Normal stress dependence of ice-ice friction coefficients

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ABSTRACT

The ice-ice friction coefficient was measured as a function of normal stress (1.2 ~ 17.3 kPa), temperature (−0.5 ~ −20°C) and velocity (4 × 10⁻⁴, 4.5 × 10⁻², 4 × 10⁻¹ m/s). The friction coefficient was found to decrease with increasing normal stress and became almost constant at larger normal stresses than about 6 kPa at all the temperatures and velocities studied. Though the overall dependence could be expressed as \( \mu \propto P^{-0.32} \) where \( \mu \) and \( P \) are respectively the ice-ice friction coefficient and normal stress, the Mohr-Coulomb relation, \( \mu = \mu_0 + A/P \), was utilized in our data analysis, where \( \mu_0 \) and \( A \) are the Coulomb friction coefficient and adhesion strength, respectively. The temperature dependence of \( \mu_0 \) showed a minimum around −5°C. The value of \( \mu_0 \) decreased with increasing velocity, suggesting the formation of water film by friction heat at these low sliding velocities. The value of adhesion strength was of the order of 10⁻²⁵ Pa and increased with lowering temperature or increasing velocity.

Key Words: ice, friction coefficient, adhesion strength, water lubrication

1. Introduction

Among a number of physical mechanisms proposed to explain low ice friction (Joly, 1887, Reynolds, 1901; Bowden and Hughes, 1939; Bowden and Tabor, 1950; Tusima, 1977), the two mechanisms of ice adhesion and water lubrication have been widely recognized. At lower sliding velocities than about 10⁻² m/s, ice adhesion is predominant at the friction interface and the friction force is mainly caused by shear plastic deformation in the thin ice layer near the interface (Barnes et al. 1971; Tusima, 1977). At higher velocities than about 10⁻² m/s, the friction force is attributed to viscous resistance of water film produced by frictional heat (Bowden & Hughes, 1939). The latter model was more extensively discussed by Evans et al. (1976); Oksanen & Keinonen (1982) constructed a quantitative model to obtain the ice frictional coefficient as a function of velocity, temperature and normal stress.

Most past ice friction measurements have been performed between ice and other materials with practical applications to skates, skis and various structural interactions in ice environments. Experimental data of ‘ice-ice friction’ are required to gain true understanding of the ice friction mechanism since ice is mechanically soft and its surface is quite reactive compared to other materials. Such studies of ice-ice friction experiments have only been reported by Bowden & Hughes (1939), Oksanen & Keinonen (1982), Beaman et al. (1988), Jones (1989) and Casassa et al. (1991).

Based on these research papers, we began a systematic study of ice-ice friction and measured ice-ice friction coefficients in a velocity range from 10⁻⁴ to 10⁻¹ m/s and temperature range from −1 to −27°C (Yasutome et al. 1999). Yasutome et al. found that the ice-ice friction coefficients were almost constant, roughly 0.1, showing stick-slip behavior at
velocities below $10^{-2}$ m/s, and decreasing with increasing velocity at higher velocities. The result was qualitatively explained by the model of Oksanen & Keinonen (1982), suggesting the formation of lubricant water film at velocities as low as $10^{-2}$ m/s. The present paper is the second report of our systematic study and reports the experimental result of normal stress dependence of ice-ice friction, which was not studied in detail in the first report.

2. Experiments

Friction coefficients were measured with the rotating-type friction device used by Yasutome et al. (1999) schematically shown in Fig. 1. On the surface of the ice block fixed to the $z$-axis stage, the slider ice (area $S = 7.9 \times 10^{-4}$ m$^2$) rotates at a constant angular velocity, driven by a megatorque servomotor controlled by a computer. Three constant velocities ($4 \times 10^{-3}$, $4.5 \times 10^{-2}$, $4 \times 10^{-1}$ m/s) were chosen based on the study of the velocity dependence of ice-ice friction coefficient by Oksanen & Keinonen (1982) and Yasutome et al. (1999). The three velocities correspond to the three characteristic velocity regions where the ice-ice friction coefficient is almost constant (stick-slip), decreases, and increases with increasing velocity (Yasutome et al., 1999).

The frictional force ($F_\mu$) was measured as a torque ($M = F_\mu R$) through a strain gage transducer (KYOWA TP-E, capacity; 2 Nm) and dynamic strain amplifier (KYOWA DPM-630) where $R$ is the distance between the geometrical center of the ice slider and rotation axis, $7.25 \times 10^{-2}$ m. The friction coefficient ($\mu$) was calculated as,

$$
\mu = F_\mu / F_N = \frac{M}{F_N \cdot R}
$$

where $F_N$ is the normal force.

The experimental device was installed in a thermally controlled box set in a cold room maintained at a temperature between 0 and $-25^\circ$C. The temperature of the ice specimen was kept constant with an accuracy of 0.1°C in the range from $-0.5$ to $-20^\circ$C. The slider and fixed ice were both pure polycrystalline ice made by freezing distilled and deaerated water. The contact surfaces of both the bottom and slider ice were prepared as smooth as possible with microtome planing. The normal stress, $P = F_N / S$ where $S$ is the apparent contact area, is the apparent normal stress because the true contact area is unknown. $P$ was varied between 1.2 kPa and 17.3 kPa by adjusting weights acting on the slider ice. The minimum normal stress without weights was 1.2 kPa.
3. Results

Fig. 2 is an example of torque measured when the rotating ice slider was free (A) and in contact with the bottom ice (B). The first and occasional large peaks, which appeared because of static friction and unavoidable irregularities on the ice surface, were not used in the following friction analyses. The torque due to friction was obtained from the difference between B and A by time-averaging.

Fig. 3 shows the ice-ice friction coefficient plotted against the normal stress at a temperature of $-10^\circ$C and velocity of $4 \times 10^{-3}$ m/s. Each point denotes the average of data obtained in three to four measurement runs. In the figure, the data by Yasutome et al. (1999) are also included. It is seen that the friction coefficient decreases with increasing normal stress, and becomes almost constant, about 0.1, at larger normal stresses than approximately 6 kPa. Yasutome et al. apparently could not find the normal stress dependence of the friction coefficient because their measurement range was 3–6 kPa, which was not wide enough to note the dependence.

Similar normal stress dependence was obtained at other temperatures than $-10^\circ$C. In Fig. 4, we present friction coefficients measured at $4.5 \times 10^{-2}$ m/s and six different temperatures. It is clear that at all the temperatures examined the normal stress dependence is similar to that in Fig. 3. It should be noted, however, that the absolute value of friction coefficient is larger at higher temperatures in the temperature region above $-5^\circ$C (A), and smaller in the region below $-5^\circ$C.
Fig. 5 shows the normal stress dependence of the friction coefficient at three different velocities. The temperature of measurement is $-10^\circ$C. The friction coefficient at $4 \times 10^{-3}$ m/s is larger than the values at other velocities, but the variation rate of friction coefficient at lower normal stresses is smaller.

4. Discussion

Our measurements of ice-ice friction showed a clear dependence on the normal stress at all the velocities and temperatures studied (Figs. 3, 4 and 5). As mentioned above Yasutome et al. (1999) did not notice this dependence since their stress range was not wide enough. Jones’ (1989) failure to find the normal stress dependence can be attributed to the fact that their range (7–45 kPa) corresponds to that of the constant friction coefficient.

The normal stress dependence of the ice friction coefficient has been reported by Bowden and Hughes (1939), Tusima (1977), Evans et al. (1976) and Oksanen and Keinonen (1982). Bowden and Hughes (1939) reported that the friction coefficient decreases with higher loads: the loads correspond to 3–20 kPa if their contact area is $3 \times 10^{-4}$ m$^2$. Tusima (1977) found that the friction coefficient increased rapidly with increasing loads, but the increase was mainly due to ice deformation plowed by a steel-ball slider and cannot be considered as a normal stress dependence. Evans et al. (1976) and Oksanen and Keinonen (1982) found a clear decrease of the friction coefficient with normal stress, their results being expressed as $\mu \propto P^{-1/3}$ and $\mu \propto P^{-1/4}$ respectively. The stress range of Evans et al. is not clear but that of Oksanen and Keinonen was 0.8–4 kPa.

To compare our result with those of Evans et al. and Oksanen and Keinonen, we tried power-law fits to our experimental data for friction coefficient and normal stress (Fig. 6), giving the relation,

$$\mu \propto P^n$$

where $n$ is a numerical constant. The obtained value of $n$ ranged from $-0.1$ to $-0.59$, and the arithmetical mean for all our data gave $-0.32$, which does not conflict with the results of Evans et al. and Oksanen and Keinonen. However, a physical explanation for the numerical constant cannot be given at the moment and is the subject of the future study.

Alternatively, the normal stress dependence can be analyzed in a different way. Frictional shear stress, $Q = F_\mu / S$, was plotted against normal stress at a temperature of

![Figure 5](image1.png) Friction coefficient versus normal stress. Temperature is $-10^\circ$C.

![Figure 6](image2.png) Examples of power-law fit of ice-ice friction coefficient. Temperature is $-10^\circ$C.
$-10^\circ$C and three velocities (Fig. 7), and was fitted with the linear expression,

$$Q = P \cdot \mu_0 + A$$

where $\mu_0$ and $A$ are constants. Since this relation is similar to the so-called Mohr-Coulomb equation for the failure criterion of soils and other granular materials, $\mu_0$ can be regarded as the internal friction coefficient or Coulomb friction coefficient, and $A$ as the adhesion or cohesion strength. Then the friction force is explained by the sum of the two forces, namely the Coulomb friction force that is proportional to normal force and the adhesion strength that is independent of the normal force.

From the definition of friction coefficient, $\mu$ is given by

$$\mu = \frac{\mu_0 + A}{P}.$$  \hspace{1cm} (4)

This equation implies that the friction coefficient is sensitive to the adhesion force at lower normal stresses. The variation rate of friction coefficient at lower normal stresses is determined by the adhesion strength. As the normal stress increases, the value of friction coefficient approaches the Coulomb friction coefficient.

Temperature and velocity dependences of Coulomb friction coefficient are plotted against temperature for three velocities in Fig. 8. At all the velocities, $\mu_0$ showed a minimum at a temperature around $-5^\circ$C. Similar temperature dependences were reported by Kobayashi et al. (1970), Tusima (1977) and Yasutome et al. (1999). The decrease of $\mu_0$ with rising temperature below $-5^\circ$C can be explained by frictional melting. On the other hand, the increase at temperatures above $-5^\circ$C is considered to be caused by softening of ice and production of thick water films near the melting point of ice. According to our visual observation, the track of the bottom ice was covered by a shining water film at $-0.5^\circ$C (Fig. 9). The water generated increases real contact area and leads to higher viscous resistance. The larger values of $\mu_0$ at lower velocities can also be explained by the water lubrication mechanism.

The adhesion strength is plotted against temperature for three velocities in Fig. 10. The values of adhesion, which acts between the moving slider ice and the bottom ice, are 10 to 250 Pa, which are two to four orders of magnitudes smaller than the usual fracture strength of ice. But the tendency for the adhesion to decrease with increasing velocity and increase with lowering temperature is

Fig. 7 Shear stress versus normal stress. Temperature is $-10^\circ$C.

Fig. 8 Coulomb friction coefficient versus temperature.
similar to the dependence of the ice fracture strength on strain rate and temperature respectively.

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References


氷・氷摩擦係数の垂直応力依存性

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要 旨
氷・氷摩擦係数の垂直応力依存性を1.2～17.3 kPa の垂直応力範囲で調べた。実験は -0.5 ～ -20℃の温度領域、また 3 種類の一定速度（4 × 10^{-3}, 4.5 × 10^{-2}, 4 × 10^{-2} m/s）で行われた。測定が行われた全ての温度、速度において、摩擦係数は垂直応力とともに減少し、約 6 kPa 以上の垂直応力で、ほぼ一定値をとった。摩擦係数(μ)と垂直応力(P)の関係は平均的に μ ∝ P^{-0.52} で表せた。しかし、本報告では、測定されたせん断応力と垂直応力の直線関係に基づいて得られた Mohr-Coulomb の式 (μ = µ_0 + A/P) を用いて解析が行われた。µ_0 はクーロン摩擦係数、A は付着力である。µ_0 は速度の増加とともに減少した。この結果は摩擦熱による水膜発生がこのような低速度でも起っていることを示唆する。µ_0 には温度依存性が確認され、-5℃ 附近で極小を示した。付着力(A) は、10 ～ 250 Pa という小さな値で、温度の低下または速度の増加とともに増加した。