speed adjustment. Bottleneck is activated after the accumulation of speed adjustment of following drivers in the platoon passing the sag section and causes congestion occurrence.

The difference in individual car-following behavior not only contains the difference of drivers' reaction to driving condition change of leading vehicle, e.g. acceleration and deceleration, but also the difference of drivers' reaction to vertical slope change. Some drivers are not affected by the unnoticed deceleration caused by vertical slope change as much as other drivers. By analyzing the difference of individual car-following behavior, it is possible to reveal the mechanism of the probabilistic congestion occurrence at sag section.

Car-following behavior of every single driver during congestion formation with observation data from a famous sag section in central Japan is modeled in this research. A car-following model considering the vertical grade effect is utilized and calibrated for this purpose. Distributions of parameter values reflect the difference in individual car-following behavior at sag section.

2. Data

Two datasets were obtained through two observations carried out at the outer lane on a two lane basic segment in Tomei expressway in Japan. The Tomei Expressway connects Tokyo and Nagoya, which are the two major metropolitan areas in Japan. Connecting with Meishin Expressway, it is on the most important artery with the heaviest traffic volume in Japan. A total of 875 cars were observed.

Dataset 1 is a trajectory dataset obtained by Muto and Akahane from a 1.2-km section of roadway, including a sag section called Yamato sag, on the Tomei Expressway during congestion formation. The trajectories were processed through several fixed-site videos along the observation area using video recognition technology. Position, speed, and acceleration data were obtained from the trajectories for all vehicles within the observation area, every 1/30s and were properly smoothed for analysis. A total of 393 trajectories were observed. System errors occur in the video recognition, capture, and data smoothing processes. In addition, human driving behavior exhibits randomness. This is especially noticeable when there is no specific stimulus to drivers to change their driving status. Therefore, a study area for follow-the-leader behavior modeling was further extracted from the overall trajectory to focus on car-following behavior at a sag section as well as minimizing the errors that occurred in the video recognition, capture, and data smoothing processes. The study area in the dataset is a 500–650m area.

Dataset 2 is a dataset of 482 trajectories during congestion formation at the same location obtained by NILIM. The observation was carried out on weekends during Nov. 2010 to Aug. 2011. The data consisted of distance for every 0.1s and speed and acceleration for every 1s. The speed was filled with linear interpolation for every 0.1s. The study area in the dataset is a 1000–1200m area.

There were a few vehicles changing lane to overpass the preceding vehicle during observation. The spacing of vehicle decreases severely while speed increases when vehicle change lane to overpass the preceding vehicle. The lane-changing behaviors are thus recognized from dataset and are excluded from analysis as error data. Two kinds of vehicles that are not under car-following state are also excluded from analysis. One is vehicles under independent driving state, which are far enough from their preceding vehicles that their driving behavior is not affected by the behavior of preceding vehicles. Highway Capacity Manual suggests that vehicles travelling with a headway smaller than 3s is considered as following vehicles. Pasanen and Salmivaara also suggested 3s be the threshold for following vehicles and non following vehicles. Vogel found that the time headway for urban roads is around 6s and the spacing is 50-70m. Vehicles having spacing bigger than 80m or time headway bigger than 3s with preceding vehicle are regarded as vehicles driving independently and eliminated from car-following behavior analysis.

The other is vehicles already involved in congestion. Vehicles already involved in congestion are forced to maintain a low speed equal to its preceding vehicle and a close spacing. Instead of following the leading vehicle and pursuing desired spacing, the following vehicle is simply force to maintain the same congestion speed as the leading vehicle in the congestion. Considering the research objective is the car-following behavior of vehicles during free flow to the formation of congestion, this kind of vehicles involved in congestion is eliminated from analysis. There are many definitions of congestion occurrence. A simple and practical definition of congestion occurrence at sag section is two adjacent vehicles...
speed smaller than 40km/h at the same time. This threshold is used in this research. As long as there are two adjacent vehicles' speed smaller than 40km/h, the later vehicles are regarded as involved in the congestion and eliminated from car-following behavior analysis.

3. Car-following behavior modeling

In this research, a model first proposed by Oguchi and Konuma\(^{12}\) that has a vertical force effect component added to Helly’s model\(^{13}\) as shown in function (1) is used.

\[
a_1(t + \Delta t) = \begin{cases} 
    a_1(v_0(t) - v_1(t)) + a_2(x_0(t) - x_1(t) - s_1^*) & t < t_0 \\
    a_1(v_0(t) - v_1(t)) + a_2(x_0(t) - x_1(t) - s_1^*) - \beta g[\sin \theta(t) - \sin \theta_*] & t > t_0
\end{cases}
\]

\[s_1^* = \delta + r v_1(t)\]  \hspace{1cm} (2)

where

- \(\Delta t\): Driver’s reaction time
- \(a_1(t)\): Acceleration of following vehicle
- \(v_0(t)\): Speed of leading vehicle
- \(v_1(t)\): Speed of following vehicle
- \(x_0(t)\): Position of leading vehicle
- \(x_1(t)\): Position of following vehicle
- \(s_1^*\): Desired spacing of following vehicle
- \(g\): Gravity acceleration
- \(\theta(t)\): Vehicle slope
- \(\theta_*\): Initial vertical slope before sag section
- \(\alpha_v, \alpha_x, \beta\): Coefficients
- \(\delta\): Standstill spacing
- \(r\): Spacing change with speed change
- \(t_*\): Time of entering the sag section

There are two reasons for using Helly’s model as the basic model. One is that the physical relationships of acceleration, speed difference and spacing represent the actual experience in observation well. The other reason is that a simple linear model with limited numbers of variables will make estimation and analysis of variables easier as well as manifest clear physical relationships in parameters distribution and simulation.

The parameters of driver showed significant difference at different vertical gradient sections. Oguchi and Konuma\(^{12}\) used the Helly’s model among other classic car-following models to analyze the car-following behavior at sag sections in Japan with experiment data. Helly’s model showed good fitness in these researches. Sag effect parameter \(\beta\) reflects the unnoticed deceleration caused by the vertical change. The inter-driver variations are described by the different values of parameters of different drivers.

The desired spacing and reaction time range can be directly obtained from observed data. On the other hand, \(\alpha_v, \alpha_x, \beta\) and the exact value of \(\Delta t\) is unobservable and therefore obtained with cross-entropy method. The ranges of the parameter values were set as follows: \(\Delta t \in [0,5]\) s, \(\alpha_1 \in [0,1]\), \(\alpha_2 \in [0,0.3]\), \(\beta \in [0,1]\), \(\delta \in [0,100]\), and \(r \in [0,5]\). The analysis of driving behavior variability is carried out with 3 steps\(^{14}\).

1) The desired speed \(s_1^*\) is estimated with regression of vehicle’s speed spacing relationships under steady driving state \((a_1(t) < 0.01g)\). The approaches taken in desired spacing estimation are (an example is shown in Figure 1):

1a) Several steady states exist, desired spacing function parameters are estimated by Tobit model with maximum likelihood estimator.

1b) Only one steady state exists, \(r=0\), and \(\delta\) is the mean spacing of the steady state.

1c) No steady state, use the desired spacing parameters for all the trajectories: \(s_1^* = 5 + 1.2078 v_1(t)\).

2) The range of reaction time \(\Delta t\) is estimated with correlation analysis within 0~5 seconds (an example is shown in Figure 2).

2a) The maximum correlation of acceleration with relative speed > 0.8, the range of reaction time with correlation of acceleration with relative speed > 0.8 is taken as the range of \(\Delta t\).

2b) The maximum correlation of acceleration with relative speed < 0.8, and the maximum correlation of acceleration with spacing difference > 0.8, the range of reaction time with correlation of acceleration with spacing difference > 0.8 is taken as the range of \(\Delta t\).

2c) Both the maximum correlation of acceleration with relative speed and spacing difference < 0.8, the reaction time is to be searched by cross-entropy method within \(r \in [0,5]\) s.

3) The other parameters are estimated with cross-entropy method, where objective function is the relative percentage

![Figure 1: Desired spacing estimation with individual trajectory (several stable states)](image-url)
error of spacing and 3 performance indicators are also studied.

A heuristic search method called cross-entropy method is used to search for the best combination of $\alpha_1, \alpha_2, \Delta t$ and $\beta$ fitted to the trajectory. It is based on Kullback-Leiber cross-entropy, importance sampling, Markov chain and Boltzmann distribution. The core concept in this method is to adaptively adjust the occurrence of the events more likely in the vicinity of a global extremum by using important sampling.

Mixed error measure of the relative error and the absolute error is used as objective function to balance the weight of distance in error. Specifically, the objective function is defined as function (3), which was proposed by Kesting et al.:

$$\sqrt{\frac{1}{E(|s_{obs}(t)|)} \cdot \mathbb{E}\left(\frac{(s_{obs}(t) - s_{sim}(t))^2}{|s_{obs}(t)|}\right)}$$

(3)

where $s_{obs}(t)$ is the observed spacing and $s_{sim}(t)$ is the simulated spacing.

The root mean square error (RMSE) of the spacing, speed, and acceleration were used as performance indicators (PI) to reflect whether or not the actual driving conditions were properly described by the estimated parameter values (see function (4)). The vehicle behavior entering sag section are time series with a natural temporal order. So the dynamic change of car-following behavior is also very important for congestion formation. This dynamic in driving behavior is reflected by P12 and P13.

$$P12 = \sqrt{\frac{1}{n} \sum (v_{obs}(t) - v_{sim}(t))^2}$$

$$P13 = \sqrt{\frac{1}{n} \sum (a_{obs}(t) - a_{sim}(t))^2}$$

The search range of $\alpha_1, \alpha_2$ is estimated with several times of heuristic search. Initially a wide range of $\alpha_1, \alpha_2$ are set: $\alpha_1 \in [0,1.5], \alpha_2 \in [0,1]$, most estimated $\alpha_1, \alpha_2$ are found to be within the range of $\alpha_1 \in [0,1], \alpha_2 \in [0,0.3]$ while a few cases are at both boundary. The cases with either $\alpha_1, \alpha_2$ value at the boundary is a failure of the heuristic search because no extremum value is reached inside the range. In order to eliminate the failure of heuristic search and obtain reasonable parameter estimation values, the range of $\alpha_1, \alpha_2$ for heuristic search are finally shorten to $\alpha_1 \in [0,1], \alpha_2 \in [0,0.3]$.

The performance of objective function and PIs of both datasets are studied. This performance indicates how well the car-following model and its parameters can reflect the actual condition. The estimated trajectories cannot well reflect the dynamic behavior are also excluded from analysis since the car-following behavior of vehicle entering sag-section is a time series data. The final result of effective samples in field observation data is shown in Table 1.

### 4. Individual difference

The joint cumulative frequency of $\tau$ and $\delta$ is shown in Figure 3. It is consisted with a distribution of $\delta$ along the $\tau<0.2$ range and a distribution of $\tau$ and $\delta$ at other value ranges. The former composition is a single distribution of $\delta$ of Gamma (1.8478,0.1019) with an intercept of 16m, which is the minimum spacing of vehicles in congestion. The latter composition is a joint distribution of $\tau$ and $\delta$ which is a bivariate gamma distribution (See Figure 4). Most drivers have an adequate desired spacing consisting with desired headway around 1.2s and standstill spacing around 5m. $(5-\tau)/5$ and $\delta$ obeys a bivariate Mckay’s gamma distribution of $p = 0.4954, q = 4.4171, a = 0.1476$.

<table>
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<tr>
<th>Cases</th>
<th>$\delta$ (m)</th>
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Figure 3 Joint cumulative frequency of $\delta$ and $\tau$
The distribution of other parameters is separately estimated. $\alpha_1$ is of a Gamma distribution: $\text{Gamma}(1.689480,4.266007)$. $\alpha_2$ is of a negative exponential distribution: Exp (26.05566). Spacing with preceding vehicle has a minor effect on most drivers. $\beta$ is of a beta exponential distribution: Beta (0.3568652, 0.8675302). $\Delta t$ is of a negative exponential distribution: Exp (26.05566). The distribution of $\beta$ has two peaks concentrating on the value range, which indicates that there are drivers who are not affected by the vertical grade while others are. $\Delta t$ is of a Gamma exponential distribution: Gamma (1.7081664, 0.8675302). Most drivers have a reaction time around one second.

5. Discussion

This research presents an empirical study of difference in individual car-following behavior at expressway basic segment bottlenecks well known as sag section in Japan with trajectory data obtained from an actual sag section. In this research, a car-following behavior model considering both the grade effect and individual difference of drivers is utilized and calibrated with field observation data.

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Reference


