The Simultaneous Block Triangular Decoupling, Disturbance Rejection and Pole Assignment Problem for Linear Multivariable Systems[†]

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This paper studies the problem of simultaneously achieving block triangular decoupling, disturbance rejection and pole assignment by state feedback for linear multivariable systems in the so-called geometric approach, and presents necessary and sufficient conditions for this simultaneous problem to be solvable.

Key words: block triangular decoupling, disturbance rejection, pole assignment, geometric approach

1. Introduction

The block triangular decoupling problem for linear multivariable systems was first studied in the so-called geometric approach by Morse and Wonham²⁾, and necessary and sufficient conditions for the problem to be solvable were obtained. Furthermore, they investigated the problem of simultaneous block triangular decoupling and pole assignment, and gave necessary and sufficient conditions for its solvability. On the other hand, the problem of simultaneous disturbance rejection and pole assignment was first discussed in the same approach by Chang and Rhodes¹⁾, and necessary and sufficient conditions for its solvability were obtained. Recently, the problem of simultaneous block triangular decoupling and disturbance rejection was studied in the geometric approach by Otsuka³⁾ and necessary and sufficient conditions for its solvability were obtained.

The objective of this paper is to study the problem of simultaneous block triangular decoupling, disturbance rejection and pole assignment by state feedback for linear multivariable systems in the geomet-

ric approach, and to give necessary and sufficient conditions for this simultaneous problem to be solvable. Section 2 gives preliminaries and the problem formulation. In Section 3, the main results are given. Some concluding remarks are given in Section 4.

2. Preliminaries and the Problem Formulation.

We consider the following time invariant linear system :

$$(2.1) \quad \Sigma: \frac{d}{dx}x(t) = Ax(t) + Bu(t), \quad x(0) = x_0 \in X,$$

where $x(t) \in X := \mathbf{R}^n$ is the state vector, $u(t) \in U := \mathbf{R}^m$ is the input vector, A and B are matrices with appropriate sizes. First, we give some definitions and notations which will be used throughout the following discussion.

Let $\Gamma \subset X$ be a subspace. Then, Γ is said to be (A, B) -invariant if there exists an $F \in \mathbf{R}^{m \times n}$ such that $(A + BF)\Gamma \subset \Gamma$. We denote by $\mathbf{F}(A, B; \Gamma)$ the set of all those F satisfying the above inclusion. Further, we introduce the following notations: $\langle A|\Gamma \rangle := \Gamma + A\Gamma + \cdots + A^{n-1}\Gamma$, $\text{Im } A :=$ the image of A , $\sigma(A) :=$ the set of eigenvalues of A , and $A|_{\Gamma} :=$ the restriction A to Γ as a linear map. The subspace $\langle A|\text{Im } B \rangle$ is called the controllable subspace for System (2.1), and System (2.1) is controllable if and only if $\langle A|\text{Im } B \rangle = X$. A subspace $\Phi \subset X$ is called a *controllability subspace* for (2.1) if there exist matrices $F \in \mathbf{R}^{m \times n}$ and $G \in \mathbf{R}^{m \times m}$

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so as to satisfy $\Phi = \langle A + BF | \text{Im } BG \rangle$. Finally, we introduce two families of subspaces as follows :

$$\mathbf{V}(A, B; \Gamma) := \{V \subset \Gamma | V \text{ is an } (A, B) \text{--invariant subspace}\},$$

and

$$\mathbf{C}(A, B; \Gamma) := \{V \subset \Gamma | V \text{ is a controllability subspace for } (2.1)\}.$$

Now, we quote the following two lemmas.

(2.2) **Lemma⁶⁾**. If Φ is a controllability subspace for (2.1), then

$$\Phi = \langle A + BF | \text{Im } B \cap \Phi \rangle \text{ for all } F \in \mathbf{F}(A, B; \Phi).$$

□

(2.3) **Lemma⁶⁾**. $\mathbf{V}(A, B; \Gamma)$ and $\mathbf{C}(A, B; \Gamma)$ have their supremal elements $V^*(\Gamma)$ and $\Phi^*(\Gamma)$, respectively. □

The following lemma plays an important role to prove one of the main theorems in Section 3.

(2.4) **Lemma⁷⁾**. Let System (2.1) be controllable and let $V \in \mathbf{V}(A, B; X)$ with $\dim(V) = \alpha$. If F_0 is in $\mathbf{F}(A, B; V)$ and Λ is a symmetric set of $n - \alpha$ complex numbers, there exists a matrix $F \in \mathbf{R}^{m \times n}$ such that

$$\begin{aligned} F|_V &= F_0|_V \text{ and } \sigma(A + BF) \\ &= \sigma[(A + BF_0)|_V] \cup \Lambda \end{aligned}$$

where $F|_V$ indicates the restriction of F onto V . □

Next, consider System (2.1) with $k (\geq 2)$ block-outputs $z_1(t), \dots, z_k(t)$ and unknown disturbance $\xi(t)$ given by

$$(2.5) \quad \begin{cases} \frac{d}{dt}x(t) = Ax(t) + Bu(t) + K\xi(t), \\ x(0) = x_0 \in X, \\ z_i(t) = D_i x(t) \quad (i=1, \dots, k), \end{cases}$$

where A and B are as in (2.1), $K \in \mathbf{R}^{n \times q}$, and $D_i (\neq 0) \in \mathbf{R}^{p_i \times n}$ (For simplicity $D := [D_1^T \dots D_k^T]^T$). For this system, apply a state feedback of the form

$$(2.6) \quad u(t) = Fx(t) + \sum_{i=1}^k G_i v_i(t),$$

where $F \in \mathbf{R}^{m \times n}$, $G_i \in \mathbf{R}^{m \times m}$ and $v_i(t)$ are the new external inputs. Then, we have the following closed loop system :

$$(2.7) \quad \begin{cases} \frac{d}{dt}x(t) = (A + BF)x(t) + \sum_{i=1}^k BG_i v_i(t) \\ \quad + K\xi(t), \quad x(0) = x_0 \in X, \\ z_i(t) = D_i x(t) \quad (i=1, \dots, k). \end{cases}$$

Our *simultaneous block triangular decoupling, disturbance rejection and pole assignment problem* is to find (if possible) a state feedback of the form (2.6),

i.e., matrices F and $G_i (i=1, \dots, k)$, such that the corresponding closed loop system (2.7) achieves simultaneously the following three requirements:

(i) the block-outputs $z_1(t), \dots, z_k(t)$ are triangularly decoupled through the corresponding inputs $v_1(t), \dots, v_k(t)$, i.e., each $v_i(t)$ controls the corresponding block-output $z_i(t)$, but does not affect the previous $z_1(t), \dots, z_{i-1}(t)$, (ii) disturbance $\xi(t)$ is rejected from all block-outputs $z_i(t)$, i.e., all $z_i(t)$ are independent of $\xi(t)$, and (iii) the poles of (2.7) coincide with an arbitrarily given symmetric set of n complex numbers.

The requirement (ii) above has been shown to be equivalent to existence of an (A, B) -invariant subspace $V \subset X$ such that $\text{Im } K \subset V \subset \text{Ker } D^q$. Noticing that the controllability subspace Φ_i generated by the input v_i is given by

$$(2.8) \quad \Phi_i = \langle A + BF | \text{Im } BG_i \rangle \quad (i=1, \dots, k),$$

it is not difficult to see that our problem above can be formulated in the following compact form^{3),5)}.

(2.9) **Simultaneous Block Triangular Decoupling, Disturbance Rejection and Pole Assignment Problem.**

Given $A, B, K, D_i (i=1, \dots, k)$ of (2.5) and a symmetric set $\Lambda = \{\lambda_1, \dots, \lambda_n\} \subset \mathbf{C}$ of n complex numbers, find (if possible) controllability subspaces $\Phi_1, \dots, \Phi_k$ for (2.5) and an (A, B) -invariant subspace $V \subset X$ such that

- (i) $\Phi_i + \text{Ker } D_i = X \quad (i=1, \dots, k)$,
- (ii) $\Phi_i \subset \bigcap_{j=1}^{i-1} \text{Ker } D_j =: \Gamma_i \quad (i=2, \dots, k), \quad \Gamma_1 := X$,
- (iii) $\text{Im } K \subset V \subset \text{Ker } D =: \Gamma$,
- (iv) $\bigcap_{i=1}^k \mathbf{F}(A, B; \Phi_i) \cap \mathbf{F}(A, B; V) \neq \emptyset$ and
- (v) there exists

$$F \in \bigcap_{i=1}^k \mathbf{F}(A, B; \Phi_i) \cap \mathbf{F}(A, B; V)$$

such that $\sigma(A + BF) = \Lambda$. □

We call a set $\{\Phi_1, \dots, \Phi_k, V\}$ of subspaces satisfying the above conditions (i)–(v) a *solution* of Problem (2.9). The problem of finding controllability subspaces $\Phi_1, \dots, \Phi_k$ for (2.5) and an (A, B) -invariant subspace $V \subset X$ satisfying the conditions (i)–(iv) is called the *simultaneous block triangular decoupling and disturbance rejection problem*. Further, the problem of finding an (A, B) -invariant subspace $V \subset X$ satisfying condition (iii) and the conditions (iv), (v)

replaced with $F(A, B; V) \neq \emptyset$ and $\exists F \in F(A, B; V)$ such that $\sigma(A + BF) = \Lambda$ is called the *simultaneous disturbance rejection and pole assignment problem*. Moreover, the problem of finding controllability subspaces $\Phi_1, \dots, \Phi_k$ for (2.5) satisfying the conditions (i) and (ii), and the conditions (iv), (v) replaced with $\bigcap_{i=1}^k F(A, B; \Phi_i) \neq \emptyset$ and $\exists F \in \bigcap_{i=1}^k F(A, B; \Phi_i)$ such that $\sigma(A + BF) = \Lambda$ is called the *simultaneous block triangular decoupling and pole assignment problem*.

Now, let us use the following notations: $V^* := a$ supremal element of $V(A, B; \Gamma)$, $\Phi^* := a$ supremal element of $C(A, B; \Gamma)$, $V_i^* := a$ supremal element of $V(A, B; \Gamma_i)$ ($i=1, \dots, k$) and $\Phi_i^* := a$ supremal element of $C(A, B; \Gamma_i)$ ($i=1, \dots, k$). Then, the following theorems have been shown.

(2.10) **Theorem**^{1),6)}. The disturbance rejection problem is solvable if and only if $\text{Im } K \subset V^*$. Further, the simultaneous disturbance rejection and pole assignment problem is solvable if and only if $\text{Im } K \subset \Phi^*$ and $X = \langle A | \text{Im } B \rangle$. $\square$

(2.11) **Theorem**²⁾. The block triangular decoupling problem is solvable if and only if $\Phi_i^* + \text{Ker } D_i = X$ ($i=1, \dots, k$). Further, the simultaneous block triangular decoupling and pole assignment problem is solvable if and only if $\Phi_i^* + \text{Ker } D_i = X$ ($i=1, \dots, k$) and $X = \langle A | \text{Im } B \rangle$. $\square$

Recently, the following theorem has been shown.

(2.12) **Theorem**³⁾. The simultaneous block triangular decoupling and disturbance rejection problem is solvable if and only if

- (i) $\Phi_i^* + \text{Ker } D_i = X$ ($i=1, \dots, k$), and
- (ii) $\text{Im } K \subset V^*$.

And in this case $\{\Phi_1^*, \dots, \Phi_k^*, V^*\}$ is a solution of the problem. $\square$

Using Theorems (2.10), (2.11) and (2.12), the following corollary can be easily obtained.

(2.13) **Corollary**. The simultaneous block triangular decoupling and disturbance rejection problem is solvable if and only if

- (i) the block triangular decoupling problem is solvable,
 - (ii) the disturbance rejection problem is solvable.
- $\square$

3. The Main Results

The following theorem is our main result.

(3.1) **Theorem**. Simultaneous block triangular decoupling, disturbance rejection and pole assignment problem (2.9) is solvable if and only if

- (i) $\Phi_i^* + \text{Ker } D_i = X$ ($i=1, \dots, k$),
- (ii) $\text{Im } K \subset \Phi^*$, and
- (iii) $X = \langle A | \text{Im } B \rangle$ (i.e., The system is controllable), and in this case $\{\Phi_1^*, \dots, \Phi_k^*, \Phi^*\}$ is a solution of Problem (2.9).

Proof. (Necessity) The proof of necessity easily follows from Theorems (2.10) and (2.11).

(Sufficiency) It suffices to show that under the conditions (i)–(iii) the set $\{\Phi_1^*, \dots, \Phi_k^*, \Phi^*\}$ is a solution of Problem (2.9).

First, it is clear that the set satisfies the condition (i) of (2.9). Further, since each Φ_i^* belongs to $C(A, B; \Gamma_i)$, we have

$$\begin{aligned} \Phi_i^* \subset \Gamma_i &:= \bigcap_{j=1}^{i-1} \text{Ker } D_j \quad (i=2, \dots, k), \\ \Phi_1^* \subset \Gamma_1 &:= X, \end{aligned}$$

which shows that the condition (ii) of (2.9) is satisfied. The condition (iii) of (2.9) is also satisfied because

$$\text{Im } K \subset \Phi^* \subset \Gamma := \text{Ker } D.$$

What remains to be shown is that the set $\{\Phi_1^*, \dots, \Phi_k^*, \Phi^*\}$ satisfies the conditions (iv), (v) of (2.9), i.e., that for an arbitrarily given symmetric set $\Lambda = \{\lambda_1, \dots, \lambda_n\}$ of n complex numbers there exists an $F \in R^{m \times n}$ satisfying

$$\begin{aligned} F \in \bigcap_{i=1}^k F(A, B; \Phi_i^*) \cap F(A, B; \Phi^*) &\neq \emptyset \\ \text{and } \sigma(A + BF) &= \Lambda. \end{aligned} \quad (1)$$

To show this, first set $\Phi_{k+1}^* := \Phi^*$, and note that from the supremarity of Φ_i^* ($i=1, \dots, k+1$) and the inclusions $\Gamma \subset \Gamma_k \subset \dots \subset \Gamma_1 = X$ it follows that

$$\Phi_{k+1}^* \subset \Phi_k^* \subset \dots \subset \Phi_1^* = \langle A | \text{Im } B \rangle = X. \quad (2)$$

Next, define subspaces $X_1, \dots, X_k, X_{k+1}$ of X by the following relations:

$$X_{k+1} := \Phi_{k+1}^*, \quad \Phi_i^* = X_i \oplus \Phi_{i+1}^* \quad (i=1, \dots, k) \quad (3)$$

where $\oplus$ means a direct sum of subspaces. Then, it is not difficult to see that $X_1, \dots, X_{k+1}$ are linearly independent, and hence noticing $\Phi_1^* = X$ it follows that

$$X = X_1 \oplus \dots \oplus X_k \oplus X_{k+1}. \quad (4)$$

Now, introduce the projection matrix P_i of X onto X_i

along $\sum_{j=1, j \neq i}^{k+1} X_j$. Further, let $F_i \in \mathbf{F}(A, B; \Phi_i^*)$ ($i=1, \dots, k+1$), and define a matrix $F \in \mathbf{R}^{m \times n}$ by

$$F := \sum_{i=1}^{k+1} F_i P_i \quad (5)$$

which is well-defined by virtue of (4).

Then, we claim that

$$F \in \bigcap_{i=1}^{k+1} \mathbf{F}(A, B; \Phi_i^*) \quad (6)$$

which will be verified with the following two steps (a) and (b).

(a) Since $X_{k+1} = \Phi_{k+1}^* = \Phi^*$ it follows from (5) that

$$(A + BF)\Phi_{k+1}^* = (A + BF_{k+1})\Phi_{k+1}^* \subset \Phi_{k+1}^*,$$

proving $F \in \mathbf{F}(A, B; \Phi_{k+1}^*)$.

(b) Let $1 < q \leq k+1$ and suppose that the inclusion

$$(A + BF)\Phi_q^* \subset \Phi_q^* \quad (7)$$

holds. Then, by (2), (3), (5) and (7), we obtain

$$\begin{aligned} (A + BF)\Phi_{q-1}^* &= (A + BF_{q-1})X_{q-1} + (A + BF)\Phi_q^* \\ &\subset \Phi_{q-1}^* + \Phi_q^* = \Phi_{q-1}^*. \end{aligned}$$

Consequently, our claim (6) follows from (a) and (b).

Next, we set

$$W_i := \{u \in \mathbf{R}^m | Bu \in \Phi_i^*\} \quad (i=1, \dots, k+1).$$

Then, (2) implies that

$$W_{k+1} \subset W_k \subset \dots \subset W_1 = \mathbf{R}^m.$$

Now, introduce subspaces $U_2, \dots, U_{k+1}$ of $\mathbf{R}^m$ by the relations $W_i = U_{i+1} \oplus W_{i+1}$ ($i=1, \dots, k$). Then, noticing the relation $\mathbf{R}^m = W_1 = U_2 \oplus U_3 \oplus \dots \oplus U_{k+1} \oplus W_{k+1}$, we can introduce the projection matrix G_i of $\mathbf{R}^m$ onto W_i along $U_2 \oplus U_3 \oplus \dots \oplus U_i$ ($i=2, \dots, k+1$) and $G_1 := I$. Then it is clear that $G_1 G_2 \dots G_i = G_i$ and $\text{Im } B \cap \Phi_i^* = \text{Im } B G_i$ ($i=1, \dots, k+1$). Thus, it follows from (6) and Lemma (2.2) that

$$\Phi_i^* = \langle A_F | \text{Im } B G_i \rangle \quad (i=1, \dots, k+1) \quad (8)$$

where $A_F := A + BF$.

We first note that by virtue of (8) the pair $(A_F, B G_{k+1})$ can be interpreted as a controllable system with its state space Φ_{k+1}^* , and hence that there exists a matrix $F_{k+1}^* \in \mathbf{R}^{m \times n}$ such that

$$\sigma[(A_F + B G_{k+1} F_{k+1}^*)|_{\Phi_{k+1}^*}] = \{\lambda_1, \dots, \lambda_{\sigma_{k+1}}\} =: \Lambda_{k+1} \quad (9)$$

where σ_{k+1} is the dimension of Φ_{k+1}^* . Similarly, interpreting the pair $(A_F, B G_k)$ to be a controllable system with its state space Φ_k^* , we can consider Φ_{k+1}^* to

be an $(A_F, B G_k)$ -invariant subspace of Φ_k^* , i.e., $\Phi_{k+1}^* \in V(A_F, B G_k; \Phi_k^*)$. Furthermore, we also note that $F_{k+1}^* := G_{k+1} F_{k+1}^* \in \mathbf{F}(A_F, B G_k; \Phi_{k+1}^*)$.

Now, it can be seen from Lemma (2.4) and (9) that there exists a matrix $F_k^* \in \mathbf{R}^{m \times n}$ such that

$$F_k^*|_{\Phi_{k+1}^*} = F_{k+1}^*|_{\Phi_{k+1}^*} \quad (10)$$

and

$$\begin{aligned} \sigma(A_F + B G_k F_k^*|_{\Phi_k^*}) &= \sigma(A_F + B G_k F_{k+1}^*|_{\Phi_{k+1}^*})|_{\Phi_{k+1}^*} \cup \Lambda_k^- \\ &= \sigma(A_F + B G_{k+1} F_{k+1}^*|_{\Phi_{k+1}^*})|_{\Phi_{k+1}^*} \cup \Lambda_k^- \\ &= \Lambda_{k+1} \cup \Lambda_k^- \\ &= \{\lambda_1, \dots, \lambda_{a_{k+1}}, \lambda_{a_{k+1}+1}, \dots, \lambda_{a_k}\} =: \Lambda_k \end{aligned} \quad (11)$$

where $\Lambda_k^- := \{\lambda_{a_{k+1}+1}, \dots, \lambda_{a_k}\}$ and a_k is the dimension of Φ_k^* . Again, interpreting Φ_k^* to be an $(A_F, B G_{k-1})$ -invariant subspace of Φ_{k-1}^* , we have $F_k^* := G_k F_k^* \in \mathbf{F}(A_F, B G_{k-1}; \Phi_k^*)$.

Thus, we have extended the mapping F_{k+1}^* to F_k^* so as to satisfy (10) and (11) with $F_{k+1}^* = G_{k+1} F_{k+1}^* \in \mathbf{F}(A_F, B G_k; \Phi_{k+1}^*)$ and $F_k^* = G_k F_k^* \in \mathbf{F}(A_F, B G_{k-1}; \Phi_k^*)$. Repeating this extension we finally obtain a matrix $F_1^* \in \mathbf{R}^{m \times n}$ such that

$$F_1^*|_{\Phi_2^*} = F_2^*|_{\Phi_2^*}, \quad (12)$$

$$\sigma(A_F + B G_1 F_1^*) = \Lambda_2 \cup \Lambda_1^- = \{\lambda_1, \dots, \lambda_n\} = \Lambda. \quad (13)$$

Furthermore, $F_1^* = G_1 F_1^* = F_1^* \in \mathbf{F}(A_F, B; \Phi_1^*)$ since $G_1 = I$. Noticing that $A_F + B G_1 F_1^* = A + B(F + F_1^*)$ and defining $F^* := F + F_1^*$, we see from (13) that $\sigma(A + B F^*) = \Lambda$. Moreover, it can be shown from the construction of F^* above that

$$F^* \in \bigcap_{i=1}^{k+1} \mathbf{F}(A, B; \Phi_i^*) \neq \emptyset. \quad (14)$$

In fact, using (6) with the relations $G_1 G_2 \dots G_i = G_i$ and $F_i^*|_{\Phi_i^*} = G_i F_i^*|_{\Phi_i^*}$, it follows that

$$\begin{aligned} (A + B F^*)\Phi_i^* &= (A + B F)\Phi_i^* + B F_1^* \Phi_i^* \\ &= (A + B F)\Phi_i^* + B G_i F_i^* \Phi_i^* \\ &\subset \Phi_i^* \quad (i=1, \dots, k+1) \end{aligned}$$

showing that (14) holds. Hence, (13) and (14) show that F^* satisfies (1). Therefore, $\{\Phi_1^*, \dots, \Phi_k^*, \Phi^*\}$ is a solution of Problem (2.9). This completes the proof of Theorem (3.1). $\square$

Using Theorems (2.10) and (2.11), the following corollary can be easily obtained.

(3.2) Corollary. Simultaneous block triangular decoupling, disturbance Rejection and pole assignment problem (2.9) is solvable if and only if

(i) the block triangular decoupling problem is solvable.

(ii) the simultaneous disturbance rejection and pole assignment problem is solvable. $\square$

4. Conclusions

This paper considered the problem of simultaneously achieving block triangular decoupling, disturbance rejection and pole assignment by state feedback, and presented necessary and sufficient conditions for this problem to be solvable.

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