An Adaptive Storage Function Method for Rainfall–Runoff Forecasting

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The storage function method (SFM) has been the main method of rainfall–runoff forecasting used in Korea. However, it has a major drawback in that the SFM’s parameters are difficult to calibrate exactly. They have large degrees of uncertainty and may also be time varying. To cope with these difficulties, we present an adaptive storage function method (ASFM). Under the assumption that a small basin can be modelled by a single watershed and channel pair, a multiple model adaptive estimation (MMAE) for the parameter adaptation is introduced. In ASFM, measurements of outflow level are required every sample time. Applying ASFM to two catchment datasets, namely for the Pyungchang River in 1988 and the Chungju Basin in 1995, we show that the proposed ASFM has better performance than the conventional SFM.

Key Words: storage function method, multiple model adaptive estimation, rainfall, runoff

1. Introduction

Flood routing techniques are widely used in the planning and design of flood control and in real-time flood forecasting for river systems. There are two approaches for rainfall-runoff models, namely conceptual models and mathematical models (or input-output models). The former is represented by such examples as the Stanford Watershed Model1) and the Storage Function Method (SFM). The latter include the Constrained Linear Systems2) and Auto-Regressive Moving Average (ARMA) models3,4).

In Korea, since two thirds of the total rainfall every year is generally concentrated from June to September, it is very important to manage water resources. For the past decade, the SFM developed by Kimura has been used for flood routing of the Han River systems and others in Korea.

However, the main drawback of SFM is that its parameters are difficult to calibrate. The parameters of storage functions such as inflow coefficient, storage constant and time delay need to be changed according to ground configuration, rainfall pattern and surface condition, etc. There are different methods5,6) for calibrating conceptual rainfall–runoff models. Those schemes are not adaptive.

This paper will consider an adaptive algorithm for SFM, in which several parameters of the storage function will be adjusted in real-time by multiple model adaptive estimates (MMAE).

The MMAE is not new in estimation theory. In fact, multiple Kalman filters were proposed in the 1970s by Lainiotis and Athans et al.7,8) to improve the accuracy of the state estimate in control problems. Many interesting practical applications of this approach have appeared in the following two decades. Kaufman and colleagues9) applied it to medical control, while Moose and co-workers used these ideas in maneuverable target tracking. Application in air traffic control was proposed by BarShalom and co-workers, and Maybaek and Pogoda10) adapted it to fault detection and control in aircraft. In such MMAE, the model structure of the plant is given and each uncertain parameter is partitioned, resulting in a set of corresponding dynamic models. This is followed by a procedure which estimates the optimal model by using hypothesis probabilities.

Thus, the proposed adaptive storage function method (ASFM) consists of N multiple storage function models, which correspond to the set of partitioned parameters, and a real-time identification algorithm, which can estimate the time-varying storage function model by means of a conditional probability evaluator. This method requires measurements of the outflow discharge as well as rainfall data at every sample time. Some simulation results are shown in which ASFM is applied to real data of the Pyungchang River in 1988 and the Chungju Basin in 1995 in Korea.
The paper is organized as follows. Section 2 explores the existing SFM. Section 3 is devoted to establishing the ASFM. Applying ASFM to two real datasets, section 4 examines the performance of ASFM in comparison with SFM. Finally, the concluding remark is presented in section 5.

2. Storage Function Method (SFM)

The SFM is a conceptual rainfall-runoff model that is possible to consist of two components: a watershed and a channel connected to it. The relationships between the inflow and the outflow at each component is expressed by the storage equation and the balance equation.

A basin is generally divided into several smaller areas, called sub-basins. Each area is modelled in a pair of watershed and channel as shown in Fig. 1, and then those sub-basins are connected along the stream line. It is known that the SFM is appropriate when the catchment area includes mostly mountains. In this section, we summarise the SFM. We will extend it to the ASFM in Section 3.

The two basic equations relating to the flood-runoff model in SFM are shown below:

\[ S(t) = K \cdot Q_l(t)^p \]  \hspace{1cm} (1)

\[ \frac{dS(t)}{dt} = R_{ave} - Q_l(t) \]  \hspace{1cm} (2)

where \( S(t) \): the storage [m³], \( K \cdot P \): storage constants, \( Q_l \): the direct discharge under time delay [m³/sec], \( R_{ave} \): average rainfall [mm/hr], \( A \): the area of watershed (or channel) [Km²] and \( f_l \): inflow coefficient (or runoff ratio) which is dependent upon percolation and saturation rainfall, etc.

Based on equations (1) and (2), we first represent the discharge equation at a watershed. Dividing (2) by \( f_l A/3.6 \) and considering time delays, the normalized discharge \( Q_b(t) \) can be obtained below:

\[ \frac{dS(t)}{dt} = R_{ave}(t-T_b) - q_b(t) \]  \hspace{1cm} (3)

\[ s_b(t) = K_b \cdot q_b(t)^p \]  \hspace{1cm} (4)

\[ Q_b(t) = \frac{1}{3.6} \cdot f_l(t) \cdot A \cdot q_b(t) + q_o \]  \hspace{1cm} (5)

Similarly, we can derive the discharge \( Q_l(t) \) at the outlet of a channel as follows:

\[ \frac{dS(t)}{dt} = Q_b(t - T_c) - Q_l(t) \]  \hspace{1cm} (6)

\[ S(t) = K_c \cdot Q_l(t)^p - T_c \cdot Q_l(t) \]  \hspace{1cm} (7)

where \( s_c \): unit storage level [mm], \( q_l \): change of depth of runoff [mm/hr], \( q_o \): base runoff [m³/sec] and \( T_b, T_c \): Time delay in watershed and channel. Subscript B and C denote the watershed and channel, respectively.

To solve the discharge \( Q_l(t) \) at the outlet of a sub-basin, the Newton–Raphson method is usually applied to equations (3) to (7). The flow chart for calculating this value is shown in Fig. 2. However, the most difficult problem is the calibration of the storage function parameters \( K_b, P_b, T_b, f_l, K_c, P_c, T_c \). These parameters can be time varying according to such environmental conditions as rainfall patterns, configuration of mountains and ground surface, etc. To cope with these time variations, we now propose an adaptive algorithm of SFM.

3. Adaptive Storage Function Method (ASFM)

In general, a small basin may consist of several watersheds and channels, connected in a sequential structure. As mentioned in the previous section, seven parameters of each storage function model have uncertainty. The magnitudes of these uncertainties can be determined a priori by past data of rainfalls and discharges of the catchment area. Here, we assume that the runoff level at the outlet of a basin as well as average rainfall is observed every sample time. We can now introduce the MMAE so that
the original SFM may be adapted to variations in environmental conditions.

First, it is assumed that a small basin can be modeled by only one pair of watershed and channel, instead of the sequential model having several watersheds and numbers of channels in SFM. This assumption will be examined in the next section. The seven parameters of the model are partitioned individually. We will call such a set of models 'multiple models'. For a watershed, the partitioned parameters are

\[ K_B \in [K_{B1}, K_{B2}, \ldots, K_{B6}] \] (8)

\[ P_B \in [P_{B1}, P_{B2}, \ldots, P_{B6}] \] (9)

\[ T_B \in [T_{B1}, T_{B2}, \ldots, T_{B6}] \] (10)

\[ f_B \in [f_{B1}, f_{B2}, \ldots, f_{B6}] \] (11)

Similarly, the partitioned parameters of a channel are

\[ K_c \in [K_{c1}, K_{c2}, \ldots, K_{c6}] \] (12)

\[ P_c \in [P_{c1}, P_{c2}, \ldots, P_{c6}] \] (13)

\[ T_c \in [T_{c1}, T_{c2}, \ldots, T_{c6}] \] (14)

\[ f_c \in [f_{c1}, f_{c2}, \ldots, f_{c6}] \] (15)

Thus, their combination results in \( N = (6^3) \) multiple models. Let us define the constant parameter vector as

\[ x := [K_B P_B T_B f_B K_c P_c T_c] \] (16)

and let the \( i \)th model parameter be \( x_i \), and

\[ X_N := \{x_i\}_{i=1,2,\ldots,N} \] (17)

The MMAE considers \( x \) to be a random vector. In order to handle the uncertainty in \( x \), assume that the true value of \( x \) is one of \( x_i \in X_N \) for all time.

Under the assumption stated above, equations (3) to (7) can be written, for \( i=1,2,\ldots,N \),

\[ \frac{dS(t)}{dt} = Ra(t) - q(t) \] (18)

\[ S(t) = K_B \cdot q(t)^{\alpha} \] (19)

\[ Q_{Bn}(t) = \frac{a}{b} \cdot f_t(t) \cdot A \cdot q(t) + q_n \] (20)

\[ \frac{dS(t)}{dt} = Q_{Bn}(t) - Q_{Bn}(t) \] (21)

\[ S(t) = K_B \cdot Q_{Bn}(t) - T_B \cdot Q_{Bn}(t) \] (22)

By solving these equations numerically, the output \( Q_{Bn} \) of the \( i \)th model \( x \) will be obtained.

The problem of parameter identification can be viewed as an hypothesis testing problem, with \( N \) hypotheses \( H_i \), where random variable \( H \) is such that

\[ H = H_i \text{ if } x = x_i, \text{ for } x_i \in X_N \] (23)

Now define the conditional probability

\[ P(t) = \text{Prob}(H = H_i | Ra(t), Q(t)), \text{ for } i=1,2,\ldots,N \] (24)

where \( Q(t) \) is the observed discharge.

These \( P(t) \) can be calculated in a recursive manner using Bayes' rule

\[ P_i(t) = \frac{\exp\left(-r_i(t)^2/2\sigma^2\right)}{\sum_{j=1}^{N} \exp\left(-r_j(t)^2/2\sigma^2\right)} P(t-1) \] (25)

where \( r_i(t) \) is the residual of the \( i \)th model, that is

\[ r(t) = Q(t) - Q_{Bn}(t) \] (26)

It is important to select the \( V \), the covariance of the residuals, properly since it related to the convergence rate of \( P(t) \). The smaller value of \( V \) makes the convergence be rapidly. On the other hand, the excessive reduction in \( V \) could cause a computational overflow.

The initial weighting factors \( P(0) \) must be determined a priori. Since the actual model may match with any model in partitioned multiple models, \( P(0) \) can be assumed to be uniform \( P(0) = 1/N \).

The model \( x^* \) with the highest probability is the one nearest to the true system. At every sample time conditional probabilities of \( N \) multiple models should be calculated. Then the estimated discharge of the given basin is determined by

\[ \hat{Q}(t) = \sum_{i=1}^{N} P(t) \cdot Q_{Bn}(t) \] (27)

The overall configuration of ASFM using MMAE is shown in Fig. 3.

4. Comparative Applications of ASFM

The aim of this section is to show the validity of ASFM and to compare it with conventional SFM. For this purpose, we will consider two catchment basins, firstly, the Pyungchang River system (519.69 km²), and secondly, the No.2 sub-basin of the Chungju catchment (1404.4 km²), both of which are situated in north-east Chungchungbuk-do, Korea. Rainfall and runoff data from storm events have been used. Observations began on July 21, 1987 for Pyungchang basin, and August 8, 1995 for the No. 2 sub-basin of Chungju catchment. For both cases, data was observed hourly.
Fig. 4 shows the rain gauge station and water stage gauge station of the No. 2 sub-basin of Chungju catchment and Pyungchang basin. Chungju catchment has been modelled by four watersheds and two channels for the conventional storage function model. However, in ASFM, it has only one watershed and one channel with real-time parameter adaptation.

In this simulation, the parameter partition shown in Table 1 is considered, where both lower and upper bounds of parameters have been obtained by numbers of storage function models identified by using the past rainfall-runoff data. Thus, the Pyungchang Basin results in $N=864$ models and Chungju No. 2 sub-basin has $N=144$ models for ASFM. The value of $V$ should be chosen properly. It depends on the rainfall-runoff model. In these cases, the acceptable range of $V$ was $[0.1 \ 0.008]$, which was found by the simulations to many cases. Here, $V$ was chosen as 0.01.

We first show the validity of the assumption that a small basin can be modelled in a single watershed-channel pair for a storage function runoff model.

Fig. 5 shows the observed and the estimated discharge for Pyungchang Basin. It shows excellent agreement between the two values.

Fig. 6 shows the adaptation profiles, where it is applied to the same case as Fig. 5. Weighting values at specific time, 15 hr, 27 hr, and 60 hr, is illustrated since it is difficult to describe those, caused by many models used in this simulation, for the models with time change. It shows how well the models are identified to estimate output discharge. The values at each time are applied to models that closely represent the current rainfall-runoff relation. We can observe that the adaptation of ASFM is effectively executed as the pattern of weighting values varied at each time.

Fig. 7 shows the comparative results of SFM and ASFM applied to Chungju sub-basin No. 2. We see that ASFM gives a much better solution. Therefore, these results support that there exists such a small basin as it can be modelled in a single watershed-channel pair with ASFM.

| Table 1. The partitioned parameter set corresponding to multiple models of ASFM |
|---------------------|---|---|---|---|---|---|---|
|                  | $K_r$ | $P_b$ | $f_r$ | $T_a$ | $K_c$ | $P_c$ | $T_c$ |
| Pyungchang Basin | 38    | 0.4   | 0.4   | 1.5   | 40    | 0.6   | 0.5   |
|                   | 45    | 0.5   | 0.6   | 2.5   | 35    | 0.75  | 1.5   |
| Chungju sub-basin| 24.2  | 0.4   | 0    | 0.2   | 2.7   | 0.6   | 0.1   |
| No. 2             | 35.8  | 0.6   | 0.25  | 2.5   | 41.9  | 1.6   |

Fig. 4 Rain gauge station and water stage gauge station of (A) Pyungchang basin, (B) No. 2 sub-basin of Chungju catchment.

Fig. 5 Observed and estimated discharge by ASFM (the case of Pyungchang River at 7. 1987)
Secondly, we discuss the forecasting performance of ASFM. In evaluating this, we take into account that ASFM does not use output data (i.e., runoff data) after the starting time for prediction has elapsed, but uses only rainfall data. Fig. 8 depicts the prediction results of ASFM obtained in simulation using real datum for Pyungchang basin. The starting time of forecasting was decided to consider the sudden variation of discharge as after the concentrated average rainfall. The prediction result after 17, 28, and 43 hours is shown in Fig. 8. Fig. 8 shows ASFM has rainfall input with complex pattern during 50 hours. These results show that ASFM as proposed here has satisfactory prediction performance. Prediction error in Fig. 8 is \( e_r = \frac{(Q(t) - \hat{Q}(t))}{Q(t)} \times 100 \% \).

5. Conclusion

An adaptive storage function method (ASFM) for the rainfall-runoff model has been represented. The conventional SFM has a major drawback in that it is difficult to calibrate the storage constants accurately. These parameters have a large degree of uncertainty and are also time varying according to such environmental conditions as rainfall patterns, ground configuration and surface condition. Under the assumption that a small basin can be modelled by a single watershed-channel pair, a multiple model adaptation estimate (MMAE) has been introduced in the standard SFM. It is called ‘ASFM’ here.

The proposed ASFM was applied to data for two observed catchments in north-east Chungbuk, Korea. Seven parameters of the runoff model were partitioned to 2–4 values and this resulted in hundreds of multiple models.

Simulation results showed that ASFM performed well. While it will be necessary to investigate further, (1) what number of partitions of model parameters is appropriate, and (2) the maximum size of basin to which ASFM can be applied, we expect that ASFM can lead to even greater improvements in SFM.

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