A Unified Linear Camera Calibration Method Using Two Parallel Calibration Planes

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In this paper, we examine the two plane camera calibration model in terms of perspective mapping, and present a new effective camera calibration method by taking the advantage of the conventional method and modifying it based on the exact perspective imaging. It also takes the advantage that each of intrinsic and extrinsic camera parameters can be obtained by solving linear equations only. So that the solutions will be provided uniquely and stably under some conditions, and which can be definitely specified.

In our method *a priori* knowledge on camera intrinsic parameters is only physical pixel sizes. Exact focal length of the lens, the position of image center or optical center, and the camera pose are unknown and to be calibrated. This is a practical assumption for common commercial TV camera systems.

For the cases where non-linear lens distortions must be taken into account, our method can be extended and their parameters are calibrated also using only linear solutions.

It is shown that higher performance and accurate calibration can be achieved by this method theoretically and experimentally.

Key Words: computer vision, camera calibration, pose estimation, two-plane method

1. Introduction

Martins *et al.*¹⁾ proposed an effective camera calibration method named “Two plane method”, and Gremban *et al.*²⁾ improved it.

The authors of both papers claimed that their methods were effective under some nonlinear lens distortions because they did not employ the exact pin-hole camera model. However, it has been pointed out for this method that “since the formulas for the transformation between image and object coordinates is empirically based, it is not clear what kind of approximation is assumed”³⁾.

Actually, as is shown later, their model which their techniques based on does not exactly hold physically. Moreover, their method is based on interpolation concept, so that it makes the error analysis difficult, and reliability of the methods is uncertain.

On the other hand, their method consists of only linear procedures. It is a great practical advantage that each intrinsic and extrinsic camera parameters can

be obtained by solving linear equations only, because the solutions are provided uniquely and stably, if conditions are definitely specified^{4),5)}, and then the linear algorithm has a possibility of a definite analysis of error propagations in computations.

In this paper, we examine the two plane calibration model in terms of perspective mapping, and present a new effective method by taking the advantage of the two plane method and modifying it based on the exact perspective imaging.

In the new method, our *a priori* knowledge on camera intrinsic parameters is only physical pixel sizes. They are given by design specification data of CCD LSI and image digitizer. Exact focal length of the lens and the position of image center or optical center are unknown and to be calibrated. This is a practical assumption for common commercial TV camera.

In this method, two parallel calibration planes are prepared and a set of calibration points is collected in each plane (Fig. 1). At least four points on one of the planes, whose positions on the plane are exactly known, and at least two points on the other plane are needed, respectively.

This setup of calibration points may be realized by firstly setting the calibration plane at the first position

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and taking its first image, then, shifting it to the second position keeping it parallel to the plane at the first position, and taking its second image. Or, it may be effective to prepare a block-like object which has two parallel plane surfaces each of which contains at least four and two known points. It should be also noted that the positions of these calibration points in 3D space are given with sufficient accuracies practically required for individual systems. The extrinsic parameters of the camera are determined with respect to the coordinates based on these objects.

The total minimum number of calibration points required for our new algorithm is six, which is the same as the conventional method. It should be noted that any additional conditions are not needed for the improvement of the algorithm.

In the followings, in the Section 2, we review briefly the conventional two plane method, and point out the reason why the conventional method cannot achieve higher accuracies. Then, in the Section 3 and after, we present the new model, its geometric implication, and total camera calibration procedures. Finally, experimental results will be shown.

2. Conventional Two-Plane Models

Firstly, we summarize the conventional two plane method provided in 1) and 2). Their method did not assume a unique lens center. Their basic assumption is that, given an image point, two points in the 3D space, each of which is in each of two calibration planes, respectively, and both of which make their

Fig. 1 Set-up of the calibration planes for the camera in the two-plane calibration method

image at the given point in the image plane, are calculated. These two points define a line of sight of the given image point in the 3D space as shown in Fig. 2.

In their method, two parallel calibration planes are prepared and a set of calibration points is collected in each plane. Then, two so-called *back-projection* matrices which transform the position of a given point in the image plane into two positions of points in respective two calibration planes are determined. Using these two back-projection matrices, a line of sight of the given image point is defined as the line in the 3D space passing through these two points.

Many types of interpolation techniques have been proposed to define and determine the *back-projection* matrices for the given image point from the set of calibration points in the planes. Here, we summarize the simplest one called the *linear* or the *linear spline* interpolation.

Firstly, x , y and z axes of the base frame in 3D space are employed as both of the calibration planes are parallel to the x - y plane. The position of the first calibration plane is given as $z=d_1$, and the second $z=d_2$. The position of the image plane may be left unknown.

For the determination of a line of sight of a given image point q , which has a homogeneous representation of $(r, c, 1)^t$ of image coordinates, n calibration points are selected in the first calibration plane G_1 whose images are located around q in the image plane. And also for the second calibration plane G_2 , n

Fig. 2 Camera model in the two-plane method

calibration points are selected similarly.

Let the 3D coordinates of the calibration points be denoted as

$$\mathbf{p}_{ij} = (x_{ij}, y_{ij}, d_i)^t \quad (1)$$

for $i=1$ and 2 according that the point belongs to the planes G_1 or G_2 , and where j is the index of points from 1 to n . And let the position of the image of the point $\mathbf{q}_{ij}$ be denoted as

$$\mathbf{q}_{ij} = (r_{ij}, c_{ij}, 1)^t \quad (2)$$

In the *linear* and *linear spline* method, the transformations between $\mathbf{p}_{ij}$ and $\mathbf{q}_{ij}$ are assumed to be expressed linearly as

$$\mathbf{p}_{ij} = A_i \mathbf{q}_{ij} \quad (j=1, 2, \dots, n) \quad (3)$$

where $A_i (i=1, 2)$ are 3×3 matrices.

For the set of n collected points, the relation can be expressed as

$$P_i = A_i Q_i \quad (i=1, 2) \quad (4)$$

where $P_i = [\mathbf{p}_{i1}, \mathbf{p}_{i2}, \dots, \mathbf{p}_{in}]$ and $Q_i = [\mathbf{q}_{i1}, \mathbf{q}_{i2}, \dots, \mathbf{q}_{in}]$.

At least, three points per plane are needed, respectively, to determine the A_1 and A_2 ($i.e. n \geq 3$). After A_i 's are determined, an image point $\mathbf{q}$ in the image plane can be "back-projected" into the planes G_1 and G_2 , respectively, as

$$\mathbf{u}_i = A_i \mathbf{q} \quad (i=1, 2) \quad (5)$$

Then, using the $\mathbf{u}_1$ and $\mathbf{u}_2$, the line of sight corresponding to the image point $\mathbf{q}$ is determined as the line passing through both of them.

To back-project the given point $\mathbf{q}$, some selection methods of $\mathbf{q}_{ij}$'s for the determination of A_i 's were proposed. One of the methods the *linear spline* interpolation method, is to use three nearest $\mathbf{q}_{ij}$'s to the $\mathbf{q}$ (when $n=3$) as shown in Fig. 3. Then, A_i is given as

$$A_i = P_i Q_i^{-1} \quad (6)$$

When more points are collected and a least square method is employed, the solution will be,

$$A_i = P_i Q_i^{-1} = P_i Q_i^t (Q_i Q_i^t)^{-1} \quad (7)$$

and this is called the *linear* interpolation method.

When lines of sight for several image points have been obtained, the optical center of the imaging system is determined as the point where the bundle of lines of sight encounter. The details of these procedures are skipped here and will be described later, because these stages are effective for our new algorithm.

There is a total of 18 coefficients in the matrices A_i (nine for each i) to be determined. In the case where

Fig. 3 Interpolation in the Linear Spline Method

the planes are parallel to the x - y plane so that the third row of the two matrices is known to be $(0, 0, d_i)$, the number of coefficients is reduced to 12 (six for each i). That is, the matrix A_i has the form of

$$A_i = \begin{pmatrix} A_{fi} & x_{i0} \\ 0 & 0 & d_i \end{pmatrix} \quad (8)$$

This means that in this scheme the mapping between 2D coordinates of the image point $(r, c)^t$ and the 2D coordinates of calibration point (x_i, y_i) in the calibration plane G_i is assumed to be given as

$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} = A_{fi} \begin{pmatrix} r \\ c \end{pmatrix} + \begin{pmatrix} x_{i0} \\ y_{i0} \end{pmatrix} \quad (9)$$

This mapping is of Affine transformation. Therefore, in the conventional two plane methods, the imaging of a pattern on a plane in 3D space is assumed to be an Affine transformation.

However, it is known that the perspective transformation between 2D planes in 3D space does not agree with Affine transformation. Only if the object plane and the image plane are nearly parallel, or if the set of points span within a local small spatial region, the perspective transformation is well approximated by the Affine transformation. For the former case, the approximation error depends on the slant of the image plane to the global z -axis. For the latter case, it depends on the arrangement and the density of the calibration points given on the calibration planes. These are the reasons why the performance and accuracy of the conventional two plane method becomes uncertain.

It is also known⁽⁶⁾⁽⁷⁾ that a perspective transforma-

tion between two given planes in 3D space will be defined uniquely when at least four pairs of point correspondences are given. The model on which the conventional method based, where the relational matrix A_i are determined with three points for each, lacks in the degree of freedom to express the mapping sufficiently. Therefore, it is clear that a new formulation of the two plane model is needed.

3. A Unified Model of Two Plane Camera Calibration

To modify the two plane method and present a new algorithm, here we formulate a unified camera model.

We define three coordinate systems; world coordinates, camera coordinates, and image coordinates, as shown in Fig. 4.

The world coordinate system O_o-XYZ is based on calibration planes. The calibration planes are placed so as to be $Z=Z_i$, and calibration points are given with (X, Y) coordinates.

The camera coordinate system is represented by O_i-xyz . O_i is the optical center and the z -axis is the optical axis of the camera. The image plane is corresponding to $z=F$, where F is the focal length. On the image plane, image coordinates are given as (u, v) independently with (x, y, z) coordinates, but the directions of x and y -axes are coincident with u and v -axes, respectively. The optical axis, z -axis, intersects the image plane at (u_0, v_0) , which corresponds to the image center on the image plane.

Of course, the position of O_i and the direction of z -axis with respect to O_o-XYZ system, as well as F and (u_0, v_0) , are unknown to be calibrated.

The two O_o-XYZ and O_i-xyz systems can be related by a rotation matrix R and a translation vector t as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + t \quad (10)$$

Then the perspective image of the point (x, y, z) in camera coordinates is given as

$$\begin{aligned} u &= (1/\rho_u) F \frac{x}{z} + u_0 \\ v &= (1/\rho_v) F \frac{y}{z} + v_0 \end{aligned} \quad (11)$$

where ρ_u and ρ_v are scale factors based on the physical pixel sizes of the image plane.

Fig. 4 Coordinate systems of camera, object and image plane

Substituting (10) in (11) and rearranging and introducing the new variables, the transformation from the object coordinates (X, Y, Z) to the image coordinates (u, v) can finally be expressed as

$$\begin{aligned} u &= \frac{a_1 X + a_2 Y + a_3 Z + T_1}{c_1 X + c_2 Y + c_3 Z + T_3} \\ v &= \frac{b_1 X + b_2 Y + b_3 Z + T_2}{c_1 X + c_2 Y + c_3 Z + T_3} \end{aligned} \quad (12)$$

For all the calibration points in G_1 , $Z=Z_1$, and for the points in G_2 , $Z=Z_2$, which are constants and given. Therefore, denoting the 2D coordinates of the points in G_1 as (X_1, Y_1) and their image coordinates as (u_1, v_1) , and the 2D coordinates in G_2 as (X_2, Y_2) and their image coordinates as (u_2, v_2) , we have

$$\begin{aligned} u_1 &= \frac{a_1 X_1 + a_2 Y_1 + a_{31}}{c_1 X_1 + c_2 Y_1 + c_{31}}, \\ v_1 &= \frac{b_1 X_1 + b_2 Y_1 + b_{31}}{c_1 X_1 + c_2 Y_1 + c_{31}} \end{aligned} \quad (13)$$

$$\begin{aligned} u_2 &= \frac{a_1 X_2 + a_2 Y_2 + a_{32}}{c_1 X_2 + c_2 Y_2 + c_{32}}, \\ v_2 &= \frac{b_1 X_2 + b_2 Y_2 + b_{32}}{c_1 X_2 + c_2 Y_2 + c_{32}} \end{aligned} \quad (14)$$

Because c_{31} in (13) is never equal to zero, setting $c_{31}=1$, we have equations for the variable $\{a_1, a_2, a_{31}, b_1, b_2, b_{31}, c_1, c_2\}$,

$$\begin{pmatrix} X_1 & Y_1 & 1 & 0 & 0 & 0 & -u_1 X_1 & -u_1 Y_1 \\ 0 & 0 & 0 & X_1 & Y_1 & 1 & -v_1 X_1 & -v_1 Y_1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_{31} \\ \vdots \\ c_2 \end{pmatrix} = \begin{pmatrix} u_1 \\ v_1 \end{pmatrix} \quad (15)$$

If we have at least four points in G_1 , and if any three of which are not co-linear, we can solve (15) and obtain the coefficients $\{a_1, a_2, a_{31}, b_1, b_2, b_{31}, c_1, c_2\}$ to transform G_1 to the image plane.

Then, the remaining unknown parameters $\{a_{32}, b_{32}, c_{32}\}$ for G_2 are obtained by substituting above obtained coefficients into (14) and reformulating it as

$$\begin{aligned} -a_{32} + u_2 c_{32} &= a_1 X_2 + a_2 Y_2 - u_2(c_1 X_2 + c_2 Y_2) \\ -b_{32} + v_2 c_{32} &= b_1 X_2 + b_2 Y_2 - v_2(c_1 X_2 + c_2 Y_2) \end{aligned} \quad (16)$$

At least two calibration points in G_2 and their image points are needed to obtain $\{a_{32}, b_{32}, c_{32}\}$.

Therefore, at least six points, four of which are in one plane and two in the other plane, are required to determine the transformation from the object planes to the image. If more points are available, the least square solution can be employed to reduce the errors.

After the coefficients of transformation of "projection" into the image plane are determined, we can "back-project" a given image point (u, v) into the planes G_1 or G_2 by using the next relation derived from (13) and (14);

$$\begin{aligned} X_i &= \frac{A_{1i}u + A_{2i}v + A_{3i}}{C_1u + C_2v + C_3}, \\ Y_i &= \frac{B_{1i}u + B_{2i}v + B_{3i}}{C_1u + C_2v + C_3} \end{aligned} \quad (17)$$

where $i=1, 2$ and

$$\begin{aligned} A_{1i} &= c_{3i}b_2 - b_{3i}c_2, \quad A_{2i} = a_{3i}c_2 - c_{3i}a_2 \\ A_{3i} &= b_{3i}a_2 - a_{3i}b_2, \quad B_{1i} = -c_{3i}b_1 + b_{3i}c_1, \\ B_{2i} &= -a_{3i}c_1 + c_{3i}a_1, \quad B_{3i} = -b_{3i}a_1 + a_{3i}b_1 \\ C_1 &= b_1c_2 - b_2c_1, \quad C_2 = c_1a_2 - c_2a_1, \quad C_3 = a_1b_2 - a_2b_1 \end{aligned} \quad (18)$$

The 3D coordinates (X_1, Y_1, Z_1) and (X_2, Y_2, Z_2) given by (17) are the corresponding back-projected points on G_1 and G_2 , respectively.

4. Camera Calibration by Two Plane Back Projection

4.1 Optical Center

For a given image point, we just provided the corresponding back-projected points in G_1 and G_2 . Let those points be denoted as $\mathbf{u}_1(X_1, Y_1, Z_1)$ and $\mathbf{u}_2(X_2, Y_2, Z_2)$, respectively. The line passing those points $\mathbf{u}_1$ and $\mathbf{u}_2$ in 3D space means the line of sight of the image point (u, v) .

The optical center O_i of the imaging system is defined to be the point where all the lines of sight pass

through. The coordinates of O_i with respect to the object coordinate system O_0-XYZ can be computed through the two step computation. Determining the lines of sight firstly and then the point they are passing, the optical center is obtained by solving a set of linear equations.

This can be done by following the technique proposed in 2). Here we summarize it.

Theoretically, the optical center is determined by two lines of sight as their crossing point. But this computation is sensitive to the computational round-off errors. So, we define the "effective" optical center as the nearest point from each line of sight (Fig. 5).

Let the distance in 3D space between a point $(x, y, z)^t$ and a line which passes $(x_1, y_1, z_1)^t$ with a direction vector $(a, b, c)^t$ (where $a^2 + b^2 + c^2 = 1$) be denoted as d . Then d is given as

$$d^2 = \left| \frac{y - y_1}{b} - \frac{z - z_1}{c} \right|^2 + \left| \frac{z - z_1}{c} - \frac{x - x_1}{a} \right|^2 + \left| \frac{x - x_1}{a} - \frac{y - y_1}{b} \right|^2 \quad (19)$$

Now, let the calibration points in G_1 and G_2 be used as (x_i, y_i, z_i) 's, and the directional vectors of their lines of sight be (a_i, b_i, c_i) 's. Also, let the coordinates of the effective optical center be (f_x, f_y, f_z) , and the sum of square of the distances from it to all the lines of sight be $D_c (D_c = \sum d_i^2)$. Then, (f_x, f_y, f_z) can be obtained by minimizing D_c , so that,

$$\frac{\partial D_c}{\partial x} \Big|_{x=f_x} = \frac{\partial D_c}{\partial y} \Big|_{y=f_y} = \frac{\partial D_c}{\partial z} \Big|_{z=f_z} = 0 \quad (20)$$

Rewriting these equations in matrix-vector forms, we have

$$\mathbf{h} = \mathbf{H}\mathbf{f} \quad (21)$$

where

$$\begin{aligned} \mathbf{h} &= (\sum (ck_2 - bk_3), \sum (ak_3 - ck_1), \sum (bk_1 - ak_2))^t \\ (k_1 &= z_1b - y_1c, \quad k_2 = x_1c - z_1a, \quad k_3 = y_1a - x_1b) \\ \mathbf{H} &= \begin{pmatrix} \sum (b^2 + c^2) & -\sum ab & -\sum ac \\ -\sum ab & \sum (a^2 + c^2) & -\sum bc \\ -\sum ac & -\sum bc & \sum (a^2 + b^2) \end{pmatrix} \end{aligned} \quad (22)$$

$$\mathbf{f} = (f_x, f_y, f_z)^t$$

(Subscripts i are omitted for the simplicity.)

Therefore, the $(f_x, f_y, f_z)^t$ will be given by

$$\mathbf{f} = \mathbf{H}^{-1}\mathbf{h} \quad (23)$$

4.2 Optical Axis and Pose of Image Plane

The optical axis and the pose of the image plane are given as to fit the image points to the points at which the corresponding lines of sight across the image plane (Fig. 6). However, the actual available

Fig. 5 Determination of the effective optical center

algorithm to determine it has not been presented yet in 1)~5). Here, we present a new scheme for it.

In this subsection, all 3D coordinate vectors are defined based on the object coordinate system O_o -XYZ. Firstly, we define and list the additional vectors and parameters used here.

- $\mathbf{p} = (p_x, p_y, p_z)^t$: a point in 3D space,
- $\mathbf{l}$: unit directional vector from optical center to the point $\mathbf{p}$,
- $\mathbf{P}$: crossing point of the line-of-sight of $\mathbf{l}$ onto the image plane,
- $\mathbf{P}'$: spatial position of image point (u, v) of $\mathbf{p}$,
- $\mathbf{g}$: unit directional vector of optical axis,
- $\mathbf{r} = (r_x, r_y, r_z)^t$, $\mathbf{c} = (c_x, c_y, c_z)^t$: vectors defining spatial sizes and directions of one pixel in the image plane in u and v directions, respectively.

These are shown in Fig. 7.

The unit directional vector of the line of sight for a point $\mathbf{p}$ is given as

$$\mathbf{l} = \frac{\mathbf{p} - \mathbf{f}}{|\mathbf{p} - \mathbf{f}|} \quad (24)$$

Now, from $(\mathbf{P} - \mathbf{f}) \cdot \mathbf{g} = F$, which is the focal length,

$$\mathbf{P} = \mathbf{f} + \frac{F}{(\mathbf{l} \cdot \mathbf{g})} \cdot \mathbf{l} \quad (25)$$

On the other hand, $\mathbf{P}'$ is given as

$$\mathbf{P}' = \mathbf{f} + F\mathbf{g} + (u - u_0)\mathbf{r} + (v - v_0)\mathbf{c} \quad (26)$$

$\mathbf{P}$ does not necessarily coincide with $\mathbf{P}'$. Then, we denote the difference between $\mathbf{P}$ and $\mathbf{P}'$ on the image plane with Δ , that is,

$$\Delta = |\mathbf{P} - \mathbf{P}'| \quad (27)$$

Then, Δ^2 is given as

$$\Delta^2 = F^2 \left(\frac{1}{(\mathbf{l} \cdot \mathbf{g})^2} - 1 \right) - 2F(u - u_0) \frac{(\mathbf{l} \cdot \mathbf{r})}{(\mathbf{l} \cdot \mathbf{g})} - 2F(v - v_0) \frac{(\mathbf{l} \cdot \mathbf{c})}{(\mathbf{l} \cdot \mathbf{g})} + (u - u_0)^2 |\mathbf{r}|^2 + (v - v_0)^2 |\mathbf{c}|^2 \quad (28)$$

Fig. 6 Determination of the effective optical axis

$$= \left((u - u_0) |\mathbf{r}| - F \frac{(\mathbf{l} \cdot \mathbf{r}')}{(\mathbf{l} \cdot \mathbf{g})} \right)^2 + \left((v - v_0) |\mathbf{c}| - F \frac{(\mathbf{l} \cdot \mathbf{c}')}{(\mathbf{l} \cdot \mathbf{g})} \right)^2 \quad (29)$$

(where $\mathbf{r}' = \mathbf{r}/|\mathbf{r}|$, and $\mathbf{c}' = \mathbf{c}/|\mathbf{c}|$).

Because $\mathbf{g}$, $\mathbf{r}'$ and $\mathbf{c}'$ make up an orthonormal coordinate system and $|\mathbf{l}|=1$,

$$(\mathbf{l} \cdot \mathbf{g})^2 + (\mathbf{l} \cdot \mathbf{r}')^2 + (\mathbf{l} \cdot \mathbf{c}')^2 = 1 \quad (30)$$

For all the test point $\mathbf{p}_i (i=1, \dots, N)$, let the total sum of their squared differences be D_p , that is, letting $\Delta_i = |\mathbf{P}_i - \mathbf{P}'_i|$,

$$D_p = \sum_i \Delta_i^2 \quad (31)$$

$$= \sum_i \left((u_i - u_0) |\mathbf{r}| - F \frac{(\mathbf{l}_i \cdot \mathbf{r}')}{(\mathbf{l}_i \cdot \mathbf{g})} \right)^2 + \left((v_i - v_0) |\mathbf{c}| - F \frac{(\mathbf{l}_i \cdot \mathbf{c}')}{(\mathbf{l}_i \cdot \mathbf{g})} \right)^2$$

Fig. 7 Definitions and relations of spatial vectors used in the determination of effective optical axis and image coordinates

$$+ \sum_i \left((v_i - v_0) |c| - F \frac{(l_i \cdot c')}{(l_i \cdot g)} \right)^2 \quad (32)$$

Then, introducing new temporal unknown variables $R = Fr/|r|^2 + u_0 g$ and $C = Fc/|c|^2 + v_0 g$, (32) becomes

$$D_p = |r|^2 \sum_i \left(u_i - \frac{(l_i \cdot R)}{(l_i \cdot g)} \right)^2 + |c|^2 \sum_i \left(v_i - \frac{(l_i \cdot C)}{(l_i \cdot g)} \right)^2 \quad (33)$$

If we employ the test points p_i so as their images are within region whose size is sufficiently small compared to the focal length F (which is called "small image assumption" and this is a usual case), we may consider that $(l_i \cdot g)$ is constant for all the test point. For these case, the minimization of D_p is established when $\sum_i (u_i(l_i \cdot g) - (l_i \cdot R))^2 \rightarrow \min$. and $\sum_i (v_i(l_i \cdot g) - (l_i \cdot C))^2 \rightarrow \min$.

Now we need the ratios of the components of g to determine the direction of the optical axis. So, this can be achieved by setting $g_z = 1$ and solving the next linear equation by the least square error method: Denote the components of R and C with (R_x, R_y, R_z) and (C_x, C_y, C_z) , respectively, and l_i with (l_{ix}, l_{iy}, l_{iz}) . Then, solve

$$\begin{bmatrix} u_1 l_{x1} & u_1 l_{y1} & -l_{x1} & -l_{y1} & -l_{z1} & 0 & 0 & 0 \\ v_1 l_{x1} & v_1 l_{y1} & 0 & 0 & 0 & -l_{x1} & -l_{y1} & -l_{z1} \\ u_2 l_{x2} & u_2 l_{y2} & -l_{x2} & -l_{y2} & -l_{z2} & 0 & 0 & 0 \\ v_2 l_{x2} & v_2 l_{y2} & 0 & 0 & 0 & -l_{x2} & -l_{y2} & -l_{z2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_n l_{xn} & u_n l_{yn} & -l_{xn} & -l_{yn} & -l_{zn} & 0 & 0 & 0 \\ v_n l_{xn} & v_n l_{yn} & 0 & 0 & 0 & -l_{xn} & -l_{yn} & -l_{zn} \end{bmatrix} \times \begin{bmatrix} g_x \\ g_y \\ R_x \\ R_y \\ R_z \\ C_x \\ C_y \\ C_z \end{bmatrix} = \begin{bmatrix} u_1 l_{z1} \\ v_1 l_{z1} \\ u_2 l_{z2} \\ v_2 l_{z2} \\ \vdots \\ u_n l_{zn} \\ v_n l_{zn} \end{bmatrix} \quad (34)$$

for given n test points. The resulted solution g must be normalized as $|g| = 1$.

Using these R , C , and their definitions, and the fact that $(g \cdot r) = (g \cdot c) = 0$, the image center coordinates are given as,

$$u_0 = (R \cdot g) \quad (35)$$

$$v_0 = (C \cdot g) \quad (36)$$

This (u_0, v_0) is the crossing point of the optical axis

on the image plane.

Then, using (u_0, v_0) , the focal length can be obtained as

$$F = \sqrt{(u - u_0)^2 / |r|^2 + (v - v_0)^2 / |c|^2} / (l \cdot g) \quad (37)$$

5. Lens Distortion Models

Our imaging model of (10), (11) and (12) are based on the exact pinhole model. The proposed technique is to determine the "effective" optical center and optical axis with the least mean square techniques. Therefore, it works well under some lens distortions as shown in the experiments later.

But, there may be cases where non-linear lens distortions must be taken into account. It has been believed that, when non-linear distortions were assumed, the determination of camera intrinsic parameters must involve non-linear iterative search⁹⁾, even in the two plane method by Martins *et al.*¹⁾ In their method, to avoid this, two reference calibration planes must be parallel to image plane, which was not realistic as was pointed out.

However, our two plane camera calibration method can be carried out using only linear solutions while considering all lens distortions.

If there exist lens distortions, the left hand sides of the perspective imaging of (11) must be replaced as

$$u \rightarrow u + \delta u, \text{ and } v \rightarrow v + \delta v \quad (38)$$

where $(\delta u, \delta v)$ are corrections to compensate for those distortions.

For well designed and manufactured lenses, it was reported and verified to be sufficient to take account the following components^{8),9)}:

$$\begin{aligned} \delta u &= \underbrace{k_1 u(u^2 + v^2)}_{\text{radial}} + \underbrace{p_1(3u^2 + v^2) + 2p_2 uv}_{\text{decentering}} + \underbrace{s_1(u^2 + v^2)}_{\text{thin prism}} \\ \delta v &= \underbrace{k_1 v(u^2 + v^2)}_{\text{radial}} + \underbrace{2p_1 uv + p_2(u^2 + 3v^2)}_{\text{decentering}} + \underbrace{s_2(u^2 + v^2)}_{\text{thin prism}} \end{aligned} \quad (39)$$

Substituting (u, v) by $(u + \delta u, v + \delta v)$ into (17), we arrive at following general forms;

$$\begin{aligned} X_1 &= \sum_{i+j=0}^3 A_{ij}^{(1)} u^i v^j / \sum_{i+j=0}^3 C_{ij}^{(1)} u^i v^j \\ Y_1 &= \sum_{i+j=0}^3 B_{ij}^{(1)} u^i v^j / \sum_{i+j=0}^3 C_{ij}^{(1)} u^i v^j \end{aligned} \quad (40)$$

$$\begin{aligned} X_2 &= \sum_{i+j=0}^3 A_{ij}^{(2)} u^i v^j / \sum_{i+j=0}^3 C_{ij}^{(2)} u^i v^j \\ Y_2 &= \sum_{i+j=0}^3 B_{ij}^{(2)} u^i v^j / \sum_{i+j=0}^3 C_{ij}^{(2)} u^i v^j \end{aligned} \quad (41)$$

as the *back-projection* transformation.

Equations (40) and (41) constitute a set of *linear* equations on the coefficients $\{A_{ij}^{(k)}, B_{ij}^{(k)}, C_{ij}^{(k)}\} (k=1, 2)$, and we can directly solve for them by linear computations. It is worth of noting that

$$C_{ij}^{(1)}=0, C_{ij}^{(2)}=0 : \text{for } 0 < i+j \leq 3 \quad (42)$$

$$B_{ij}^{(1)}=0, A_{ij}^{(2)}=0 : \text{for } i+j=3 \quad (43)$$

With these transformation coefficients we can also map each image point onto the two calibration planes G_1 and G_2 to get two corresponding object points. Connecting those two points will result in the line of sight for the given image point. This computation does not involve any nonlinear operation while considering nonlinear lens distortions.

6. Experimental Results

Two experimental results will be shown in this section. One is for synthesized images of experimental simulation, and the other is for actual images taken with a commercial CCD camera.

Fig. 8 shows a synthesized image assuming the positional relation between the object coordinates and the camera coordinates are given as the rotation angle with respect to x, y and z axes are $\pi/8, \pi/4$ and $\pi/6$, respectively, and translation in x, y and z directions are (20, 30, 500) in mm. The focal length of the camera is 50 mm. The calibration plane G_1 is given at $Z_1=0$, and G_2 at $Z_2=20$ mm. Two sets of 25 test points are given in each plane at 5×5 grid points.

From this synthesized image without image reading noise, by using the present method, the coordinates of the optical center $(-355.2, 101.4, -338.9)$ in mm with respect to the object coordinate system and the focal length 50.0 mm which are exact values used in the synthesis of the image are obtained.

In this method, the main errors were introduced in readings of image points. Then, to the readings of image points we added Gaussian noises with a standard deviation of 1 pixel size. The average calibration errors in standard deviation for 100 samples were as follows: The error of the focal length was 0.26 mm for the true value of 50 mm, those of the x and y components of the effective center were about 0.7 mm and that of z component was 5.3 mm for a distance of about 500 mm, and that of the optical center was about 5 pixels in both u and v directions.

Fig. 9 shows images of a block total sizes of which are $5.05 \text{ cm} \times 5.05 \text{ cm} \times 12.65 \text{ cm}$. They are sequence of 20 images taken by a moving camera, and the first, 10th and 20th images are shown. In each image 25 vertices are visible, which are labeled from 1 to 25, and the bold lines show the sequence of those vertex points. The image coordinates of those points were read out manually.

The origin of the image coordinates $(u, v)=(0, 0)$ was set to be random and off the screen to show that this technique does not depend on the set-up of the image coordinates.

The origin of the object coordinates $(X, Y, Z)=(0, 0, 0)$ was set at the vertex 25, X -axis was on the edge including vertices 1 and 25, and Y -axis on the edge including vertices 18 and 25. This block can be interpreted to have four parallel planes to X - Y plane, the first of which consists of the points {1, 8, 9, 12, 13, 16, 18, 19, 21, 22, 24, 25}, the second {10, 11, 14, 15, 20, 23}, the third {4, 5}, and the fourth {2, 3, 6, 7}.

In the experiments, we used the first plane as the first calibration plane, which includes 12 test points, and the fourth as the second calibration plane, which includes four test points. The physical pixel pitches on the CCD image sensor were given as 0.009836 mm in u and 0.0092784 mm in v .

We solved the calibration problem with non-lens-distortion model given in Section 3 and 4, and also with lens distortion model given in Section 5.

Fig. 8 Image of two calibration planes used in an experimental simulation

Table 1 is a list of a part of calibration results for the image sequence. The images in the sequence which had not good quality to read out the vertex positions were skipped in the experiment and the result of which are not listed.

The effective center and the effective axis listed were obtained using non-distortion model. The results shows that the camera movement had constant rotational and translational velocities, which was the case of this experiment. To verify the positioning results, the calculated effective centers were fitted to the true movement trajectory of the camera, and

evaluated the errors. Their standard deviations were about 3 mm in both X and Y directions and 10.1 mm in Z direction with object distance of about 75 cm. The values of effective center and axis obtained using the distortion model were slightly different from these values, but within 1% for all values. This is because the lens used in the experiments has high quality and small distortion, and also because these "effective" values are robust for small lens distortions.

Two sets of resulted values of focal lengths are listed, one was with non-distortion model and other was with distortion model. They must be constant through out the results within each respective set. The calibrated values with non-distortion model had a standard deviation of 0.29 mm for the average of 19.3 mm, which was 1.5%. That with distortion model has 0.11 mm of standard deviation for 19.2 mm of the average value (0.6%). The value with distortion model was slightly improved, which also means the lens had good quality.

These calibrated values are fairly good in comparison with the results reported in the literatures.

7. Conclusions

In this paper, we examine the two plane calibration model in terms of perspective mapping, and present a new effective method by taking the advantage of the two plane method and modifying it based on the

(a) First image in sequence

(b) 10th image in sequence

(c) 20th image in sequence

Fig. 9 Image sequence by an actual moving camera. (a) First image in the sequence, (b) 10th image, and (c) 20th image

Table 1 Camera calibration results from images of Fig. 9. Focal lengths with (non-dist.) were by no-lens-distortion model, and those with (distortion) were by lens-distortion model

Image number	Effective center (cm)			Effective axis			Focal length (non-dist.) (mm)	Focal length (distortion) (mm)
	X	Y	Z	g_x	g_y	g_z		
1	37.27	40.46	-70.44	-0.38	-0.39	0.87	19.62	19.53
2	37.24	40.01	-70.97	-0.39	-0.39	0.87	19.70	19.57
3	37.07	39.67	-71.63	-0.40	-0.38	0.87	19.66	19.41
4	38.41	39.15	-71.93	-0.41	-0.38	0.85	19.81	19.53
9	38.30	38.40	-72.92	-0.43	-0.36	0.84	19.31	19.20
10	35.46	38.06	-73.25	-0.44	-0.35	0.84	19.10	19.01
12	36.57	37.70	-73.75	-0.45	-0.35	0.84	19.23	19.28
13	39.41	37.18	-74.54	-0.46	-0.35	0.84	18.90	19.10
14	39.17	37.04	-75.02	-0.46	-0.34	0.83	19.52	19.37
15	34.97	36.85	-75.84	-0.46	-0.33	0.83	19.13	19.20
18	34.54	35.94	-77.19	-0.48	-0.31	0.82	18.79	19.08
20	34.14	35.46	-77.83	-0.48	-0.31	0.81	19.30	19.20

exact perspective imaging. It also takes the great advantage of the conventional two plane method that each of intrinsic and extrinsic camera parameters can be obtained by solving linear equations only, so that the solutions will be provided uniquely and stably.

As mentioned above, the main errors are introduced in readings of image points. The analysis of this kind of error and the stability conditions are the next problem. They must be verified under well-defined experimental protocols.

The performance tests under large lens distortions are also next problem.

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