An Ultrasonic Tomography for Reconstructing the Velocity Profile of Air Flow from Nonuniformly Spaced Projection Data[†]

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The technique of computed tomography (CT) is used to reconstruct velocity profile of air flow from its average velocities from ultrasonic velocimeter measuring over a specific set of paths. Although the scanning method for collecting data can be considered as a special case of diverged beam scan, this set of data can be rearranged into the form of interlaced parallel scan. A knowledge of zero velocity at circumference of the pipe is also introduced and the data are interpolated in each view to obtain uniformly spaced data. This is then applied in the convolution algorithm for parallel scan data. This method is tested with the data from the experiment and the result is reported.

Key Words: computed tomography, flow measurements, image reconstruction

1. Introduction

The technique of computed tomography (CT) is known as a method of reconstructing the internal details of a structure from its projections. These projections are obtained by measuring, at the receiver array, the beam which is radiated from an external source and transmits through the structure. Since measured data are obtained without affecting the original nature of the objects, the method of CT becomes of great interest in noninvasive medical diagnosis, remote sensing and other scientific fields.

The most simplified model of projection data obtained from measurement is to interpret it as the line integrals along straight path through the test object. This is also called straight-path tomography to distinguish it from diffraction tomography where diffraction and refraction within an inhomogeneous object are also taken into account. The problem of this simplified model is then, given a set of line integrals, reconstruct the internal details of a structure

from these line integrals.

Our work has been conducted to apply the technique of CT to reconstruct the velocity profile of air flow. It is known that the ultrasonic flowmeter operates basically as a kind of velocimeters. It measures the average velocity of flow over the path that the sound wave propagated. The sound wave from the transmitter at upstream will be sent to the receiver at downstream and vice versa. The difference of time of propagation of sound wave in upstream and downstream is proportional to the average velocity of the flow over the path of propagation. By this fact, it means that the product of this average velocity with the distance, between the transmitter and the receiver, projected onto the cross section of the pipe can be used as the projection data in algorithm of CT to reconstruct the velocity distribution of flow over the cross section of the pipe. However, an important restriction is that the profile of flow between the sections where the transmitter and the receiver are located must be constant.

In our previous work, some experiments were conducted with air flow and the velocity profile were reconstructed under the assumptions of axisymmetrical case¹⁾ and, more generally, unaxisymmetrical case²⁾. The result of the latter was compared

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with the velocities from a hot wire anemometer measured at four different sections passed through the center of the pipe. However, since the reconstruction algorithm used in that work does not take into account a priori knowledge that the velocity of flow at the wall of the pipe is zero, large deviation from the reference values near the circumference of the pipe was then observed.

In this work, the data from diverged scanning method, which are used in our experiment, are rearranged into the form as the data obtained from parallel scan. However, the sampling points in each view of this rearranged data are nonuniformly spaced and, this then prevent us from applying it directly in the algorithms for parallel scan data. The Lagrangian interpolation is then used to interpolated for uniformly spaced sampling points from this interlaced parallel scan data. The interpolated data are then used in a reconstruction algorithm for parallel scan data. Note that, the knowledge of zero value at the circumference can be directly introduced into the interpolation step by this method.

In the next section, the fundamental theory of CT and the ultrasonic velocimeter are briefly stated. Some scanning methods and reconstruction algorithms are investigated, and among them, the one which seems suitable to use in our work is then chosen. This is to be discussed in the next. Finally, to test the idea proposed, an experiment is conducted and the result is reported.

2. Theory

2.1 Fundamental theory of CT

Let us define $f(x, y)$ as a function which corresponds to the distribution property of the object under interest. Any straight lines, L , on the plane $x-y$ can be specified by 2 variables, t and θ , where t is the distance from the lines to the origin and θ is the angle which the other line, which is perpendicular to L and passed through the origin, made with the x -axis. This can be expressed as follow,

$$L(t, \theta): x \cos \theta + y \sin \theta = t \quad (1)$$

Let us also define $P_f(t, \theta)$ as the line integral of f over the line $L(t, \theta)$. This can be expressed as follow,

$$P_f(t, \theta) = \int_L f(x, y) ds \quad (2)$$

The variable of integration, s , in the above equation is an ordinate in the other coordinate, $t-s$, which can be obtained by rotating coordinate $x-y$ by an angle θ . To reconstruct f from $P_f(t, \theta)$, an algorithm called convolution algorithm is a widely used one. This algorithm was developed based on the Fourier's slice theorem. This theorem states that the Fourier transform (F. T.) of the projection at angle θ yields a central cross section of the F. T. of the object function at the same angle. Let us define $S(\omega, \theta)$ as the F. T. on variable t of $P_f(t, \theta)$ and ω as the radial frequency, the Fourier slice theorem can then be expressed as follow,

$$S(\omega, \theta) = F(\omega, \theta) \quad (3)$$

where $F(\omega, \theta)$ is the 2-D F. T. of f on polar coordinate. Reconstruction can then be performed on either frequency space or real space. In the former, it becomes the direct Fourier transformation method³⁾, while in the latter, becomes the convolution method⁴⁾, which is the one used in this work. However, the filter in the algorithm which is used here is the one later proposed by Shepp L. A. et. al.⁵⁾.

2.2 The ultrasonic velocimeter

Let us define that the distance between the transmitter and the receiver is ' L ', the angle between the axis of the pipe and the direction of propagation of sound wave is ' θ ', the velocity of sound when there is no flow is ' C ' and the average velocity over the path of propagation of sound wave is ' v '. This is illustrated in Fig. 1(a). The difference of the inverted time for the sound to propagate from the transmitter to the receiver in downstream (t_d) and upstream (t_u) can be expressed in term of v as follow,

$$\begin{aligned} \frac{1}{t_d} - \frac{1}{t_u} &= \frac{[(C + v \cos \theta) - (C - v \cos \theta)]}{L} \\ &= \frac{2v \cos \theta}{L} \end{aligned} \quad (4)$$

It is noted that, since the path of propagation of sound wave has to make some angle with the pipe axis, the problem should be considered as a 3-D reconstruction problem. However, this needs more data, and hence, more complex scanning structure to obtain all data needed. Our experiment was then performed under the following assumptions.

Firstly, the profile of flow from the upstream to the downstream is spatially constant in the direction of

(a) Outline of the Ultrasonic Velocimeter

(b) View from the cross section of the pipe

Fig. 1 The Ultrasonic Velocimeter

the pipe axis. The problem then becomes a 2-D reconstruction problem. For the measured path as in Fig. 1, the projection data for CT reconstruction algorithm can be expressed as follow,

$$\left(\frac{1}{t_d} - \frac{1}{t_u} \right) \frac{LD}{2 \cos \theta}$$

where D is as shown in Fig. 1(b).

Secondly, the flow is steady so that manually recorded data can be used in the algorithm.

3. Consideration of Scanning Method and Reconstruction Algorithm for This Work

3.1 Scanning method

Scanning methods for collecting data can be categorized into 2 types, parallel scan and diverged scan. The former obtains the data by moving the source and detector along parallel lines enclosing the object, while the latter obtains the data which its source fixed in a position relative to a line of detectors. It is clear that the time needed for collecting data can be made shorter in diverged scan than in parallel scan. By this fact, diverged scan is widely used as the scanning method in commercial CT machine developed in these recent years. Furthermore, its shorter collecting time is regarded as of

great advantage when the property of the object under interest is time-varying.

An advantage point to measure velocity in water flow by ultrasonic velocimeter is that it is a non-intrusive measurement. Ultrasonic transducers are commonly installed outside at the wall of pipe. In ultrasonic tomography, where measurements must be done over many directions through mechanically scanning part, coupling media such as water can be used between scanning part and the pipe. This is for matching acoustic impedance between these two parts and also introduce a free layer between two rigid layers to allow complete scanning⁶⁾.

This is not the case for the measurement in air flow. Acoustic wave highly loss its power when propagates through the boundary of air and the other media. In our work, the transducers are then installed directly in the holes open at the wall of the pipe. Since now the scanning part is regarded as being a part of the wall of pipe, it seems impractical to rotate the pipe to obtain the data by parallel scan. By above reasons, the geometry called PET (Positron Emission Tomography) geometry⁷⁾ was then used as the scanning method in our work. It is called such that since this geometry arises in positron emission tomography. Scanning by this method collects the data over $N(N-1)/2$ chords connecting any two points of N points equally spaced along the circumference of the circle which, in this case, is the pipe in our experiment. This scanning method can be considered as a special case of diverged scan. Moreover, the data in PET geometry scan can be rearranged into the form as if it is obtained from parallel scan of N views which the data in each view nonuniformly spaced, in other words, interlaced parallel scan. The case when $N=8$, which is the case in our experiment, is shown in Fig. 2(a) for all the scanning paths and in Fig. 2(b) when it is rearranged into interlaced parallel scan.

3.2 Reconstruction algorithm

Reconstruction algorithms can be generally categorized into 2 types. The first one is in the form of an explicit formula for an estimation of the original function in term of its projections. The second one is in the form of searching a function whose projections agree with the measured data, using iterative relaxation⁸⁾. Our attention was formerly paid for the

(a) Measured paths

(b) Rearrangement into a parallel geometry
(shown in the figure are 2 views from 8 views)**Fig. 2** PET geometry with $N=8$

first one, since the second one seems to have the problems in calculation time and convergence property. The former can be further separated into the ones which the verification of their formulae are based on the Fourier slice theorem, which was mentioned earlier, and the other ones which are not. These are, for example, the ones which are based on orthogonal expansion of functions⁹⁽¹⁰⁾. The formula in Ref. 10) was used in our previous work since the data in the discrete form has the form of PET geometry. The function under interest is expanded by a system of orthogonal polynomials. The coefficient of the expansion is then obtained explicitly from the projection data. However, reconstructed result near the circumference of the pipe was observed to be highly corrupted by error. This can be expected since the algorithm was developed without incorporating with a priori knowledge that speed of flow will decrease to zero at the wall of the pipe.

Other algorithms were then reconsidered. The con-

volution algorithm, which was stated earlier, seems preferable in this case. The algorithms for both parallel scan data and diverged scan data¹¹⁾ has been already developed. Unfortunately, diverged scan version seems not suitable with such PET geometry scan. This is because when the source of the beam comes closer to the region of interest, the reconstruction result becomes more and more severely increase near the boundary and recall that, PET geometry scan is a special type of diverged scan with the source of beam located at the boundary of region under interested.

As was stated in the previous section, the data from PET geometry scan can be rearranged in the form of interlaced parallel scan. However, since the data are nonuniformly spaced, it prevents us from simply applying any ordinary reconstruction algorithms for parallel scan. Our idea is then first to interpolate for the data on uniformly spaced points from the data at hand before apply them in the convolution algorithm for parallel scan. The interpolation method used in our work is Lagrangian interpolation. Its formula is shown here for completeness as follow. Let $\{t_n\}$ denote a set of sample locations for a function $x(t)$. The corresponding Lagrangian interpolation from these samples is

$$y(t) = \sum_n x(t_n) k_n(t) \quad (5)$$

where

$$k_n(t) = \prod_{m \neq n} \frac{t - t_m}{t_n - t_m} \quad (6)$$

Let us note that a priori knowledge as stated earlier was also introduced in interpolation. The diagram for the step of our proposed method is shown in **Fig. 3**.

Before go further to the next section, let us discuss in some details about interpolation by Lagrangian method and its relation to cardinal series. Although it is known that Lagrangian interpolation becomes cardinal series when sample locations are uniformly spaced, i. e. ' t_n ' becomes ' n ' in above equations, it is not true when sample locations are nonuniformly spaced as in the case in our work. The cardinal series is a formula which represents a band-limited signal by a set of its samples. This representation is exact if the rate in sampling is higher than Nyquist rate.

Fig. 3 The diagram of proposed method

4. Experiment

4.1 Description of the experimental set up

To test the idea mentioned above, an experiment was conducted. Its arrangement is shown in Fig. 4 (a). A motor fan is used as the air source. The size of pipes used are 490 mm in diameter. The vinyl pipe on the left is 2000 mm length. Inside it is filled with many small 20 mm ϕ ×150 mm paper tubes which function as a flow straightener. The pipe on the right,

which is the part where the data are measured, is an acrylic pipe with 1100 mm length. It is placed on four casters to make it be rotatable. This is shown again in Fig. 4(b). One transducer is installed at upstream while other 7 transducers are installed at downstream. This 2 planes are 50 cm apart. The transducers used in ultrasonic velocimeter are 40kHz speaker type. The time resolution of the velocimeter is 0.25 μ s.

After finished measuring in each view, the pipe is then rotated manually for next measuring in other view. Although the data were measured manually, it is noted that this can be further developed to be an automatic one. Eight sets of data from 8 views with 7 values in each view are then recorded. By this way, every path of measurements is then measured 2 times and the value used in reconstruction algorithm is the average one of these 2 values. In interpolation, the number of points, which are interpolated for, is 15 in each view. Moreover, since the views of even number, as shown by dotted lines in Fig. 2(b), do not have a sample point at the center, the average value of data at the center from other 4 points from the odd number of view was inserted. Without doing this, comparatively flat patterns were observed near the centers of interpolated results in those views, which

Fig. 4 Experimental arrangement

seems inconsistent to use in reconstruction. Although not shown here, the consistency of the interpolated results were checked by reprojecting the reconstruction result and more deviation of the reprojected value from the real projection data was observed in interpolating without inserting the center point as compared with the inserting case. To check the reconstructed result, the velocities at 4 sections with 7 points in each section were measured at the downstream plane by a hot wire anemometer. This is shown in Fig. 4(c). Mode of measurements used was 10 seconds-averaged. The data were recorded every 5 seconds, 10 times for each point.

4.2 Results and discussion

Dimension of the pipe is normalized to range from -1 to 1 in plotting all of the graphical results. The contour and 3-D graphic of the velocity profile

reconstructed from the data are shown in Fig. 5(a) and (b), respectively. The step in plotting contour is 0.1 m/s. Comparing the reconstructed results with measured velocities by a hot wire anemometer over 4 sections in Fig. 4(c) are shown in Fig. 6(a), (b),

(a) Contour lines of the reconstructed velocity profile (step in plotting is 0.1 m/s)

(b) 3-D graphic of the reconstructed velocity profile

Fig. 5 Experimental results

Fig. 6 The velocity distribution is shown for 4 sections (a), (b), (c) and (d) corresponds to as indicated in Fig. 4(c)
(---: reconstructed results; ○: measured value by a hot wire anemometer)

(c), (d), respectively. Solid lines are the reconstructed results. Measured velocity at each point by hot wire anemometer is shown by a straight line which indicated the range of 10 values measured at each point. The circle indicated the average value among them.

From the results, deviation of the reconstructed profile from the values measured by a hot wire anemometer is observed. This is expected to be due to the following reasons.

1. The length of the pipe used may be too short compare to its diameter, says, about 6 times the diameter. By this fact, the profile of flow in the volume of measurement, i.e. from upstream to downstream plane may be not constant. The problem may then be preferable to be reconsidered as a 3-D reconstruction problem. However, 3-D reconstruction problem needs more data in other directions which means that it needs a more complex scanning structure to obtain all data needed. This is a reason why 2-D reconstruction problem was considered instead of 3-D in our work.

2. Some fluctuation in the motor fan which was used as the air source may be arisen. Noted that, since the data were measured by scanning manually, this is then inconsistent to use the data measured at different time as if it is the same set of data.

3. The error is arisen in the reconstruction result

due to too few number of projection data.

However, reconstructed result near the circumference decrease to zero as desired and improvement over our previous work is also observed. This is due to the fact that, the profile of the flow in our experiment is comparatively smooth. It then seems reasonable to use polynomials of low degree, which was realized by Lagrangian interpolation in this work, to interpolate for the data needed. It is intuitively noted that more complex profile may need more measured data to allow good reconstruction result.

5. Conclusion

The technique of CT was used to reconstruct velocity profile of air flow from average velocities of flow from ultrasonic velocimeter measuring over a specific set of paths. This set of data is used to interpolate for the value at the uniformly spaced set of points. The interpolated data are then applied in convolution algorithm to reconstruct the velocity profile of the flow. An experiment with a motor fan was conducted and the result was compared with the values measured by a hot wire anemometer. The improvement in the result from our previous work was observed.

References

- 1) S. Teerawatanachai, K. Komiya and H. Sasamoto: Reconstructing the Velocity Profile of Axisymmetric Flow of Air by Means of the Ultrasonic Translation Time Computed Tomography Method, Proc. of SICE'91, 277/278 (1991) In Japanese.
- 2) S. Teerawatanachai, K. Komiya and H. Sasamoto: Estimating the Velocity Profile of Air Flow by Means of the Ultrasonic Translation Time Computed Tomography Method, Proc. of IECON'91, 2379/2384 (1991)
- 3) H. Stark, J. W. Woods, I. P. Paul and R. Hingorani: An Investigation of Computerized Tomography by Direct Fourier Inversion and Optimum Interpolation, IEEE Transactions on Biomedical Engineering, **BME-28**, 496/505 (1981)
- 4) G. N. Ramachandran and A. V. Lakshminarayanan: Three-dimensional Reconstruction from Radiographs and Electron Micrographs: Application of Convolutions Instead of Fourier Transforms, Proc. Nat. Acad. Sci. USA, **68-9**, 2236/2240 (1971)
- 5) L. A. Shepp and B. F. Logan: The Fourier Reconstruction of a Head Section, IEEE Transactions on Nuclear Science **NS-21**, 21/43 (1974)
- 6) A. Hauck: Ultrasonic Tomography for the Non-intrusive Measurements of Flow-velocity-fields, 5th Int. IMEKO-Con. on Flow Meas. FLOMEKO Dusseldorf (FRG), 361/369 (1989)
- 7) F. Natterer: The Mathematics of Computerized Tomography, John Wiley & Sons (1989)

- 8) R. Gordon, R. Bender and G. Herman : Algebraic Reconstruction Techniques (ART) for Three-dimensional Electron Microscopy and X-ray Photograph, *J. Theor. Biol.*, **29**, 471/481 (1970)
- 9) A. M. Cormack : Representation of a Function by Its Line Integrals I, *J. Appl. Phys.*, **34**, 2722/2724 (1963)
- 10) R. B. Marr : On the Reconstruction of a Function on a Circular Domain from a Sampling of Its Line Integrals, *J. Math. Anal. Appl.*, **45**, 357/374 (1974)
- 11) A. C. Kak and M. Slaney : Principles of Computerized Tomographic Imaging, IEEE Press (1988).

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