Compliance Control Considering a Robot's Manipulability[†]

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In this paper, we present a new compliance control strategy for a robot manipulator to interact with its uncertain environment. Our strategy considers the manipulability explicitly in designing the robot's compliance. Computer simulations of the crank rotation task show that, our approach results in smaller interaction forces and better positional accuracy compared to the conventional compliance control.

Key Words: robot, manipulability, compliance control, geometric uncertainty of environment, crank task

1. Introduction

Compliance control is one of the most effective approach for a robot manipulator to interact with its uncertain environment¹⁾. In order to design a robot's compliance, we should study on how to select the compliance coordinate frame, how to set the compliance center, and how to specify the compliance values along each coordinate. Usually, we are given a model of the environment constraints, and we focused on how to comply with this task-imposed constraint in designing the robot's compliance. In order to overcome the large interaction force generated by the environment's model uncertainties (as well as the control errors), we specified a large compliance along the environment constraint subspace. We then specified a small compliance along the free motion subspace to increase the robot's positional accuracy. This design however, did not considered the task performer's ability—the robot's manipulability. As we will show in this paper, when the environment has geometric uncertainties, the manipulability will influence the robot's task performance. Therefore, in this paper, we present a new compliance control

strategy which considers the manipulability²⁾ explicitly in designing the robot's compliance. We prove that the resultant compliance matrix is positive definite, thus the robot's control system is stable. Computer simulations of the crank rotation tasks show that, our approach results in smaller interaction forces and better positional accuracy compared to the conventional compliance control.

2. Compliance Control

We consider an example of crank rotation task as shown in Fig.1, where the robot is geometrically constrained by the crank. The task requirement is to turn the crank with a constant angle velocity. Usually, if we do not know the crank's real size (the black circle in Fig. 1) and center location (a, b), we design the robot's compliance on the estimated circle (the gray one). In detail, we first set the compliance center on X_d , the desired trajectory. Then, we select the compliance coordinate Σ_d along the tangent and the normal directions of the estimated gray circle.

Fig. 1 Robot's crank task

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Finally, we design a large compliance c_n along the normal $\mathbf{n}_d$ and a small compliance c_t along the tangent $\mathbf{t}_d$, respectively.

From the relations

$$\Delta \mathbf{X} = \mathbf{J}(\theta) \Delta \theta \quad (1)$$

$$\mathbf{F}_{\text{int}} = \mathbf{R} \mathbf{K}_x \mathbf{R}^T \Delta \mathbf{X} \quad (2)$$

and

$$\tau = \mathbf{J}^T \mathbf{F}_{\text{int}} \quad (3)$$

we obtain the robot's control torque as

$$\tau = \mathbf{J}^T \mathbf{R} \mathbf{K}_x \mathbf{R}^T \mathbf{J} \Delta \theta \quad (4)$$

where $\mathbf{X}$ is the robot end-effector's real position vector in the world space Σ_0 , θ is the robot's joint angle vector, $\Delta \mathbf{X} = \mathbf{X}_d - \mathbf{X}$ is the robot end-effector's position error. $\mathbf{J}(\theta)$ is the Jacobian matrix, $\mathbf{R}$ is the rotation matrix between Σ_0 and Σ_d . $\mathbf{F}_{\text{int}}$ is the contact force and τ is the robot's control torque. $\mathbf{K}_x$ is the robot's stiffness matrix as follows.

$$\mathbf{K}_x = \mathbf{C}_x^{-1} = \begin{bmatrix} c_n^{-1} & 0 \\ 0 & c_t^{-1} \end{bmatrix} \quad (5)$$

3. Robot's Manipulability and Compliance Design

When the crank has geometric uncertainties, the compliance coordinate Σ_d we selected in section 2 will not coincide with the real crank's normal and tangent $(\mathbf{n}, \mathbf{t})$ on Σ_c . If there is a large position error $\Delta \mathbf{X} = \mathbf{X}_d - \mathbf{X}$, then from eqs. (4) and (5), a small compliance c_t will result in a large motion driving force along the tangent direction of $\mathbf{t}_d$, which will be projected to the real crank's normal $\mathbf{n}$ and generate a large interaction force between the robot and the crank on $\mathbf{n}$. Conversely, a large compliance c_n on the estimated normal $\mathbf{n}_d$ will be projected to the real crank's tangent $\mathbf{t}$, which will reduce the robot's positional performance along $\mathbf{t}$, the real free motion subspace. Consequently, the conventional compliance control may also results in a large interaction force and a bad positional performance, when the environment has geometric uncertainties.

In order to realize the conventional compliance as well as to prevent the large contact force, we define a cost function as follows

$$E = \frac{1}{2} \|\tau - \mathbf{J}^T \mathbf{R} \mathbf{K}_x \mathbf{R}^T \Delta \mathbf{X}\|^2 + \frac{\lambda}{2} \|\mathbf{J}^{-T} \tau\|^2 \quad (6)$$

where λ is a weight coefficient. We propose an new approach to design the robot's control torque τ which

minimizes this cost function.

Differentiating the eq. (6) with respect to τ , we then simply get the robot's new control torque as follows.

$$\tau = [\lambda(\mathbf{J}^T \mathbf{J})^{-1} + \mathbf{I}]^{-1} \mathbf{J}^T \mathbf{R} \mathbf{K}_x \mathbf{R}^T \mathbf{J} \Delta \theta \quad (7)$$

Comparing eq. (4) with eq. (7), the equivalent new stiffness matrix with respect to the coordinate Σ_d is derived as

$$\mathbf{K}_x^{\text{equ}} = \mathbf{R}^T [\lambda(\mathbf{J}^T \mathbf{J})^{-1} + \mathbf{I}]^{-1} \mathbf{R} \cdot \mathbf{K}_x \quad (8)$$

where $\mathbf{I}$ is a identity matrix.

As studied in Ref. (3), the matrix $(\mathbf{J} \mathbf{J}^T)^{-1}$ measures the robot's manipulability and its inverse measures the robot's manipulating force. We define the singular values of $\mathbf{J}$ as σ_1, σ_2 , and the principal axes of the manipulability ellipsoid $\Delta \mathbf{X}^T (\mathbf{J} \mathbf{J}^T)^{-1} \Delta \mathbf{X} = 0$ as $\sigma_1 \mathbf{u}_1$ and $\sigma_2 \mathbf{u}_2$. Then, as larger as σ_1 is, as easier as the robot moves and as difficult as the robot generates the manipulating force along the axe $\mathbf{u}_1$. From the eq. (8), it is understood that

$$\mathbf{K}_x^{\text{equ}} \propto \mathbf{R}^T \begin{bmatrix} \frac{\sigma_1^2}{\lambda + \sigma_1^2} & 0 \\ 0 & \frac{\sigma_2^2}{\lambda + \sigma_2^2} \end{bmatrix} \mathbf{R} \cdot \mathbf{K}_x \quad (9)$$

That is, the robot's stiffness becomes larger along those directions where the robot has large manipulability and small manipulating force, this improves the robot's positional accuracy. Conversely, along those directions where the robot has small manipulability and large manipulating force, the robot's stiffness becomes smaller which prevents the large interaction force. Therefore, the manipulability can effectively be used to improve the robot's task performance.

We now study the stability problem of our control strategy. It is understood, that the robot control system is stable if the robot's stiffness matrix is positive definite⁴⁾. In our strategy, if the conventional stiffness matrix $\mathbf{K}_x$ is positive definite, then it is easy to prove that $\mathbf{K}_x^{\text{equ}}$ in eq. (8) is also positive definite. Therefore, our control system is stable. We can also justify this result using An's method in Ref. (5).

4. Computer Simulations

The simulation results of the crank rotation task performed by a two degree of freedom robot manipulator are shown in Fig. 2 and 3. Fig. 2 is the result of

(a) Contact force

(b) Crank angle

Fig. 2 Simulation results of the conventional approach

(a) Contact force

(b) Crank angle

Fig. 3 Simulation results of our approach

the conventional compliance control (eq. (4)), and Fig. 3 is the result of our approach (eq. (7)). Fig. 4 shows three kinds of ellipsoids along the crank's rotation circle, (a) shows the conventional stiffness ellipsoids, (b) is the robot's manipulability ellipsoids,

(a) Conventional stiffness ellipsoids

(b) Robot's manipulability ellipsoids

(c) Our stiffness ellipsoids

Fig. 4 Ellipsoids along the crank's rotation circle

and (c) is our stiffness ellipsoids. By considering the robot's manipulability in its compliance design, we not only obtained a smooth crank rotation, but also prevent the large interaction force.

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