$\mathcal{L}_2$ Gain Analysis of Switched Systems with Average Dwell Time[†]

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In this paper, we investigate the $\mathcal{L}_2$ gain properties of time-controlled switched systems consisting of several linear time-invariant subsystems by using an average dwell time approach incorporated with a piecewise Lyapunov function. We show that when all subsystems are Hurwitz stable and have the $\mathcal{L}_2$ gain smaller than a positive scalar, the switched system can achieve any $\mathcal{L}_2$ gain larger than such scalar if the average dwell time is chosen sufficiently large. The result is extended to the case where state jumps occur at switching points, and to a class of nonlinear switched systems by considering a Hamilton-Jacobi inequality for each subsystem.

Key Words: switched system, $\mathcal{L}_2$ gain, average dwell time, piecewise Lyapunov function

1. Introduction

By a switched system, we mean a hybrid dynamical system that is composed of a family of continuous-time subsystems and a rule orchestrating the switching between the subsystems. Recently, there has been increasing interest in stability analysis and switching control design of such systems (see, e.g., 1)~11) and the references cited therein). The motivation for studying such switched systems comes from the fact that many practical systems are inherently multimodal in the sense that several dynamical systems are required to describe their behavior which may depend on various environmental factors^{2),3)}, and from the fact that the methods of intelligent control design are based on the idea of switching between different controllers^{1),2),4),5)}.

In this paper, we consider the linear switched system

$$\begin{aligned} \dot{x}(t) &= A_\sigma x(t) + B_\sigma u(t), & x(t_0) &= x_0 \\ y(t) &= C_\sigma x(t), \end{aligned} \quad (1)$$

where $x(t) \in \mathcal{R}^n$ is the state, $u(t) \in \mathcal{R}^m$ is the input, $y(t) \in \mathcal{R}^q$ is the output, t_0 is the initial time and x_0 is the initial state. $\sigma : [t_0, \infty) \rightarrow \mathcal{I}_N = \{1, \dots, N\}$ is a piecewise constant function of time, called a *switching signal*, and thus A_σ, B_σ and C_σ are also piecewise constant functions since $(A_\sigma, B_\sigma, C_\sigma) : [t_0, \infty) \rightarrow \{(A_1, B_1, C_1), \dots, (A_N, B_N, C_N)\}$. Here, A_i, B_i, C_i ($i \in$

$\mathcal{I}_N$) are constant matrices of appropriate dimensions denoting the subsystems, and $N > 1$ is the number of subsystems.

We first introduce some notation and review several existing results concerning stability properties of the system (1) (when $u = 0$) where the dwell time approach was used. Given a positive constant τ_d , let $\mathcal{S}_d[\tau_d]$ denote the set of all switching signals with time interval between consecutive switchings being no smaller than τ_d . The constant τ_d is called the “dwell time”⁴⁾ between subsystems. It has been shown in 4) that when all subsystem matrices A_i are Hurwitz stable (A matrix or the corresponding subsystem is called *Hurwitz stable* if all its eigenvalues have negative real part), we can choose τ_d sufficiently large so that the switched system (1) without the input is exponentially stable for any $\sigma \in \mathcal{S}_d[\tau_d]$. In 6), a dwell time scheme is analyzed for local asymptotic stability of nonlinear switched systems with the activation time being used as a dwell time. In 7), Hespanha extends the concept of “dwell time” to “average dwell time”. For any switching signal σ and any $t > \tau \geq t_0$, let $N_\sigma(\tau, t)$ denote the number of switchings of σ over the interval (τ, t) . For given $N_0, \tau_a > 0$, let $\mathcal{S}_a[\tau_a, N_0]$ denote the set of all switching signals satisfying

$$N_\sigma(\tau, t) \leq N_0 + \frac{t - \tau}{\tau_a}, \quad (2)$$

where the constant τ_a is called the “average dwell time” and N_0 denotes the “chatter bound”. The idea is that there may exist consecutive switchings separated by less than τ_a , but the average time interval between consecutive switchings is not less than τ_a (if we ignore the first N_0 switchings). It has been shown in 7) that if τ_a is sufficiently large, then the switched system (1) without the input is exponentially stable for any switching signal

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$\sigma \in \mathcal{S}_a[\tau_a, N_0]$. In a recent paper⁸⁾, the authors extended the above stability results to the case where both Hurwitz stable and unstable subsystems exist, by showing that if the average dwell time is chosen sufficiently large and the total activation time of unstable subsystems is relatively small compared with that of Hurwitz stable subsystems, then global exponential stability of a desired decay rate is guaranteed.

In the present paper, we extend the above existing stability results to the considerations of $\mathcal{L}_2$ gain for the system (1). If the $\mathcal{L}_2$ gain from the input u to the output y is less than $\gamma > 0$ in the sense of

$$\int_{t_0}^{\infty} y^T(\tau)y(\tau)d\tau \leq \gamma^2 \int_{t_0}^{\infty} u^T(\tau)u(\tau)d\tau + \delta(x_0) \quad (3)$$

for all $u \in \mathcal{L}_2[t_0, \infty)$, where $\delta(x_0)$ is a real-valued function of the initial state x_0 satisfying $\delta(0) = 0$, then we say the system (1) has (or achieves) the $\mathcal{L}_2$ gain γ . The system has the $\mathcal{L}_2$ gain smaller than γ if there exists some $0 < \tilde{\gamma} < \gamma$ such that (3) holds for $\tilde{\gamma}$. Throughout this paper, we assume that all subsystems in (1) are Hurwitz stable and their $\mathcal{L}_2$ gains are smaller than a known positive scalar γ_0 (the assumption of the same $\mathcal{L}_2$ gain bound γ_0 does not lose generality since we can set $\gamma_0 = \max_i \{\gamma_i\}$ if the i -th subsystem has the $\mathcal{L}_2$ gain smaller than γ_i ($i \in \mathcal{I}_N$)). The discussion on the case where both Hurwitz stable and unstable subsystems exist can be found in another paper⁹⁾ by the authors.

In the next section, we show that if we choose the average dwell time sufficiently large, then the switched system can achieve any $\mathcal{L}_2$ gain larger than γ_0 . A piecewise Lyapunov function is used to analyze the $\mathcal{L}_2$ gain from the input u to the output y in the switched system. It is well known that Lyapunov function theory is the most general and useful approach for studying stability of various control systems. Recently, instead of traditional single Lyapunov function, the use of piecewise Lyapunov functions (or multiple Lyapunov functions) has been proposed in many references (for example, 5), 10), 11)). Since it is nearly impossible to find a single Lyapunov function for us to discuss the $\mathcal{L}_2$ gain property for the switched system (1), we construct a piecewise Lyapunov function based on the solutions of a set of matrix inequalities concerning all subsystems' $\mathcal{L}_2$ gain properties separately.

In Section 3, some remarks are given on the main theorem concerning stability and robust stability. Then, we show that the average dwell time scheme remains valid even when state jumps occur at the switching points. Another extension is made to a class of nonlinear switched systems by introducing a Hamilton-Jacobi inequality for

each subsystem, though the Hamilton-Jacobi inequalities may be difficult to solve at present. It is shown that the average dwell time scheme is also effective for the nonlinear switched systems if we can manage to find a set of Lyapunov functions satisfying the Hamilton-Jacobi inequalities. Finally, Section 4 concludes the paper.

2. $\mathcal{L}_2$ Gain Analysis

Since all subsystems in (1) are assumed to be Hurwitz stable and have the $\mathcal{L}_2$ gain smaller than γ_0 , there exist a set of positive definite matrices P_i , $i \in \mathcal{I}_N$, such that

$$A_i^T P_i + P_i A_i + \gamma_0^{-2} P_i B_i B_i^T P_i + C_i^T C_i < 0 \quad (4)$$

holds¹²⁾. It is well known that the above inequality can be converted into an LMI¹³⁾ with respect to P_i , and thus is easily solved using any existing softwares such as the LMI Control Toolbox¹⁴⁾. Then, there always exists a scalar $\lambda_0 > 0$ such that

$$A_i^T P_i + P_i A_i + \gamma_0^{-2} P_i B_i B_i^T P_i + C_i^T C_i + \lambda_0 P_i < 0 \quad (5)$$

is satisfied for all $i \in \mathcal{I}_N$. Also, we can directly solve the matrix inequalities (5) with respect to P_i 's and λ_0 in the framework of generalized eigenvalue problems^{13), 14)}.

Using the solution P_i 's of (5), we define the following piecewise Lyapunov function candidate

$$V(t) = V_\sigma(x) = x^T P_\sigma x \quad (6)$$

for the switched system (1), where P_σ is switched among the solution P_i 's of (5) in accordance with the piecewise constant switching signal σ . Then, the following properties are obtained for (6):

- (1) Each $V_i = x^T P_i x$ in (6) is continuous and its derivative along the solutions of the corresponding subsystem satisfies

$$\begin{aligned} \dot{V}_i &= \frac{\partial V_i}{\partial x} (A_i x + B_i u) \\ &= x^T (P_i A_i + A_i^T P_i) x + x^T P_i B_i u + u^T B_i^T P_i x \\ &\leq -\lambda_0 V_i - y^T y - (\gamma_0^{-1} B_i^T P_i x - \gamma_0 u)^T \\ &\quad \times (\gamma_0^{-1} B_i^T P_i x - \gamma_0 u) + \gamma_0^2 u^T u \\ &\leq -\lambda_0 V_i - y^T y + \gamma_0^2 u^T u. \end{aligned} \quad (7)$$

- (2) There exist constant scalars $\alpha_2 \geq \alpha_1 > 0$ such that

$$\alpha_1 \|x\|^2 \leq V_i(x) \leq \alpha_2 \|x\|^2, \quad \forall x \in \mathcal{R}^n, \quad \forall i \in \mathcal{I}_N. \quad (8)$$

- (3) There exists a constant scalar $\mu \geq 1$ such that

$$V_i(x) \leq \mu V_j(x), \quad \forall x \in \mathcal{R}^n, \quad \forall i, j \in \mathcal{I}_N. \quad (9)$$

The first property is based on the matrix inequality (5), while the second and third properties hold, for example, with $\alpha_1 = \inf_{i \in \mathcal{I}_N} \lambda_m(P_i)$, $\alpha_2 = \sup_{i \in \mathcal{I}_N} \lambda_M(P_i)$,

and $\mu = \sup_{k,l \in \mathcal{I}_N} \frac{\lambda_M(P_k)}{\lambda_m(P_l)}$, respectively. Here, $\lambda_M(P)$ ($\lambda_m(P)$) denotes the largest (smallest) eigenvalue of the positive definite symmetric matrix P .

Now, for an arbitrary piecewise constant switching signal σ and any given $t > t_0$, we let $t_1 < \dots < t_i$ ($i > 1$) denote the switching points of σ over the interval (t_0, t) . Using the differential inequality theory (for example, (15)) for (7), we obtain by induction that

$$\begin{aligned}
 V(t) &\leq V(t_i) e^{-\lambda_0(t-t_i)} - \int_{t_i}^t e^{-\lambda_0(t-\tau)} \Gamma(\tau) d\tau \\
 &\leq \mu V(t_i^-) e^{-\lambda_0(t-t_i)} - \int_{t_i}^t e^{-\lambda_0(t-\tau)} \Gamma(\tau) d\tau \\
 &\leq \mu \left(V(t_{i-1}) e^{-\lambda_0(t_i-t_{i-1})} \right. \\
 &\quad \left. - \int_{t_{i-1}}^{t_i} e^{-\lambda_0(t_i-\tau)} \Gamma(\tau) d\tau \right) \times e^{-\lambda_0(t-t_i)} \\
 &\quad - \int_{t_i}^t e^{-\lambda_0(t-\tau)} \Gamma(\tau) d\tau \\
 &\leq \dots \\
 &\leq \mu^i e^{-\lambda_0(t-t_0)} V(t_0) - \mu^i \int_{t_0}^{t_1} e^{-\lambda_0(t-\tau)} \Gamma(\tau) d\tau \\
 &\quad - \mu^{i-1} \int_{t_1}^{t_2} e^{-\lambda_0(t-\tau)} \Gamma(\tau) d\tau \\
 &\quad - \dots - \mu^0 \int_{t_i}^t e^{-\lambda_0(t-\tau)} \Gamma(\tau) d\tau \\
 &= \mu^{N_\sigma(t_0, t)} e^{-\lambda_0(t-t_0)} V(t_0) \\
 &\quad - \int_{t_0}^t \mu^{N_\sigma(\tau, t)} e^{-\lambda_0(t-\tau)} \Gamma(\tau) d\tau, \\
 &= e^{-\lambda_0(t-t_0) + N_\sigma(t_0, t) \ln \mu} V(t_0) \\
 &\quad - \int_{t_0}^t e^{-\lambda_0(t-\tau) + N_\sigma(\tau, t) \ln \mu} \Gamma(\tau) d\tau, \quad (10)
 \end{aligned}$$

where $\Gamma(\tau) \triangleq y^T(\tau)y(\tau) - \gamma_0^2 u^T(\tau)u(\tau)$, and the third property (9) was used at the switching points as $V(t_i) \leq \mu V(t_i^-)$ in some of the inequalities. Here, $V(t_i^-) = \lim_{\tau \uparrow t_i} V(\tau)$.

When $\mu = 1$, which is a trivial case, we get from (10) and $V(t) \geq 0$ that

$$\begin{aligned}
 \int_{t_0}^t e^{-\lambda_0(t-\tau)} \Gamma(\tau) d\tau &\leq V(t) + \int_{t_0}^t e^{-\lambda_0(t-\tau)} \Gamma(\tau) d\tau \\
 &\leq e^{-\lambda_0(t-t_0)} V(t_0), \quad (11)
 \end{aligned}$$

and thus,

$$\int_{t_0}^t e^{-\lambda_0(t-\tau)} y^T(\tau)y(\tau) d\tau \leq e^{-\lambda_0(t-t_0)} V(t_0)$$

$$+ \gamma_0^2 \int_{t_0}^t e^{-\lambda_0(t-\tau)} u^T(\tau)u(\tau) d\tau. \quad (12)$$

Note that the left side in the above inequality is integrable from $t = t_0$ to $t = \infty$ since the right side is integrable for any $u \in \mathcal{L}_2[t_0, \infty)$ in the same interval. Then, we integrate (12) from $t = t_0$ to $t = \infty$ to obtain (by rearranging the double-integral area)

$$\begin{aligned}
 &\int_{t_0}^{\infty} \left(\int_{t_0}^t e^{-\lambda_0(t-\tau)} y^T(\tau)y(\tau) d\tau \right) dt \\
 &= \int_{t_0}^{\infty} y^T(\tau)y(\tau) \left(\int_{\tau}^{\infty} e^{-\lambda_0(t-\tau)} dt \right) d\tau \\
 &= \frac{1}{\lambda_0} \int_{t_0}^{\infty} y^T(\tau)y(\tau) d\tau \\
 &\leq V(t_0) \int_{t_0}^{\infty} e^{-\lambda_0(t-t_0)} dt \\
 &\quad + \gamma_0^2 \int_{t_0}^{\infty} \left(\int_{t_0}^t e^{-\lambda_0(t-\tau)} u^T(\tau)u(\tau) d\tau \right) dt \\
 &= \frac{1}{\lambda_0} V(t_0) + \frac{\gamma_0^2}{\lambda_0} \int_{t_0}^{\infty} u^T(\tau)u(\tau) d\tau, \quad (13)
 \end{aligned}$$

which means

$$\int_{t_0}^{\infty} y^T(\tau)y(\tau) d\tau \leq \gamma_0^2 \int_{t_0}^{\infty} u^T(\tau)u(\tau) d\tau + V(x_0), \quad (14)$$

and thus the switched system (1) achieves the $\mathcal{L}_2$ gain γ_0 without having to consider any average dwell time scheme.

When $\mu > 1$, we choose a positive scalar $\lambda (< \lambda_0)$ and another scalar $\alpha > 0$ to consider the following *average dwell time scheme*: for any $\tau < t$,

$$N_\sigma(\tau, t) \leq N_0 + \frac{t-\tau}{\tau_a^*}, \quad N_0 = \frac{\alpha}{\ln \mu}, \quad \tau_a^* = \frac{\ln \mu}{\lambda_0 - \lambda}. \quad (15)$$

From (15), we know that

$$-\lambda_0(t-\tau) + N_\sigma(\tau, t) \ln \mu \leq -\lambda(t-\tau) \quad (16)$$

holds for any $\tau \leq t$. Then, from (10) and $V(t) \geq 0$, we obtain

$$\begin{aligned}
 &\int_{t_0}^t e^{-\lambda_0(t-\tau)} y^T(\tau)y(\tau) d\tau \\
 &\leq V(t) + \int_{t_0}^t e^{-\lambda_0(t-\tau) + N_\sigma(\tau, t) \ln \mu} y^T(\tau)y(\tau) d\tau \\
 &\leq e^{\alpha-\lambda(t-t_0)} V(t_0) + \gamma_0^2 \int_{t_0}^t e^{\alpha-\lambda(t-\tau)} u^T(\tau)u(\tau) d\tau. \quad (17)
 \end{aligned}$$

Similarly to (13), we integrate the above inequality from $t = t_0$ to $t = \infty$ to get

$$\begin{aligned}
 &\frac{1}{\lambda_0} \int_{t_0}^{\infty} y^T(\tau)y(\tau) d\tau \\
 &\leq \frac{e^\alpha}{\lambda} V(t_0) + \frac{e^\alpha \gamma_0^2}{\lambda} \int_{t_0}^{\infty} u^T(\tau)u(\tau) d\tau, \quad (18)
 \end{aligned}$$

which implies that the system (1) achieves the $\mathcal{L}_2$ gain $\sqrt{\frac{\lambda_0 \varepsilon^\alpha}{\lambda}} \gamma_0$ under the average dwell time scheme (15). Therefore, for any given $\gamma > \gamma_0$, we choose λ and α satisfying $\frac{\varepsilon^\alpha}{\lambda} = \frac{1}{\lambda_0} (\frac{\gamma}{\gamma_0})^2$ in the average dwell time scheme (15) so that the achieved $\mathcal{L}_2$ gain is $\gamma = \sqrt{\frac{\lambda_0 \varepsilon^\alpha}{\lambda}} \gamma_0$.

We conclude the above discussion in the following main theorem.

Theorem 1. Assume that all subsystems in (1) are Hurwitz stable and achieve the $\mathcal{L}_2$ gain smaller than γ_0 . Then, for any $\gamma > \gamma_0$, there exist finite constants τ_a^* and N_0 such that the switched system (1) achieves the $\mathcal{L}_2$ gain γ over $S_a[\tau_a, N_0]$ for any average dwell time $\tau_a \geq \tau_a^*$. $\square$

It is easy to understand from the proof of Theorem 1 that, in analyzing the $\mathcal{L}_2$ gain of the given switched system (1), there are three computational steps: (i) solve the matrix inequality (5) with respect to $\lambda_0 > 0$ and $P_i > 0$; (ii) find a small scalar $\mu \geq 1$ satisfying (9); (iii) estimate the average dwell time using a positive scalar $\lambda < \lambda_0$ in (15). We have discussed the computations in Steps (i) and (ii). In Step (iii), λ should be chosen close to λ_0 if we are to expect a small $\mathcal{L}_2$ gain. However, the average dwell time τ_a^* in (15) becomes large when λ is close to λ_0 . Thus, there is a trade off in the choice of λ in the average dwell time scheme (15).

3. Some Remarks and Extensions

We first give some remarks on the result obtained in the previous section.

Remark 1. As can be seen from (15), the proposed average dwell time switching scheme requires that the average time interval between consecutive switchings be not less than a specified constant, and does not add any other restrictive switching additions. Thus, there are infinite switching signals that are included in our switching scheme. Theorem 1 gives a general result for these infinite switching signals. $\square$

Remark 2. When $u = 0$, we get from (11) and (17) that $V(t) \leq e^{-\lambda_0(t-t_0)} V(t_0)$ and $V(t) \leq e^{-\lambda(t-t_0)} V(t_0)$, which mean that the switched system (1) without the input is exponentially stable with decay rates λ_0 and λ , respectively. In this way, the $\mathcal{L}_2$ gain analysis in this paper covers the existing results^{4),7)} concerning exponential stability of linear switched systems if these are controlled by average dwell time schemes. $\square$

Remark 3. By assuming that the switched system (1) itself is uniformly exponentially stable, the paper⁷⁾ discussed the boundness property of several input-to-state

induced norms, i.e., the induced norms of u to x . Since input-to-state norms are special cases of the input-to-output norms considered in this paper, Theorem 1 in Section 2 extended the $\mathcal{L}_2$ norm analysis result in 7). $\square$

Remark 4. It is easy to see that Theorem 1 can be used to analyze robust stability property of the following time-controlled switched system

$$\dot{x}(t) = A_\sigma x(t) + B_\sigma F(x, t) C_\sigma x(t), \quad (19)$$

where $x(t)$ and σ are the same as in (1), and $F(x, t)$ describes norm-bounded uncertainties. Roughly speaking, when each subsystem is quadratically stable for any $F(x, t)$ satisfying $\|F(x, t)\| \leq \frac{1}{\gamma_0}$, then the entire switched system (19) is robustly asymptotically stable for any $\|F(x, t)\| \leq \frac{1}{\gamma}$ with $\gamma > \gamma_0$ if the average dwell time is chosen sufficiently large. $\square$

In the proof of Theorem 1, we have assumed tacitly that there are no state jumps when switching from one subsystem to another. However, there is no difficulty considering state jumps in our framework. When state jumps occur, it is reasonable to assume that there exists a positive scalar J such that $\|x(t_j)\| \leq J\|x(t_j^-)\|$ holds at all the switching points t_j . If $J \leq 1$, the discussion up to now is the same. If $J > 1$, then $V(x(t_j)) \leq J^2 \mu V(x(t_j^-))$ holds according to (9), and the inequality (10) becomes

$$V(t) \leq e^{-\lambda_0(t-t_0) + N_\sigma(t_0, t) \ln(J^2 \mu)} V(t_0) - \int_{t_0}^t e^{-\lambda_0(t-\tau) + N_\sigma(\tau, t) \ln(J^2 \mu)} \Gamma(\tau) d\tau. \quad (20)$$

Thus, the discussion in the previous section remains valid if we replace μ with $J^2 \mu$.

Corollary 1. Assume (a) all subsystems in (1) are Hurwitz stable and achieve the $\mathcal{L}_2$ gain smaller than γ_0 ; (b) there exists a positive scalar J describing state jumps such that $\|x(t_j)\| \leq J\|x(t_j^-)\|$ holds at all the switching points. Then, for any $\gamma > \gamma_0$, there exist finite constants τ_a^* and N_0 such that the switched system (1) achieves the $\mathcal{L}_2$ gain γ over $S_a[\tau_a, N_0]$ for any average dwell time $\tau_a \geq \tau_a^*$. $\square$

In the remainder of this section, we extend our discussion to the case of the nonlinear switched system

$$\begin{aligned} \dot{x}(t) &= f_\sigma(x(t)) + G_\sigma(x(t))u(t) \\ y(t) &= h_\sigma(x(t)), \end{aligned} \quad (21)$$

where $x(t), u(t), y(t)$ and σ are the same as in (1), each $f_i(x)$ is locally Lipschitz and $G_i(x), h_i(x)$ are continuous over $\mathcal{R}^n$. The idea is to consider for each subsystem a positive definite and proper Lyapunov function $V_i(x)$ sat-

isfying the following Hamilton-Jacobi inequality^{15),16)}

$$\begin{aligned} \frac{\partial V_i}{\partial x}(x)f_i(x) + \frac{1}{2\gamma_0^2} \frac{\partial V_i}{\partial x}(x)G_i(x)G_i^T(x) \frac{\partial^T V_i}{\partial x}(x) \\ + \frac{1}{2}h_i^T(x)h_i(x) + \lambda_0 V_i(x) \leq 0, \end{aligned} \quad (22)$$

which means that all subsystems in (21) achieve the $\mathcal{L}_2$ gain smaller than γ_0 . In this case, it is easy to see that the piecewise Lyapunov function $V(t) = V_\sigma(x)$ easily satisfies the first property (7) and the second property (8), and the third property similar to (9) assumes the form

$$\mu \triangleq \sup \left\{ \frac{V_p(x)}{V_q(x)} : x \in \mathcal{R}^n, p, q \in \mathcal{I}_N \right\} < \infty. \quad (23)$$

In view of the discussion in the previous section, we obtain the following corollary immediately.

Corollary 2. Assume that for the nonlinear switched system (21) there exist a set of positive definite and proper Lyapunov functions $V_i(x)$ satisfying (22) and (23). Then, for any $\gamma > \gamma_0$, there exist finite constants τ_a^* and N_0 such that the switched system (21) achieves the $\mathcal{L}_2$ gain γ over $S_a[\tau_a, N_0]$ for any average dwell time $\tau_a \geq \tau_a^*$. $\square$

However, it should be noted here that we still lack effective methodologies for solving the Hamilton-Jacobi inequalities (22). For this purpose, the authors proposed a Lyapunov-based redesign method for a kind of nonlinear switched systems, where each subsystem is composed of a linear Hurwitz stable part and a nonlinear norm-bounded perturbation part; see 17) for the details.

4. Conclusion

We have investigated the $\mathcal{L}_2$ gain properties for time-controlled switched systems by using an average dwell time approach incorporated with a piecewise Lyapunov function. It has been shown that when all subsystems are Hurwitz stable, we can choose the average dwell time sufficiently large so that the switched system achieves a reasonable $\mathcal{L}_2$ gain. The result has also been extended to the case where state jumps exist at the switching points, and to a class of nonlinear switched systems by introducing a Hamilton-Jacobi inequality for each subsystem. We have shown that a reasonable $\mathcal{L}_2$ gain can also be achieved for these cases if we choose the average dwell time sufficiently large.

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