A Synthesis for Robust Tracking Systems Based on $\mathcal{H}_\infty$ Control

Shinji HARA*, Hisaya FUJIOKA**, Tsugumoto KOSUGIYAMA*** and Toru ASAI*

In this paper, we consider a synthesis problem of a feedback compensator which achieves the following three design specifications: 1) closed-loop internal stability, 2) desired feedback characteristics such as robust stability, sensitivity reduction and disturbance attenuation, and 3) robust tracking. We show that the Glover-Doyle's solution on $\mathcal{H}_\infty$ control problem can be directly applied to the case where the integrated error is evaluated if we modify the problem appropriately. The properties of the Riccati equations to be solved and the order reduction of the resultant controller are also investigated. The proposed method is applied to the position control of a linear AC servo motor.

Key Words: robust tracking, $\mathcal{H}_\infty$ control

1. Introduction

In this paper, we consider a synthesis problem of a feedback compensator which achieves the following three design specifications: 1) closed-loop internal stability, 2) desired feedback characteristics such as robust stability, sensitivity reduction and disturbance attenuation, and 3) robust tracking.

The $\mathcal{H}_\infty$ control is a powerful tool to accomplish the first two specifications. It was shown that the third specification can be also expressed as a norm constraint. There are several approaches to design a servo controller based on the $\mathcal{H}_\infty$ control. Sugie et al.8) developed a method using the free parameters to obtain a servo controller. The drawback of the method is its indirect specification for the low frequency characteristics. Augmented $\mathcal{H}_\infty$ control problems with integral type weighting functions have been investigated in 4), 7), 9). They, however, are not solvable with usual technique such as Glover-Doyle's solution 2), since the related generalized plants are not detectable for the modes of reference inputs and it has a j$\omega$-axis zeros caused by the integrator in the weighting function 3).

The purpose of this paper is to propose an approach to design an integral type controller based on the $\mathcal{H}_\infty$ control. To overcome the difficulty mentioned above, we transform the augmented problem to a modified problem with which we can apply the usual technique in Section 3. The method can be applied to any $\mathcal{H}_\infty$ problem and the usual package can be used without modification. In Section 4, we will show the properties of the Riccati equations to be solved and the order reduction of the resultant controller. We then apply the proposed method to the position control of a linear AC servo motor to verify the usefulness of the proposed method. The experimental results are shown in Section 5.

2. Problem Formulation

Consider a unity feedback control system shown in Fig. 1, where $P(s)$ and $K(s)$ denote the plant and controller, respectively. The purpose of the control system design is to find a feedback compensator
Fig. 1 Feedback Control System

Fig. 2 Original Problem

The integral type weighted sensitivity function \( \frac{1}{s} (I + PK)^{-1} s \) must be stable for any perturbed plant \( \tilde{P} \) for complete robust tracking, and it is well known that the integral type controller is required for the robust tracking. Also note that the smaller norm of the integral type weighted sensitivity function

\[
\left\| \frac{W_1}{s} (I + PK)^{-1} W_2 \right\|_{\infty}
\]

yields the better performance for the tracking, where \( W_1(s) \) and \( W_2(s) \) are appropriate unimodular weighting functions\(^4,9\).

It is well known that the \( \mathcal{H}_\infty \) control is useful for the first two specifications with an appropriate generalized plant \( G_o(s) \) having exogenous input \( w \), controlled output \( z_0 \), control input \( u \), and measured output \( y \). Hence, we can formulate the synthesis problem

\[
K(s) \text{ which satisfies the following three specifications:}
\]

1) closed-loop internal stability,
2) desired feedback characteristics such as robust stability, sensitivity reduction and disturbance attenuation, and
3) robust tracking.

Here, the robust tracking means that the output of the plant \( y(t) \) tracks for any step type reference command \( r(t) \) without steady-state error, i.e.,

\[
\lim_{t \to \infty} e(t) = 0,
\]

under the plant perturbation and/or step type disturbance inputs \( d(t) \).

The integral type weighted sensitivity function \( \frac{1}{s} (I + PK)^{-1} s \) must be stable for any perturbed plant \( \tilde{P} \) for complete robust tracking, and it is well known that the integral type controller is required for the robust tracking. Also note that the smaller norm of the integral type weighted sensitivity function

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It is well known that the \( \mathcal{H}_\infty \) control is useful for the first two specifications with an appropriate generalized plant \( G_o(s) \) having exogenous input \( w \), controlled output \( z_0 \), control input \( u \), and measured output \( y \). Hence, we can formulate the synthesis problem

as an \( \mathcal{H}_\infty \) control problem with an additional controlled variables \( z_e = \frac{W_e(s)}{s} y \) as depicted in Fig. 2:

**Robust Tracking \( \mathcal{H}_\infty \) Control Problem:**
Find an integral type controller \( K(s) \) which satisfies

\[ S1 \) \( K(s) \) stabilizes \( G_o(s). \)

\[ S2 \]

\[
\|T(s)\|_\infty = \left\| \frac{T_{wz_0}(s)}{T_{wz_e}(s)} \right\|_\infty < \gamma
\]

where \( T_{wz_0}(s) \) denotes the transfer function from \( w \) to \( z_0 \) and it is related to the second requirement \( \|T_{wz_0}(s)\|_\infty \). The third specification, robust tracking, is reflected by \( \|T_{wz_e}(s)\|_\infty \), where \( T_{wz_e}(s) \) denotes the transfer function from \( w \) to \( z_e \). The generalized plant \( G_o(s) \) is expressed as

\[
G_o(s) = \begin{bmatrix}
G_{11}(s) & G_{12}(s) \\
-W_e(s) G_{21}(s) & -W_e(s) G_{22}(s) \\
G_{21}(s) & G_{22}(s)
\end{bmatrix}
\]

(1)

where

\[
G_o(s) = \begin{bmatrix}
G_{11}(s) & G_{12}(s) \\
G_{21}(s) & G_{22}(s)
\end{bmatrix}
\]

(2)

We, however, have a difficulty to solve the \( \mathcal{H}_\infty \) control problem for \( G_o \), since it does not satisfy the detectability from \( y \), i.e., the state of \( \frac{W_e(s)}{s} \) is not detectable. Thus, we can not directly apply the usual Glover-Doyle's solution\(^2\) for the \( \mathcal{H}_\infty \) control problem defined above.

3. Main Results

In order to overcome the difficulty in solving the \( \mathcal{H}_\infty \) control problem for \( G_o(s) \), we modify the original problem shown in Fig. 2 to an equivalent problem depicted in Fig. 3.

**Proposition.** There exists an integral type controller \( K(s) \) satisfying \( S1 \) and \( S2 \), if and only if there exists a controller \( K(s) \) satisfying

\[ Sm1 \) \( K(s) \) stabilizes \( G_m(s). \)

\[ Sm2 \]

\[
\|T(s)\|_\infty = \left\| \frac{T_{wz_0}}{T_{wz_e}} \right\|_\infty < \gamma
\]

where
Fig. 3 Modified Problem

Fig. 4 Implementation of a Compensator

\[ G_m(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ \frac{W_a(s)}{s} G_{21}(s) & \frac{W_a(s)}{s} G_{22}(s) \end{bmatrix} \]  \hspace{1cm} (3)

and \( \alpha \) is an arbitrary positive number. \( T_{wz_o}(s) \) and \( T_{wz_e}(s) \) respectively denote the transfer function from \( w \) to \( z_o \) and \( z_e \) in Fig. 3.

**Proof:** It is readily seen from (1) and (3) that setting

\[ K(s) = \frac{s + \alpha}{s} \hat{K}(s) \]

as shown in Fig. 4 yields the equivalence. Here, we note that any integral type controller \( K(s) \) must have a form above and that any solution \( \hat{K}(s) \) for \( G_m(s) \) has no zeros at the origin for assuring the stability.

The greatest advantage for considering the modified problem instead of the original one is that we can directly apply the usual algorithms such as Glover-Doyle's solution for any \( \mathcal{H}_\infty \) problem including the mixed sensitivity problem and LQG type formulation.

In other words, no approximation technique such as \( 1/s \to 1/(s+\epsilon) \) (\( \epsilon > 0 \): small) and no specialized algorithm are required. In the rest of this section, we will show this property.

Let the state space realization of \( G_o(s) \) be given by

\[ G_o(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix} \]  \hspace{1cm} (4)

where \( A \in \mathbb{R}^{n \times n} \), \( B_1 \in \mathbb{R}^{n \times m_1} \), \( B_2 \in \mathbb{R}^{n \times m_2} \), \( C_1 \in \mathbb{R}^{p_1 \times n} \), \( C_2 \in \mathbb{R}^{p_2 \times n} \), \( D_{12} \in \mathbb{R}^{p_1 \times m_2} \) and \( D_{21} \in \mathbb{R}^{p_2 \times m_1} \). We make the following natural assumptions:

**A1** The following six conditions in 2) are satisfied for the state-space model \( G_o(s) \) given by (4):

- \( C1) \quad (A, B_2): \text{stabilizable} \)
- \( C2) \quad \text{rank}(D_{12}) = m_2 \)
- \( C3) \quad \forall \omega \in \mathbb{R}; \)

\[ \text{rank} \left( \begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} \right) = n + m_2 \]

- \( O1) \quad (A, C_2): \text{detectable} \)
- \( O2) \quad \text{rank}(D_{21}) = p_2 \)
- \( O3) \quad \forall \omega \in \mathbb{R}; \)
A2) The plant $P(s)$ satisfies the robust tracking condition for the step type reference command, i.e.,

$$\text{rank} \left( \begin{bmatrix} A - j\omega I & B_2 \\ C_2 & D_{21} \end{bmatrix} \right) = n + p_2$$

For simplicity, we also assume that $W_e(s)$ is a constant matrix of dimension $p_2 \times p_2$ with full rank and denoted by $W_e$. Then, $G_m(s)$ is realized as follows:

$$G_m(s) = \begin{bmatrix} \hat{A} & \hat{B}_1 & \hat{B}_2 \\ \hat{C}_1 & 0 & \hat{D}_{12} \\ \hat{C}_2 & \hat{D}_{21} & 0 \\ A & 0 & B_2 \\ C_2 & 0 & D_{21} \\ C_1 & 0 & 0 & D_{12} \\ 0 & W_e & 0 & 0 \\ C_2 & \alpha I_{p_2} & D_{21} & 0 \end{bmatrix}$$

The modified problem has the following property:

**Property 1.** Suppose that the assumptions A1 and A2 hold. Then the state-space model (6) for $G_m(s)$ satisfies the corresponding six assumptions in 2).

**Proof:**

- **Cm1)** From C1), for $\omega \neq 0$, we have

$$\text{rank} \left( \begin{bmatrix} A - j\omega I & 0 & B_2 \\ C_2 & -j\omega I & 0 \end{bmatrix} \right) = n + p_2$$

For $\omega = 0$, we see from (5) that

$$\text{rank} \left( \begin{bmatrix} A - j\omega I & 0 & B_2 \\ C_2 & -j\omega I & 0 \end{bmatrix} \right) = \text{rank} \left( \begin{bmatrix} A & B_2 \\ C_2 & 0 \end{bmatrix} \right) = n + p_2$$

Hence, we conclude that $(\hat{A}, \hat{B}_2)$ is stabilizable.

**Cm2)**

$$\text{rank}(\hat{D}_{12}) = \text{rank}(D_{12}) = m_2$$

**Cm3)** $\forall \omega \in \mathbb{R}$;

$$\text{rank} \left( \begin{bmatrix} A - j\omega I & 0 & B_2 \\ C_2 & -j\omega I & 0 \\ C_1 & 0 & D_{12} \\ 0 & W_e & 0 \end{bmatrix} \right) \geq \text{rank} \left( \begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} \right) + \text{rank}(W_e) = n + p_2 + m_2 \quad \text{(from C3)}$$

**Om1)** We can readily see from O1) that

$$\text{rank} \left( \begin{bmatrix} A - sI & 0 \\ C_2 & -sI \\ C_2 & \alpha I \end{bmatrix} \right) = n + p_2$$

for $\Re(s) \geq 0$. This clearly implies that $(\hat{A}, \hat{C}_2)$ is detectable.

**Om2)**

$$\text{rank}(\hat{D}_{21}) = \text{rank}(D_{21}) = p_2$$

**Om3)** $\forall \omega \in \mathbb{R}$

$$\text{rank} \left( \begin{bmatrix} A - j\omega I & 0 & B_1 \\ C_2 & -j\omega I & D_{21} \\ C_2 & \alpha I_{p_2} & D_{21} \end{bmatrix} \right) \geq \text{rank} \left( \begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} \right) + \text{rank}(\alpha I_{p_2}) = n + 2p_2 \quad \text{(from O3)}$$

4. Properties of the Modified Problem

In this section, we will discuss the properties of the corresponding Riccati equations to be solved and the resultant controllers.

Let us first focus on the properties, structure and $\alpha$-independeness, of the two Riccati equations, the control Riccati equation:

$$\hat{A}^T \hat{X} + \hat{X} \hat{A} + \hat{C}_1^T \hat{C}_1 - (\hat{X} \hat{B} + \hat{C}_1 \hat{D}_X)^T (\hat{X} \hat{B} + \hat{C}_1 \hat{D}_X) = 0$$

$$\left( \begin{array}{cccc} A - j\omega I & 0 & B_1 \\ C_2 & -j\omega I & D_{21} \\ C_2 & \alpha I_{p_2} & D_{21} \end{array} \right)$$
and the estimation Riccati equation:
\[
\dot{Y} + YAT + B_1B_1^T - (YCT + B_1D_1^TY)R_1^{-1}(YCT + B_1D_1^TY)^T = 0
\]
(8)
where
\[
\hat{Y} = \begin{bmatrix}
\hat{B}_1 & \hat{B}_2
\end{bmatrix}, \quad \hat{D}_X = \begin{bmatrix}
0 & \hat{D}_{12}
\end{bmatrix}
\]
\[
\hat{R}_X = \hat{D}_X^T \hat{D}_X - \begin{bmatrix}
\gamma^2 I & 0 \\
0 & 0
\end{bmatrix}
\]
\[
\hat{C}^T = \begin{bmatrix}
\hat{C}_1 & \hat{C}_2
\end{bmatrix}, \quad \hat{D}_Y = \begin{bmatrix}
0 & \hat{D}_{21}
\end{bmatrix}
\]
\[
\hat{R}_Y = \hat{D}_Y^T \hat{D}_Y - \begin{bmatrix}
\gamma^2 I & 0 \\
0 & 0
\end{bmatrix}
\]

Property 2. The stabilizing solutions \( \hat{X} \) and \( \hat{Y} \) of Riccati equations (7) and (8) are both independent of \( \alpha \), and \( \hat{Y} \) is given by
\[
\hat{Y} = \begin{bmatrix}
Y & 0 \\
0 & 0
\end{bmatrix}
\]
(9)
where \( Y \) is the stabilizing solution of the following Riccati equation:
\[
\dot{Y} + YAT + B_1B_1^T - (YCT + B_1D_1^TY)R_1^{-1}(YCT + B_1D_1^TY)^T = 0
\]
(10)
where
\[
\hat{C}^T = \begin{bmatrix}
\hat{C}_1 & \hat{C}_2
\end{bmatrix}, \quad \hat{D}_Y = \begin{bmatrix}
0 & \hat{D}_{21}
\end{bmatrix}
\]
\[
\hat{R}_Y = \hat{D}_Y^T \hat{D}_Y - \begin{bmatrix}
\gamma^2 I & 0 \\
0 & 0
\end{bmatrix}
\]

Proof: It is trivial that \( \hat{X} \) is independent of \( \alpha \), since all matrices in (7) are \( \alpha \)-independent.

The eigenvalue problem related to the Riccati equations (8) and (10) are given by
\[
\hat{H}_Y \begin{bmatrix}
\hat{U} \\
\hat{V}
\end{bmatrix} = \begin{bmatrix}
\hat{U} \\
\hat{V}
\end{bmatrix} \hat{\Lambda}, \hat{H}_Y \begin{bmatrix}
U \\
V
\end{bmatrix} = \begin{bmatrix}
U \\
V
\end{bmatrix} \Lambda
\]
respectively, where
\[
\hat{H}_Y := \begin{bmatrix}
A^T & 0 \\
-B_1B_1^T & -\hat{A}
\end{bmatrix} - \begin{bmatrix}
\hat{C}^T \\
-\hat{B}_1D_1^T
\end{bmatrix}
\times \hat{R}_Y^{-1} \begin{bmatrix}
\hat{D}_YB_1^T & \hat{C}
\end{bmatrix}
\]
and \( \hat{A} \) and \( \Lambda \) are appropriate Jordan form matrices made up of the stable eigenvalues of \( \hat{R}_Y \) and \( H_Y \), respectively.

We can readily verify that \( \hat{U}, \hat{V} \) and \( \hat{A} \) have the following forms:
\[
\hat{U} = \begin{bmatrix}
U & 0 \\
0 & I
\end{bmatrix}, \quad \hat{V} = \begin{bmatrix}
V & 0 \\
0 & 0
\end{bmatrix}
\]
\[
\hat{A} = \begin{bmatrix}
\Lambda & 0 \\
0 & -\alpha I_{p_2}
\end{bmatrix}
\]
Here, we note that \( \alpha > 0 \) guarantees the stability of \( \hat{A} \). Hence, we have
\[
\hat{Y} = \hat{V} \hat{U}^{-1} = \begin{bmatrix}
VU^{-1} & 0 \\
0 & 0
\end{bmatrix} = \begin{bmatrix}
Y & 0 \\
0 & 0
\end{bmatrix}
\]
Note that the Riccati equation (10) is the same one appeared in the \( H_\infty \) problem for \( G_\sigma(s) \), and hence it is independent of \( \alpha \).

We next investigate the order of the resultant controller. Let \( \hat{K}_c(s) \) denotes the central solution of the \( H_\infty \) control problem related to \( G_m(s) \). Also, let \( K_c(s) \) denotes the controller corresponding to \( \hat{K}_c(s) \), i.e., \( K_c(s) = \frac{s + \alpha}{s} \hat{K}_c(s) \). The order of \( K_c(s) \) is at most \( n + p_2 \), since the size of Riccati equations are \( n + p_2 \). Then, the order of \( K_c(s) \) seems to be \( n + 2p_2 \) in general. Fortunately, we can show that the pole-zero cancelation at \( s = -\alpha \) leads to the smaller order of the central controller \( K_c(s) \). The order is reduced to \( n + 2p_2 - p_2 = n + p_2 \). The next property shows the stronger result that such an order reduction is possible for any \( H_\infty \) controller obtained from the modified problem.

Property 3. The zeros \( -\alpha \) of \( \frac{s + \alpha}{s} I \) are completely canceled by the poles of any \( \hat{K}(s) \) in \( \hat{K} \), where \( \hat{K} \) denotes the class of \( H_\infty \) controllers for \( G_m(s) \). Hence, there exists an \( H_\infty \) controller \( \hat{K}(s) \) satisfying S1) and S2) with order \( \hat{n} \) if we have an \( H_\infty \) controller \( \hat{K}(s) \in \hat{K} \) with order \( \hat{n} \).

Proof: We first note that \( \hat{K} \) is expressed as
\( \mathcal{F}(K_k, U) \) with \( \|U\|_\infty < \gamma \) and \( K_k(s) \) is given as follows, following to the standard Glover-Doyle's solution for the \( \mathcal{H}_\infty \) control problem:

Let \( \dot{X} \) and \( \dot{Y} \) respectively denote the stabilizing solutions of the control and estimation Riccati equations corresponding to \( G_m(s) \). Property 2 states that \( \dot{X} \) and \( \dot{Y} \) are independent of \( \alpha \). Also \( \dot{Y} \) is given by (9). We also partition \( \dot{X} \) into

\[
\dot{X} = \begin{bmatrix} \dot{X}_1 & \dot{X}_3 \\ \dot{X}_3^T & \dot{X}_2 \end{bmatrix}
\]

conformably with the structure of \( G_m(s) \).

Then \( \dot{Z} := (I - \dot{Y} \dot{X})^{-1} \) has the structure

\[
\dot{Z} = \begin{bmatrix} \dot{Z}_1 & \dot{Z}_2 \\ 0 & I \end{bmatrix}
\]

\( \dot{Z}_1 := (I - Y \dot{X}_1)^{-1} \), \( \dot{Z}_2 := \dot{Z}_1 Y \dot{X}_3 \)

Straightforward but lengthy calculations using the structure of \( \dot{Z} \) lead to the following state-space form of \( K_k(s) \):

\[
\dot{K}_k(s) = \begin{bmatrix} \dot{A}_k & \dot{B}_{1k} & \dot{B}_{2k} \\ \dot{C}_{1k} & 0 & R_{X_2}^{-\frac{1}{2}} \\ \dot{C}_{2k} & R_{Y_2}^{-\frac{1}{2}} & 0 \end{bmatrix}
\]

\[
\dot{A}_k := \begin{bmatrix} A_{k11} & A_{k12} - \alpha B_{1k1} \\ 0 & -\alpha I_{p_2} \end{bmatrix}, \quad \dot{B}_{1k} := \begin{bmatrix} B_{1k1} \\ I_{p_2} \end{bmatrix}, \quad \dot{B}_{2k} := \begin{bmatrix} B_{2k1} \\ 0 \end{bmatrix}
\]

\[
\dot{C}_{1k} := \begin{bmatrix} C_{1k1} & C_{1k2} \end{bmatrix}, \quad \dot{C}_{2k} := \begin{bmatrix} C_{2k1} & C_{2k2} - \alpha R_{Y_2}^{-\frac{1}{2}} \end{bmatrix}
\]

\[
R_{X_2} := D_{12}^T D_{12}, \quad R_{Y_2} := D_{21} D_{21}^T
\]

where

\[
A_{k11} := A + B_1 B_1^T \dot{X}_1 - B_2 R_{X_2}^{-1} B_2^T \dot{X}_1 + D_{21} C_1
\]

\[
- \dot{Z}_1 Y (D_{21} B_1^T \dot{X}_1 + C_2^T \dot{X}_2)
\]

\[
- \dot{Z}_2 (C_2 + D_{21} B_1^T \dot{X}_1 + R_{Y_2} \dot{X}_3^T)
\]

(1) \( \mathcal{F}(K, U) \) denotes \( K_{11} + K_{12} U (I - K_{22} U)^{-1} K_{21} \).
where again all the matrices in the right hand side are independent of \( \alpha \). It is easy to see that the realization is uncontrollable and that canceling out of \( p_2 \) modes at \( s = -\alpha \) yields the reduced version of the realization

\[
K_k(s) = \begin{bmatrix}
A_{k11} & A_{k12} & B_{1k1} & B_{2k1} \\
0 & 0 & I_{p_2} & 0 \\
C_{1k1} & C_{1k2} & 0 & R_{X_2}^{-\frac{1}{2}} \\
C_{2k1} & C_{2k2} & R_{Y_2} & 0
\end{bmatrix}
\]

with order \( n + p_2 \). This completes the proof.

The above proof includes the following property on the independenceness with respect to \( \alpha \):

**Property 4.** The parameterization of all \( \mathcal{H}_\infty \) controllers \( K(s) \) for \( G_\alpha(s) \) is independent of the choice of \( \alpha \), which is given by \( \mathcal{F}_I(K, U) \), where \( \|U\|_\infty < \gamma \).

**Remark 1.** The results in Sections 3 and 4 ensure that we can find an integral type controller for the robust tracking \( \mathcal{H}_\infty \) control problem according to the standard Glover-Doyle’s solution, and hence we can obtain the controller by using any standard computational package.

Although the final controller after minimal realization does not depend on the value of \( \alpha \) in theory, the following situations may happen due to the numerical computation: \( \hat{Y} \) does not have the structure as in (9) and/or the order reduction of the resultant controller does not occur. Those situations occur for the fully extremely two cases: 1) \( \alpha \) too small or too large, 2) \( \alpha \) near to one of the eigenvalues of \( HY \) in (11). Hence, the reasonable choice of \( \alpha \) giving the successful result is normally possible by avoiding the above two cases.

As an another choice, we may implement the \( \alpha \)-independent parameterization of \( K(s) \) found in Property 4.

5. Application to Linear AC Servo Motor

We apply the proposed design method for robust tracking to the position control of a linear AC servo motor\(^1\).\(^6\). The nominal model \( P(s) \) of the linear AC servo motor is given by

\[
P(s) = \frac{K_p}{s T_p s + 1}
\]

![Fig. 5 Parameter Perturbation with Covering Ellipse](image)

Fig. 5 Parameter Perturbation with Covering Ellipse

or

\[
P(s) = \begin{bmatrix}
0 & 1 & 0 \\
0 & -1/T_p & K_p/T_p \\
1 & 0 & 0
\end{bmatrix}
\]

The parameters \( T_p \) and \( K_p \) are identified based on 28 step responses, and the data are given in\(^1\). Here, we consider the perturbation of \( a = 1/T_p \) and \( b = K_p/T_p \), and we estimate the uncertainty based on the data replotted in \((a, b)\) parameter space (See Fig. 5). As a model of the parameter perturbation, we employ an elliptic region defined by

\[
\begin{pmatrix}
a \\
b
\end{pmatrix} = \begin{pmatrix}
a_0 \\
b_0
\end{pmatrix} + \Delta \begin{pmatrix}
r_{aa} & r_{ab} \\
r_{ba} & r_{bb}
\end{pmatrix}
\]

where \((a_0, b_0)\) represents the center of the ellipse, and \( \Delta \in \mathbb{R}^2 \) is any row vector such that \( \Delta \Delta^T \leq 1 \) holds.

If the ellipse covers all data, we can adopt the following original generalized plant \( G_\alpha(s) \) for the robust stabilization against the parameter perturbation:

\[
G_\alpha(s) = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & -a_0 & 1 & b_0 \\
0 & r_{aa} & 0 & r_{ab} \\
0 & r_{ba} & 0 & r_{bb} \\
1 & 0 & 0 & 0
\end{bmatrix}
\]

In this paper, the above parameters are selected as
so that the corresponding ellipse covers all data. In addition, a constant weighting matrix \( W_e \) is used to tune the tracking performance.

The experimental results are illustrated in Fig. 6 with corresponding simulation results. We can confirm the efficiency of the weighting matrix \( W_e \), i.e., the tracking error is converged to be zero more quickly with the larger \( W_e \). Moreover, these results agree with the simulation results.

References


Shinji Hara (Member)
He received the B.S., M.S. and Ph.D. degrees in engineering from Tokyo Institute of Technology, Tokyo, Japan, in 1974, 1976, and 1981, respectively. From 1976 to 1980 he was a Research Member of the Electrical Communication Laboratory, Nihon Telegraph and Telephone Public Corporation, Japan. He served as Research Associate of Mechanical System Engineering at the Technological University of Nagaoka from 1980 to 1984. In 1984, he joined the faculty of Tokyo Institute of Technology, where he is currently a Professor of Department of Systems Science. His current research interests are in robust and \( H_\infty \) control, sampled-data control, and computer aided control system design.

Hisaya Fujoka (Member)
He received the B.S., M.S. and Ph.D. degrees from Tokyo Institute of Technology, Tokyo, Japan, in 1990, 1992, and 1995, respectively. He is currently a Research Associate at Osaka University.

Tsugumoto Kosugi
He received the B.S. degree in engineering from Tokyo Institute of Technology, Tokyo, Japan, in 1993.

Toru Asai (Member)
He received the B.E., M.E. and Ph.D. degrees in control engineering from Tokyo Institute of Technology, Tokyo, Japan, in 1991, 1993 and 1996 respectively. He is currently a research fellow of the Japan society for the promotion of science. His current research interests are in robust control theory and its application.